Dynamics of Sediment Transport in the Mississippi River Basin: A Temporal Scaling Analysis

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Abstract: The crucial role that the Mississippi River plays in meeting various water demands in North America necessitates adequate understanding of the dynamical changes that occur in the river system. Arguably, the phenomenon that needs the utmost and immediate attention is the sediment (bed) load transport, since the large quantity (in the order of hundreds of millions of tons per year) of sediment discharged by the river causes extensive problems to the activities that the river itself supports, such as wetland and floodplain habitats, biological and agricultural production, and commercial navigation. Furthermore, understanding the dynamics of bed load transport at different temporal scales (finer and coarser) is essential for undertaking short-term emergency measures as well as long-term river basin management. In an effort to address this issue, the present study investigates the dynamical behavior of bed load transport at five different successively doubled temporal scales (between daily and fortnightly scales), i.e. daily, 2-day, 4-day, 8-day, and 16-day. Specifically, the presence of low-dimensional deterministic behavior in the bed load dynamics is investigated, with an aim to possibly simplify the model complexity for modeling, prediction, and disaggregation purposes. The correlation dimension method is used to identify low-dimensional determinism. The bed load dynamics is represented through the reconstruction of a single-dimensional series in a multi-dimensional phase-space, and the variability (dimension) is estimated using the (proximity of the) reconstructed vectors (i.e. points) in the phase-space. The results indicate the presence of low-dimensional determinism in bed load series at each of the above five scales, suggesting the possibility of modeling their dynamics using only a few variables (in the order of 3 or 4).

Keywords: *Bed load transport; Mississippi River basin; Modeling; Scale; Chaos; Phase-space reconstruction; Correlation dimension; Number of variables*

1. INTRODUCTION

The Mississippi River is one of the world's major river systems in size, habitat diversity, and biological productivity. Of the world's rivers, the Mississippi River ranks third in length, second in watershed area, and fifth in average discharge. It is the longest and largest river in North America, originating at Lake Itasca in northern Minnesota in the United States and flowing for about 3970 km into the Gulf of Mexico in the south. The entire river basin measures about 4.76 million km^2. The drainage area of the basin that lies within the United States is about 3.22 million km^2. The main stem, together with its tributaries, extends over 31 states in the continental United States and covers about 41% of the land area (e.g. Chin et al., 1975).

The important role that the Mississippi River plays in meeting various water demands in North America necessitates adequate understanding of the dynamical changes that occur in the entire river system. Arguably, the phenomenon that requires the most immediate attention is the sediment load transport, as the Mississippi River is a dominant mover of sediment and transports more sediment than any other river in North America (e.g. Meade and Parker, 1985). In spite of the large dams that have been built across its major tributaries, the Mississippi River still ranks sixth in the world in suspended sediment discharge to the oceans (e.g. Milliman and Meade, 1983). The average annual suspended sediment discharge to the coastal zone by the Mississippi River is as large as about 230 million tons (e.g. Meade and Parker, 1985). The large quantity of sediment discharged by the river significantly affects the activities that the river itself supports, such as (wetland, open-water, and floodplain) habitats, biological and agricultural production, commercial navigation, and other human developments.

Understanding the dynamical behavior of bed load transport is an extremely challenging task, due to the following reasons:

1. Bed load occurrence and movement depend on a host of factors, such as water discharge, suspended sediment concentration, and size, shape, velocity and density of the sediment particles. The existing bed load estimation methods are designed linking bed load to water discharge and suspended sediment concentration (e.g. Einstein, 1943; Olive et al., 1996). However, there is not a simple relationship between these components. For instance, studies reveal that suspended sediment peak either lags the water discharge peak (e.g. Einstein, 1943) or arrives before the water discharge peak (e.g. Olive et al., 1996). Such linkages, therefore, may sometimes become unreliable.
2. Even if such linkages were found to be reliable, any error in water discharge and suspended sediment concentration could eventually lead to an inaccurate estimation of bed load. Since discharge and suspended sediment concentration measurements always contain errors, corresponding errors in bed load estimation are inevitable. These errors often increase in a nonlinear pattern, because of the nonlinear behavior of these components and their relationship.

