### Abstract
This paper presents a scientific and technical description of the modelling framework and the main results of modelling the long-term average sediment delivery at hillslope to medium-scale catchments over the entire Murray Darling Basin (MDB). A theoretical development that relates long-term averaged sediment delivery to the statistics of rainfall and catchment parameters will be presented. The derived flood frequency approach was adapted to investigate the problem of regionalization of the sediment delivery ratio ($SDR$) across the Basin. $SDR$, a measure of catchment response to the upland erosion rate, was modeled by two lumped linear stores arranged in series: hillslope transport to the nearest streams and flow routing in the channel network. The theory shows that the ratio of catchment sediment residence time (SRT) to average effective rainfall duration is the most important control in the sediment delivery processes. In this study, catchment SRTs were estimated using time of the concentration for overland flow multiplying by an enlargement factor which is a function of particle size. Rainfall intensity and effective duration statistics were regionalized by using long-term measurements from 195 pluviograph sites within and around the Basin. Finally, the model was implemented across the MDB by using spatially distributed soil, vegetation, topographical and land use properties under GIS environment. The results predict strong variations in $SDR$ from close to 0 in floodplains to 70% in the eastern uplands of the basin.

### Keywords:
Sediment delivery ratio; Soil erosion; Spatial Modelling.

### 1. INTRODUCTION
Observations show that sediment yields from catchments are often about an order of magnitude lower than the soil erosion rates measured from hillslope plots (Edwards 1993; Wasson et al. 1996). This implies that most of the sediment travels only a short distance (Parsons and Stromberg 1998) and is deposited before leaving the hillslope. In general, the amount of sediment deposited is intimately related to the topography, climate, soil, vegetation cover, and land use conditions, which are all closely related to the hydrological processes. Across a large regional catchment, such as the Murray Darling Basin (MDB) (a major basin located in the south-eastern Australia), sediment delivery varies spatially and temporally due to the changes in those factors.

Information on spatially distributed sediment delivery is useful in identifying relative importance between sediment sources and the effectiveness of sediment delivery. It helps to establish strategies in effective erosion control, rehabilitation planning, and achieving long-term sustainable productivity in the Basin. It is of both economic and environmental importance in MDB due to the prevalence of high-value commodities in the Basin.

This paper presents the framework and the main results of modelling the long-term average sediment delivery ratio ($SDR$) at hillslope to medium-scale catchments over the entire MDB. $SDR$, a measure of catchment response to the upland erosion rate, is modeled by two lumped linear stores arranged in series: hillslope transport to the nearest streams and flow routing in the channel network. A theory developed in hydrologic scaling (Robinson and Sivapalan 1997; Sivapalan et al. 2001) is adapted here to relate long-term averaged sediment delivery to the effective rainfall duration and catchment sediment residence time (SRT). Average rainfall intensity and effective duration were regionalized by using long-term measurements from 195 pluviograph sites within and around the Basin. SRT is estimated using time of the concentration for overland flow multiplied by an enhancement factor which is a function of particle size. The model was implemented across the MDB by using spatially distributed soil, vegetation, topographical and land use properties under a GIS environment.

### 2. BACKGROUND
Soil erosion models, such as the Universal Soil Loss Equation (USLE) (Wischmeier and Smith 1978) estimate gross soil erosion rate at plot-scale. Erosion rates estimated by USLE are often higher than those measured at catchment outlets. Sediment delivery ratio ($SDR$) is used to correct for this reduction effect. It is expressed as $SDR = Y / E$, where $Y$ is the average annual...
sediment yield per unit area and $E$ is the average annual erosion rate over that same area. In essence, $SDR$ is a scaling factor used to accommodate differences in areal-averaged sediment yields between measurement scales. It is a measure of sediment transport efficiency which accounts for the amount of sediment that is actually transported from the eroding sources to a measurement point (e.g., catchment outlet; lower end of erosion plot) compared to the total amount of soil that is detached over the same area above that point. It compensates for areas of sediment deposition that becoming increasingly important with increasing catchment area, and therefore, determines the relative significance of sediment sources and their delivery. $SDR$ also relates to deposition and sediment availability at different scales. The investigation of off-site impacts of soil erosion requires determining the significance of sediment sources in terms of their delivering efficiency. Factors influence $SDR$ including hydrological inputs (mainly rainfall), landscape properties (e.g., vegetation, topography, and soil properties) and their complex interactions at the land surface. The multitude of such interactions makes it difficult to identify the dominant controls on catchment sediment response and on catchment-to-catchment variability.

