

# A hydrologic and economic model for water trading and reallocation using linear programming techniques

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**Abstract:** With the advent of water reform framework instigated by the Council of Australian Governments (COAG), water trading on a temporary and permanent basis has become a prominent feature in all major irrigation areas in Australia. Hydrologic network models, such as the Integrated Quantity and Quality Model (IQQM), although powerful in simulating entitlement-based water allocation at the catchment scale, are unable to deal with the water reallocation through trade driven by economic conditions such as crop and water price, variable production costs. To simulate water trading, linear programming techniques are used to maximize aggregate net return subject to land, water, and crop constraints. The volume of water traded is the difference between water allocated and water required for a given simulation period. The water trading model, known as WRAM, is coupled with IQQM for the Murrumbidgee basin. IQQM represents the irrigation area in the Murrumbidgee with 49 regulated irrigation nodes that grow a variety of summer, winter and perennial crops. The water trading model runs whenever a planting decision is required, taking into account water availability, crop growth stages, crop yield and price, variable production costs, fixed and variable water charges on the potential water movement through the distribution network. WRAM provides a dynamic link with IQQM in order to assess the impacts of water management policies at the whole-of-catchment scale. The result reported in this paper is part of a CRC Catchment Hydrology project on hydrologic and economic modelling for sustainable water allocation.

**Keywords:** Water market, Murrumbidgee, WRAM, IQQM.

## 1. INTRODUCTION

Water markets are developing as part of a Council of Australian Governments initiative to promote an efficient use of Australia's water resources. Full evaluation of the benefits and spatial externalities of trade and agricultural extraction in general requires integrated hydrologic and economic modelling. Such modelling is in its infancy. Most economic models of the benefits and costs of trade to date rely on static exogenous hydrologic constraints. Hall *et al.* (1994) considered water trading, changes in water use and their effect on the salinity in the Murray River. McClintock *et al.* (2000) modelled structural adjustment in the southern Murray Darling basin. Heaney and Beare (2001) modelled the impact of water trading on return flows in the Murray. Beare and Heaney (2002) have also modelled the benefits and costs of water trading in the southern regions of the Murray Darling basin. Elsewhere in the world, Rosegrant *et al.* (2000) simulated inter-sectorial water trade in Chile and assessed the economic gains through demand management instruments such as markets in tradable water rights. Mahan *et al.* (2002) modelled the economic gains through inter-sectorial water trade in southern Alberta, Canada. While all the economic models used in

these studies require agronomic and hydrologic data for water demand and water availability, none interacts dynamically with hydrologic models.

Integrated Quantity and Quality Model (IQQM) was developed for water resources planning and management. IQQM operates on a daily basis and is used to assess the impacts of changes in water management policies on water users and the environment. The model contains complex river management rules that allow it to simulate the delivery and allocation of water resources. For the past ten years, IQQM has been progressively implemented for all the major river basins in NSW and Queensland. IQQM is purely a hydrologic model, and only in a limited sense takes into account temporary water trade among irrigation nodes driven by market processes. In IQQM, water trade or adjustment to the existing water allocation and crop patterns is treated as given rather than unknown. To be able successfully model the movement of water within systems, particularly in resource constrained years, it is important to adjust crop patterns on an economic basis. As discussed previously, optimisation techniques are widely used for resources allocation studies. Water trade occurs when the buyers perceive positive returns while the sellers are adequately compensated to maximise some form of

aggregate social welfare subject to a wide range of biophysical and social economic constraints. For example, in resource constrained years water will be traded from the less profitable crops to the higher return crops.

To develop integrated tools to assess the impact of changes to climate, land-use, and policy instruments on regional economy at the whole-of-catchment scale, a hydrologic and economic modelling project was initiated through the Collaborative Research Centre for Catchment Hydrology (CRCCH) to provide a dynamic link with hydrologic water allocation models such as IQQM and economic water trading models. A water reallocation model, WRAM, was developed for this purpose. This paper reports on this integrated hydrologic and economic modelling, and some preliminary results from this effort to develop and implement WRAM for the Murrumbidgee River. The following sections formulate water trading as a linear programming (LP) problem, give an overview of IQQM and WRAM for the Murrumbidgee, and present some preliminary water trading results.

## 2. IMPLEMENTATION

### 2.1 Formulation of a linear programming problem to simulate water trading

Let decision variables  $x_{ij}$  be the amount of land at node  $i$  to grow crop  $j$  (ha), the constraints due to area availability is

$$\sum_j \delta_{ij} x_{ij} \leq L_i, \quad i = 1, \dots, n \quad (1)$$

where  $n$  is the number of irrigation nodes,  $L_i$  is the total amount of irrigable land at node  $i$  (ha), and  $\delta_{ij}$  is a binary variable to indicate crop feasibility so that  $\delta_{ij} = 1$  if it is feasible to grow crop  $j$  at the node  $i$ ,  $\delta_{ij} = 0$  otherwise.