The effects of these problems become much more prominent at finer temporal scales (e.g. less than daily) because: (1) discharge and sediment concentration measurements are usually not available at these scales; and (2) errors in measurements increase at finer scales, since the existing technology and equipment are generally insufficient. Unfortunately, estimation of bed load at finer scales is crucial, since much of the transport occurs during heavy flood events, which last only a very short period of time (a few hours or at the most a few days). On the other hand, bed load estimation at coarser scales may also be significantly affected by the errors in discharge and suspended sediment concentration measurements, since the data required at such coarser scales are obtained by simply adding the available data (giving rise to additional errors).

In an attempt to avoid the above problems, Sivakumar (2002) and Sivakumar and Jayawardena (2002, 2003) introduced the concept of phase-space reconstruction (e.g. Takens, 1981) to the sediment transport problem. Such a concept uses the reconstruction (or embedding) of an available single-variable time series in a multi-dimensional phase-space to represent the underlying dynamics. The physics behind such a reconstruction is that a nonlinear system is characterized by self-interaction, so that a time series of a single variable can carry the information about the dynamics of the entire multi-variable system. Based on such a reconstruction and employing a local approximation prediction method (e.g. Farmer and Sidorowich, 1987), they attempted future predictions of suspended sediment concentration (Sivakumar, 2002) and bed load (Sivakumar and Jayawardena, 2003), respectively, at the daily scale in the Mississippi River basin. The outcomes revealed the usefulness of these methods, as very good predictions were obtained. The study by Sivakumar and Jayawardena (2002) provided further support to these studies, as the application of the correlation dimension method (e.g. Grassberger and Procaccia, 1983) indicated the possible presence of low-dimensional chaotic dynamics in the daily discharge, suspended sediment concentration, and bed load phenomena.

Even though the above studies are encouraging, they have addressed only part of the overall problem, because of the following. Understanding sediment transport dynamics only at the daily scale (or any other single temporal scale) is not sufficient to address either the short-term emergency measures or the long-term river basin management, as the transport dynamics change with time scale. This is particularly the case in the Mississippi River basin due to its large spatial extent and to the occurrence of frequent heavy floods, which influence the time scale (e.g. concentration time) at which the river flow and sediment transport occurs at the point of interest. As the sub-basin (at St. Louis, Missouri) studied in the above studies falls in the Lower Mississippi River basin and also as it has a drainage area of as large as 251,230 km^2, the daily scale analyzed may be too short for any realistic interpretation. Therefore, investigating the behavior of the sediment transport dynamics at (many) different temporal scales is essential to provide reliable and realistic interpretations.

In an effort to address this issue, the present study investigates the dynamical behavior of bed load transport phenomenon at five different successively doubled temporal scales (i.e. daily, 2-day, 4-day, 8-day, and 16-day) in the Mississippi River basin (at St. Louis, Missouri). These scales, between daily and fortnightly, are selected in such a way that they cover a fairly wide time scale and, therefore, represent the overall dynamical changes in the river system. Specifically, the presence of low-dimensional deterministic chaotic behavior in the bed load dynamics at these scales is investigated. The correlation dimension method is used as an indicator to identify the non-linear determinism (or to distinguish between chaotic and stochastic behaviors). The outcomes of this analysis could also provide important information on: (1) the predictability of bed load dynamics at these scales (and possibly others); (2) the presence of scaling that may exist between the dynamics at these scales; and (3) the appropriate framework for transformation of data between these scales.

The organization of this paper is as follows. Section 2 presents a brief account of the correlation dimension method. Details of the data used, analyses performed, and results obtained are presented in Section 3. A discussion of the results is made in this section. Conclusions from the present study are reported in Section 4.

2. CORRELATION DIMENSION

In the present study, the correlation dimension is estimated using the Grassberger-Procaccia algorithm (e.g. Grassberger and Procaccia, 1983), which uses the phase-space reconstruction of a time series to represent the underlying dynamics. For a scalar time series X_i, where $i = 1, 2, ..., N$, the phase-space can be reconstructed using the method of delays, according to:

$$\mathbf{Y}_j = (X_j, X_{j+\tau}, X_{j+2\tau}, ..., X_{j+(m-1)\tau}) \qquad (1)$$