At regional scale, the most widely used method to estimate $SDR$ is through a $SDR$-area power function $SDR = \alpha A^E$, where $A$ is the catchment area (in km$^2$), the constant $\alpha$ and a scaling exponent $\beta$ are empirical parameters (Maner 1958; Roehl 1962). Field measurements suggest $\beta$ is in the range -0.01 to -0.025 (Walling 1983; Richards 1993), which means that $SDR$ decreases with increasing catchment area. The scaling exponent $\beta$ contains key physical information about catchment sediment transport processes and its close linkage to rainfall-runoff processes. Lower value of $\beta$ (up to $-0.7$) were found in the Sicilian region and in former USSR catchments (Ferro and Minacapilli 1995). Field data (Fig. 1) show that the relationships between $SDR$ and drainage area changes considerably between different catchments over the world. Extrapolation of those empirical relationships can be misleading and results $SDR$ exceeding 100%. There are other empirical relationships which show that $SDR$ varies with various physiographic attributes but the data that went into these relationships are few and of only local extent (Khanbilvardi and Rogowski 1984). This limits the usefulness of such a lumped empirical approach.

The traditional $SDR$ methods are often data-driven. They depend on the existence of long periods of sediment yield records at the stream gauging stations and a sensible measure or estimation of hillslope erosion rate. However, there are few consistent long periods of sediment yield data available in the MDB to allow such an analysis to be carried out. In addition, approaches based on analyzing sediment yield records cannot identify the separate effects of changing climate, land use and management practices on sediment delivery as catchment response to change is often longer than the record length.

There are other methods to predict sediment delivery and deposition through calculation of sediment transport capacity, avoiding the need for a lumped $SDR$ (EUROSEM, Morgan et al 1998; Van Rompaey et al. 2001). Although those methods were based on improved physical understanding of sediment transport processes, they require high resolution DEMs to route the flow and sediment. They also rely on detailed sediment transport or runoff data to calibrate parameters, such as sediment transport capacity coefficient. For large scale applications, those methods are limited by availability of model inputs and parameters.

3. MODELLING METHOD

One important aim of this paper is to develop a $SDR$ model that incorporates the key elements of the catchment storm response and sediment delivery process. Sivapalan et al. (2001) showed that the interactions between time scales, namely between rainfall duration and catchment response lay at the heart of the regional flood frequency estimations. The way that catchment response time varies with catchment area depends on the relative dominance of hillslope response, channel hydraulic response, and network geomorphology.

A simple linear model of catchment response (Sivapalan et al. 2001) is used in this study. Instead of using the model for studying catchment response of flood, we use the same concept to model $SDR$. The model consists of two
independent components: sediment transport on hillslopes and sediment routing in the channel network. As shown in Fig. 2, these are represented through two linear stores, arranged in series. The hillslope store is supplied with sediment by soil erosion at a rate \( e \) [mass/area/time] over an effective storm duration \( t_{er} \) (erosion only occurs during this time period). The hillslope stores part of the eroded sediment and delivers the rest to the channel network store, located downstream of it, at a rate \( y_h \) [mass/area/time]. \( y_h \) is assumed to be a linear function of the mass of sediment stored in the hillslope per unit area, denoted by \( S_h \) [mass/area]. The area specific sediment yield from the network store, \( y \) [mass/area/time], which is the same as the area specific sediment yield from the catchment outlet, is assumed to be a linear function of the sediment stored in the channel network, denoted by \( S_n \) [mass/area]. The continuity equation of sediment for the two stores can be expressed as:

\[
\frac{dS_h(t)}{dt} = e(t) - y_h(t) \]

\[
y_h(t) = \frac{S_h(t)}{t_h}
\]

\[
\frac{dS_n(t)}{dt} = y_h(t) - y(t) \]

\[
y(t) = \frac{S_n(t)}{t_n}
\]

where \( t_h \) is the mean hillslope residence time and \( t_n \) is the mean channel residence time.

At event basis, we assume \( SDR = \frac{y_p}{e} \). The peak sediment yield \( Y_p \) [mass/time] can be estimated by multiplying area specific sediment yield \( y_p \) [mass/area/time] by the catchment area \( A \). Eqs. (1) and (2) were firstly derived by Sivapalan et al. (2001) for studying the scaling effects on regional flood frequency under different rainfall and catchment conditions.

\[
\frac{y_p}{e} = \frac{t_h}{t_h-t_n} \left[ 1 - \exp \left( -\frac{t_h}{t_n} \right) \right] \quad t_h > 0 \quad t_n = t_h
\]

\[
\frac{y_p}{e} = \frac{1}{2} \left( \frac{t_h}{t_n} \right)^2 - \frac{1}{3} \left( \frac{t_h}{t_n} \right)^3 + \ldots \quad t_n = t_h
\]

\[
\frac{y_p}{e} = 1 - \exp \left( -\frac{t_h}{t_n} \right) \quad t_n = 0
\]

Fig. 3. Comparison of SDR (%) measurements (Roehl 1962), modeled average SDR and flow response (Robinson and Sivapalan 1997). It shows that flow response represents the upper envelope of the SDR.

Sivapalan et al. (2001) showed that eq. (2) is capable of explaining the power low relationship between flow response and catchment area and changing value of the scaling exponent which is caused by a change of hydrological processes. Similar, eq. (2) can be used to explain the obtained SDR vs area relationships. As shown in Fig. 3, SDR measurements gathered by Roehl (1962) in several American catchments including Blackland Prairies, the Red Hills of Texas and Oklahoma, the Missouri Basin Loess Hills, the Mississippi Sand Clay Hill, and the Southeastern Piedmont (shown in dots) suggested that, in general, SDR decreases with catchment area. The dark line, which is the average flow response (the scaling factor of mean flood discharge defined as the ratio between average rainfall input rate and runoff at the catchment outlet during flood events) calculated using the equation (28a) of
Robinson and Sivapalan (1997), represents the upper envelope of SDR. The averaged modeled SDR estimated by eq. (2) is shown as the dashed line. The reason that SDR is often smaller than flow response is due to the settling velocity of soil particles (comparing with water particles) and other effects such as sediment transport capacity. For a given catchment area, the large variations in SDR measurements (vary up to two orders of magnitude) are due to heterogeneity in catchment properties (e.g. rainfall, catchment slope and curvature, soil texture, etc). The combination of above physical properties results in differences in the time variables $t_{th}$, $t_{th0}$ and $t_{th}$ in eq. (2). Therefore, eq. (2) can be used to model spatially distributed SDR if the time variables $t_{th}$, $t_{th0}$ and $t_{th}$ can be spatially differentiated.

The above analysis was based on a single storm. The derived flood frequency method (Eagleson 1972; Robinson and Sivapalan 1997) can be used to deal with multiple storms. In this study, for simplicity, we treat effective storm duration as a random variable and calculate $t_{th}$ and $t_{th0}$ as catchment averaged values. By knowing the probability distribution of $t_{th}$, we derive a probability distribution of SDR.

3.1. Effect of Particle Size

The residence time of sediment can be estimated as a function of particle size and the travel time of water particles.

Suppose we can estimate the travel time of water particles as a function of local slope, roughness, rainfall intensity, etc. For very small clay particles which are characterized by their smaller settling velocity, most of the time they remain suspended in the water and their trajectories of travel differ little compared to the trajectory of water particles. For silt particles, due their relatively large settling velocity, they travel with water particles during high speed flow and settle to the soil bed during low flow. Large sand particles saltate near the soil bed with slow overall velocity. These different particle movements with water flow can be modeled as follows:

$$t_{th}(d) = t_{th0}F_{th}(d)$$
$$t_{th}(d) = t_{th0}F_{th}(d)$$

where $t_{th}(d)$ and $t_{th}(d)$ are the hillslope and channel residence time for particles with diameter $d$, respectively, and $t_{th0}$ and $t_{th0}$ are the hillslope and channel travel times of water particles, respectively. $F_{th}(d)$ and $F_{th}(d)$ are the enlargement functions describing the influence of particle size $d$. The function forms of $F_{th}(d)$ and $F_{th}(d)$ are modeled as:

$$F_{th} = \exp(\gamma_{th}w(d))$$
$$F_{th} = \exp(\gamma_{th}w(d))$$

where $w(d)$ is the settling velocity for particles with diameter equal to $d$, and $\gamma_{th}$ and $\gamma_{th}$ are the parameters inversely relating to water depth. In general, $\gamma_{th}$ is larger than $\gamma_{th}$ as the typical water depth in overland flow is in the order of centimeters and the water depth in channel are in the order of meters. The settling velocity can be calculated as:

$$w(d) = \left( \frac{Ap_{p}gd}{3\rho p C_{D}(Re_{p})} \right)^{1/2}$$

where $\rho_{p}$ is the particle density, $\rho$ is the water density, $g$ is gravity acceleration, $Re_{p} = wd/\nu$ is the particle Reynolds number at the settling velocity and

$$C_{D}(Re_{p}) = \frac{24}{Re_{p}} \left( 1 + 0.15 Re_{p}^{0.67} \right)$$

is the drag coefficient (Durst et al. 1984).

Finally, SDRs are calculated for each particle size group and then weighted by the particle size distributions to get overall SDR as follow:

$$SDR = \sum_{i=1}^{N} w_{i}SDR_{i}$$

where $N$ is the total number of particle groups, $w_{i}$ and $SDR_{i}$ are the mass percentage and SDR for particle size group $i$, respectively.

Three particle size groups are considered in this study. These are: $d \leq 4 \mu m$ (clay), $4 \leq d \leq 50 \mu m$ (silt), and $50 \leq d \leq 1000 \mu m$ (sand). Particles with diameter larger than 1000 $\mu m$ are considered too large to be transport far away from their source areas.