where $j = 1, 2, ..., N-(m-1)\tau/\Delta t$, m is the dimension of the vector $\mathbf{Y}_j$, called as embedding dimension; and τ is a delay time taken to be some suitable multiple of the sampling time Δt (e.g. Takens, 1981). For an m-dimensional phase-space, the correlation function, $C(r)$, is:

$$C(r) = \lim_{N\to\infty} \frac{2}{N(N-1)} \sum_{(1\le i<j\le N)} H\left(r - \left|\mathbf{Y}_i - \mathbf{Y}_j\right|\right) \qquad (2)$$

where H is the Heaviside step function, with $H(u) = 1$ for $u > 0$, and $H(u) = 0$ for $u \le 0$, where $u = r - |\mathbf{Y}_i - \mathbf{Y}_j|$, r is the radius of sphere centered on $\mathbf{Y}_i$ or $\mathbf{Y}_j$, and N is the number of points. If the time series is characterized by an attractor (a geometric object which characterizes the long-term behavior of a system in the phase-space), then the correlation function $C(r)$ and radius r are related according to

$$\underset{\substack{r\to 0\\ N\to\infty}}{C(r)} \sim \alpha r^{\nu} \qquad (3)$$

where α is constant; and ν is the correlation exponent or the slope of the log $C(r)$ versus log r plot. If the correlation exponent saturates with an increase in the embedding dimension, then the system is generally considered to exhibit deterministic chaos. The saturation value of the correlation exponent is defined as the correlation dimension of the attractor. The nearest integer above the saturation value provides the minimum number of phase-space or variables necessary to model the dynamics of the attractor. If the correlation exponent increases without bound with increase in the embedding dimension, then the system is generally considered as stochastic.

3. ANALYSES AND RESULTS

3.1 Data

Throughout the Mississippi River basin, sediment (and flow) data are measured at a number of locations. For the present study, bed load data observed in a sub-basin station of the Mississippi River basin at St. Louis, Missouri (US Geological Survey station no. 07010000) are considered. The sub-basin is situated between 38°37′03″ latitude and 90°10′47″ longitude, on downstream side of west pier of Eads Bridge at St. Louis, 24.1 km downstream from Missouri River. The drainage area of this sub-basin is 251,230 km^2 (e.g. Chin et al., 1975). The natural flow of stream at the gaging station is affected by many reservoirs and navigation dams in the upper Mississippi River basin and by many reservoirs and diversions for irrigation in the Missouri River basin.

Daily bed load measurements for the above station have been made available by the USGS from April 1948. However, there were some missing data before 1960. In order to avoid the uncertainties that could arise on the outcomes due to such missing data, only the period that has continuous data is considered. The data used herein spans over a period of about 22.5 years (amounting to 8192 values) starting on January

1, 1961. Figure 1 shows the variation of the daily bed load series observed at the above station.

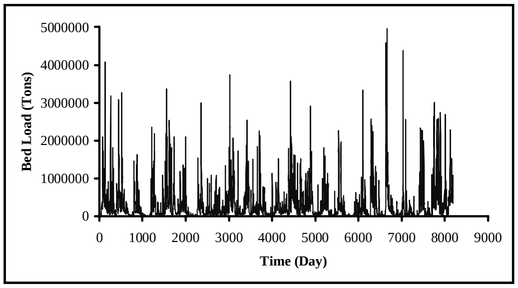

Figure 1. Variation of daily bed load data in the Mississippi River basin (at St. Louis, Missouri).

In order to obtain bed load data corresponding to other temporal scales for the present analysis, the daily bed load values are aggregated (by simply adding) to four successively doubled coarser resolutions (i.e. 2-day, 4-day, 8-day, and 16-day). Table 1 presents some of the important statistics of all the above five series.

Table 1. Statistics of bed load data of different temporal scales from the Mississippi River basin (at St. Louis, Missouri). [Values in 10^6 tons]

	Daily	2-day	4-day	8-day	16-day
Data	8192	4096	2048	1024	512
Mean	0.300	0.600	1.200	2.401	4.802
Std.	0.439	0.858	1.652	3.058	5.468
Max.	4.960	8.140	13.92	21.15	32.83
Min.	0.003	0.006	0.012	0.027	0.077
CV	1.463	1.430	1.376	1.274	1.139
Skew	3.084	2.872	2.625	2.274	1.950
Kurt	13.21	10.67	8.377	5.987	4.214
Zeros	0	0	0	0	0

*Data = Number of Data; Std = Standard Deviation; Max = Maximum Value; Min = Minimum Value; CV = Coefficient of Variation; Skew = Skewness; Kurt = Kurtosis

As Table 1 shows, there are no zero values in the five series. This eliminates the problem of underestimation of correlation dimension of these series, as the presence of a large number of zeros may significantly underestimate the dimension (e.g. Sivakumar, 2001). It is believed that data over a period of 22.5 years are long enough to represent the changes in the system, and data size may not be an issue in dimension estimation (e.g. Sivakumar, 2000).