3.2. Effective Rainfall Duration, Intensity

Rainfall varies both in space and time. Both variables are interrelated, and have significant impacts on sediment generated at plot scale and sediment transport at catchment level. High temporal resolution rainfall data provide the means of discovering the possible controls on the spatial variability of sediment delivery due to temporal variability of rainfall intensity.

Pluviograph rainfall data with 6-min interval was collected for 195 sites from Bureau of Meteorology (BoM). The analyses of rainfall data is divided in two parts: 1) Statistical analysis of rainfall data at single site to search for a suitable probability distribution function for effective
rainfall duration and maximum 30-min rainfall intensity; 2). Regionalization of the necessary parameters of the suitable probability distribution functions for both duration and maximum 30-min rainfall intensity. It was found that exponential probability distributions fit reasonably well with the rainfall data, so were used in this study. Detailed analysis and results of effective rainfall duration and intensity are presented elsewhere (Lu et al. 2003b).

We calculate the mean values of SDR for each sub-catchment according to standard statistical procedure. Modelling spatially distributed SDR at sub-catchment level was implemented using ERSI Arc/Info software (ERSI 2003). 9” DEM (Hutchinson et al. 2001), BRS land use (BRS 2000) and ASRIS soil particle distributions (Carlile et al. 2001) were the major input data. Details of the implementation can be found in Lu et al. (2003b).

4. RESULTS

Fig. 4 shows estimated spatially distributed SDR for subcatchments with area around 50 km². It is estimated that the averaged SDR is about 5.2%, which is lower than the average estimated from measurements from similar catchment size in other countries (around 10%) (Edwards 1988). In general, larger SDRs are obtained at eastern edge of the Basin, with the Australian Alps having the highest SDR values, followed by central Murrumbidgee and Bathurst regions. Relatively high SDR values are estimated for the Armidale, Tamworth and Inverell regions. In the Liverpool Plain, although the SDR for individual size groups is relatively low compared with its surrounding areas due to its relative flat landscape, the overall SDR is medium due to its higher clay content soils. In the central Basin and the west, most of the sub-catchments have SDR smaller than 5%, suggesting inefficiency of sediment transport in broad areas of the Basin. As sediments generated from slopes have to pass through the massive flat areas, it implies that the sediment delivery to the basin outlet is even more inefficient.

Area specific sediment yield (ASSY) is calculated by multiplying estimated hillslope erosion rate averaged at each sub-catchment by the overall SDR. Hillslope erosion rate was estimated by a previous study (Lu et al. 2003a). The spatially distributed ASSY is shown in Fig. 5. It is estimated that upper Murrumbidgee (upstream of Wagga Wagga), Wellington, Tamworth, Inverell, and Dalby regions export relatively larger amount of sediments each year (around 5-10 t ha⁻¹ yr⁻¹). Sediment export rate is low for majority of sub-catchments with ASSY smaller than 0.1 t ha⁻¹ yr⁻¹. The average ASSY for the Basin from the subcatchments is around 0.1 t ha⁻¹ yr⁻¹.

5. CONCLUSIONS

In this paper, we develop a SDR model which argues that sediment delivery can be closely linked
to temporal hydrological control. The model allows quantitative estimates of the non-linear effects on sediment delivery due to changes in climate and land use. It expresses the spatial variability of catchment-averaged SDR in terms of the statistical time variables and particle size distributions. It relies on rainfall intensity (6-min interval) and daily rainfall records (which cover larger area) instead of stream flow records. It offers a means to understand the dominate processes which control sediment delivery. The model has a simple analytical form which can be implemented in a GIS environment.

Applying the model to the MDB, we found: 1) sediment delivery ratio and sediment yield are low for most part of the Basin except some upland in the east and north part of the Basin; 2) combined effects of topography, rainfall intensity and rainfall duration make the system inefficient in transporting sediment to the basin outlet. However, the sediment transport can be very effective at sub-catchment level, especially in the area Australia Alps, South West Slopes, Brigalow Belt South, and Darling Down regions; 3) only about 5% of sheet and rill erosion are transported out from sub-catchments in to the streams. The average area specific sediment yield at sub-catchment level is around 0.1 t ha⁻¹ yr⁻¹. About 14 million tones in total of sediment generated from sheet and rill erosion is delivered from the sub-catchments to the major streams each year.

The quantitative, spatially distributed estimations of SDR have important implications not only for the study of off-site environment impact due to exported sediment but also to on-site erosion control. It has been demonstrated that there is economic advantage from identifying the areas that have a higher potential to deliver sediment and prioritizing control implementation in those areas (Dickinson et al. 1990). The spatially distributed SDR map contributes to the development of cost-effective strategies which aid the decision makers.

6. ACKNOWLEDGEMENTS

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7. REFERENCES


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