3.2 Analyses and Results

The phase-spaces of the above five bed load series are reconstructed according to Eq. (1). Figure 2, for instance, shows the phase-space diagram of the daily series in two dimensions (m = 2) with delay time $\tau = 1$, i.e. the projection of the attractor on the plane $\{X_i, X_{i+1}\}$. As may be seen, the projection yields a reasonably well-defined attractor suggesting the possibility of deterministic dynamics [the projections of the other four series also yield reasonably well-defined attractors, but, occupy slightly increasingly larger spaces in the phase-space].

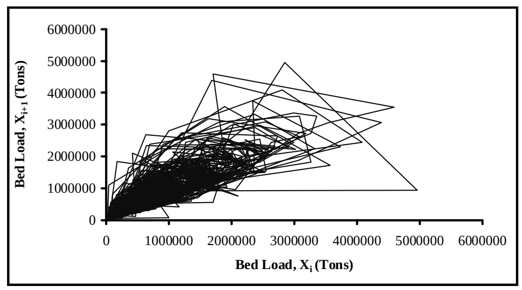

Figure 2. Phase-space diagram of daily bed load data in the Mississippi River basin.

It must be noted, however, that an appropriate τ for phase-space reconstruction is necessary because only an optimum τ gives best separation of neighboring trajectories within the minimum embedding space, whereas an inappropriate selection of τ may lead to underestimation or overestimation of correlation dimension. In this study, the optimum values of τ for the phase-space reconstruction are computed using the autocorrelation function method [see, for instance, Sivakumar (2000) for details of τ selection], and is taken as the lag time at which the autocorrelation function first crosses the zero line (e.g. Holzfuss and Mayer-Kress, 1986). The first zero value of the autocorrelation function attained is at lag times 99, 50, 25, 13, and 7 respectively, for the daily, 2-day, 4-day, 8-day, and 16-day series, as presented in Table 2.

The correlation functions and the exponents are now computed for the five bed load series. Figure 3(a) shows, for instance, the relationship between the correlation integral, $C(r)$, and the radius, r, for embedding dimensions, m, from 1 to 15, for the daily bed load series, and Figure 3(b) presents the relationship between the correlation exponent values and the embedding dimension values for this series [the figure also includes the relationships for the other four series].

Table 2. Correlation dimension results for bed load data from the Mississippi River basin.

	Daily	2-day	4-day	8-day	16-day
Data	8192	4096	2048	1024	512
Acf	99	50	25	13	7
CD	2.41	2.54	2.74	3.15	3.62
Var	3	3	3	4	4
CV	1.46	1.43	1.37	1.27	1.13

*Data = Number of Data; Acf = Autocorrelation Function; CD = Correlation Dimension; Var = Number of Variables; and CV = Coefficient of Variation

As can be seen, the correlation exponent value increases with the embedding dimension up to a certain point, and saturates beyond that point, indicating the possible existence of deterministic dynamics. The saturation value of the correlation exponent (or correlation dimension) is about 2.41, which is an indication that the dynamical behavior of bed load process at the daily scale may be low-dimensional chaotic, dominantly influenced by 3 variables.

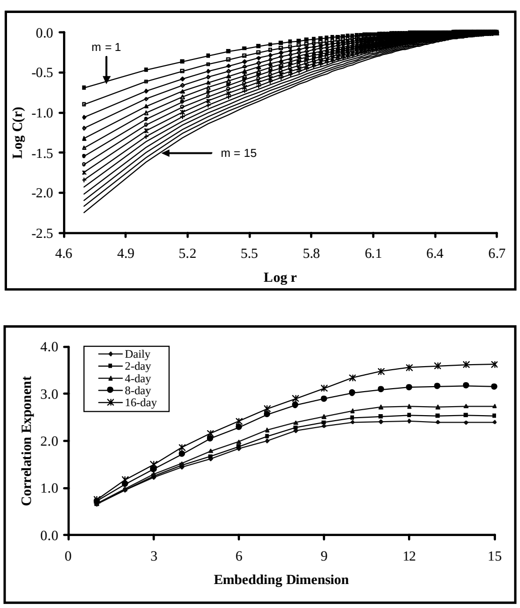

Figure 3. Correlation dimension results for bed load data in the Mississippi River basin: (a) Log C(r) versus Log r relationship for daily series; and (b) Correlation exponent versus embedding dimension for all series.

Saturation of correlation exponents and low correlation dimensions are observed also for the 2-day, 4-day, 8-day, and 16-day bed load series [Figure 3(b)], with dimensions of 2.54, 2.74, 3.15, and 3.62, respectively [Table 2]. This may be an indication of the presence of chaotic dynamics in the bed load dynamics at each of these scales, with the dominant number of variables being 3 or 4.

3.3 Discussion

A possible implication of the presence of low-dimensional chaotic dynamics in the bed load phenomena at the daily, 2-day, 4-day, 8-day, and 16-day scales may be that the scaling relationship between the properties of bed load dynamics at these scales (i.e. between daily and fortnightly scales) could also be chaotic. The possibility that the bed load dynamics at these scales as well as the scaling relationship between them could be represented using a model with as few as 3 or 4 variables is certainly encouraging from modeling point of view, as this would mean less complexity of the model and, in turn, less data, time, and computational requirements. Whether this is indeed true needs to be verified using prediction and disaggregation of bed load series. The prediction results for the daily bed load dynamics presented by Sivakumar and Jayawardena (2003) using a nonlinear deterministic local approximation procedure are certainly a positive outcome in this regard.

As the dimension of a time series represents, in a way, the complexity of the underlying dynamics, the correlation dimensions (and the number of variables) indicate that the complexity of the bed load dynamics increases with increasing time scale, i.e. from finer to coarser scales. This may seem contradictory to reality, since it is generally believed that the dynamics of bed load (or any other) phenomenon at coarser scale(s) are less complex than that at finer scale(s). This, however, is not true, since the bed load dynamics at the coarser scale(s) could indeed be more complex than that at the finer scale(s). This may easily be explained based on the (flow and) bed load dynamics that occur in the Mississippi River basin. Due to the large spatial extent of the basin, it may take several days for the (flow and) bed load to reach the point of interest (St. Louis, Missouri) from the (farthest) upstream points. Therefore, the variability of (flow and) bed load transport may well be higher at 8-day or 16-day scales than that at daily or 2-day scales. The autocorrelation functions and the delay times [Table 2] for the five bed load series support this point, as a decrease in delay time, τ, is observed with aggregation of scale. In general, a slow decrease in the autocorrelation function is an indication of determinism (high correlation between successive values), whereas a rapid decrease indicates stochasticity (low correlation between successive values). The very slow decrease observed for the daily series (with τ = 99), followed by slightly increasingly faster decreases for the 2-day, 4-day, and 8-day series (with τ = 50, 25, 13) to a very rapid decrease for the 16-day series (with τ = 7) indicate decreasing determinism (or increasing stochasticity) from finer to coarser scales. Similar observations are made also in the phase-space diagrams, as the daily series occupies a smaller space in the phase-space, whereas increasingly larger spaces are occupied by data at coarser scales.

4. CONCLUSIONS

The present study made a preliminary attempt to address the temporal scaling behavior of bed load dynamics in the Mississippi River basin. Analysis of bed load phenomenon at five different successively doubled scales (between daily and 16-day) using a nonlinear deterministic method (correlation dimension method) indicated the presence of low-dimensional chaotic dynamics, dominantly influenced by 3 or 4 variables. Based on these results, chaotic dynamics in the scaling relationship between the properties of bed load dynamics at the above scales seem to be evident. Nonlinear deterministic models with 3 or 4 variables may be sufficient for modeling, prediction, and disaggregation of bed load dynamics in the Mississippi River basin, at least for/between the time scales studied herein. The usefulness of deterministic models for disaggregation of bed load dynamics is currently being studied.

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