Another set of constraints we impose on the decision variables are based on market considerations:

$$\sum_i \delta_{ij} x_{ij} \leq (1 + \lambda) L_j, \quad j = 1, \dots, m \quad (2)$$

where  $m$  is the number of crops grown in the basin,  $L_j$  is the current amount of area (ha) grown of crop  $j$  in the basin, parameter  $\lambda$  is the percentage increase in land use for crop  $j$  that is allowed in the basin. For the current version of WRAM,  $\lambda$  is set to 0.2. The constraint imposed by water availability is given by:

$$\sum_{ij} \delta_{ij} w_{ij} x_{ij} \leq \sum_i A_i \quad (3)$$

where  $w_{ij}$  is the water requirement to grow crop  $j$  at node  $i$  (ML/ha), and  $A_i$  is the water allocation for node  $i$  (ML). The right hand side of the constraint (3) is the total amount of water available for the entire basin. We recognise that due to crop rotation, we do not expect monoculture at any node, and thus we impose the following constraints for individual crop areas:

$$0 \leq x_{ij} \leq r L_i, \quad i = 1, \dots, n, \text{ and } j = 1, \dots, m \quad (4)$$

where  $r$  is a crop rotation parameter currently set to 0.6. This represents a typical scenario in the Murrumbidgee where 3-year rice is followed by 2-year pasture at a given node.

Finally the objective function is to maximise basin wide net benefit, i.e.

$$\text{Max } z = \sum_{ij} [y_{ij} p_j - (c_{ij} w_{ij} + v_{ij})] \delta_{ij} x_{ij} \quad (5)$$

where  $y_{ij}$  is the yield of crop  $j$  at node  $i$  (tonnes/ha),  $p_j$  price of crop  $j$  (\$/tonnes),  $c_{ij}$  water charge (\$/ML) at node  $i$ ,  $v_{ij}$  variable cost other than irrigation to grow crop  $j$  at node  $i$  (\$/ha).

Solutions of the LP problems in terms of amount of land at node  $i$  for crop  $j$  can be used to determine water demand at each node and subsequently simulate the water trade in the basin. Let  $W_i$  be the total amount of water required at node  $i$  (ML) under optimal conditions, i.e.

$$W_i = \sum_j \delta_{ij} w_{ij} x_{ij} \quad (6)$$

then the amount of trade  $T_i$  for node  $i$  is given by

$$T_i = A_i - W_i \quad (7)$$

Note that when water is sold from the allocation for node  $i$ ,  $T_i$  is positive according to equation (7). This is because of a net gain in monetary terms for the node. The trade volume by value at node  $i$  is given by

$$V_i = s T_i \quad (8)$$

where  $s$  is the shadow price of water (\$/ML). Shadow price of a resource is the marginal increase in the net benefit given a unit increase of the resource (Winston, 1991). In the absence

of an established water market, the shadow price is the best indicator of the marginal value of water. Shadow price of water increases as water becomes scarce (see Section 3).

Thus output from the LP can be used to inform hydrologic water allocation model such as IQQM in terms of crop pattern at each node and modified allocation based on water requirement as a result of water trade among irrigation nodes in the basin.

A number of different options were evaluated to solve this LP problem for optimal water allocation and, simultaneously, simulate water trading. For large-scale LP problems, there are essentially two viable approaches. The first is to use commercially available modelling systems such as AMPL and GAMS (Fourer *et al.*, 2002; Bruce *et al.*, 1998). These modelling systems were developed to facilitate formulation of LP problems, and to access a range of LP solvers that are widely available. GAMS is particularly popular among economists, in fact most of water allocation and water trading problems are solved in the GAMS modelling system (Rosegrant *et al.*, 2000; Mahan *et al.*, 2002). An alternative approach is to use callable routines. The advantage of using the second approach is the flexibility with and control over input and output and the ability to embed in other numerical models the LP solver and its associated pre- and post- processors in a seamless manner. IQQM is implemented in Fortran 95 and compiled with Lahey Fortran 95 version 5.7. We therefore decided to use the LP solver, E04NKF, from a well-known Fortran library to optimise the aggregate net benefit (equation 5). E04NKF is one of 1000 plus subroutines from Numerical Algorithm Group (NAG). E04NKF solves sparse linear programming or quadratic programming problems. The subroutine is based on SQOPT, which is part of the SNOPT package described in Gill *et al.* (1997), which in turn utilizes routines from the MINOS packages (see Murtagh and Sanders, 1995). It uses stable numerical methods throughout and includes a reliable basis package, a practical anti-degeneracy procedure, efficient handling of linear constraints and bounds on the variables, as well as automatic scaling of the constraints (NAG, 2000). Solvers SNOPT and MINOS, developed at Stanford University, are also available through modelling systems such as AMPL and GAMS.

## 2.2 The Murrumbidgee Valley

The Murrumbidgee River is located in south-western NSW. It is almost 1600km in length from its source in the Snowy Mountains to its junction with the Murray River. The Murrumbidgee Valley is bounded to the east by the Great Dividing Range and lies between the Lachlan River Valley to the north and the Murray River Valley to the south.

At Wagga, upstream of any significant irrigation, annual flows are of the order of 4000 GL including a 550 GL diverted from the Snowy River via the Snowy Mountains Hydro-Electrical Scheme.

Rice is the major crop grown in the valley with almost 100,000 Ha grown in most years. A major horticultural area exists within the Murrumbidgee Irrigation Area. Wheat and pasture are other major crops grown. Until the late 1980's rice was only grown in the Coleambally and Murrumbidgee irrigation areas. Deregulation meant that irrigators pumping directly from the river could grow rice on the main stem of the Murrumbidgee and the Yanco-Colombo-Billabong effluent system.

Another significant feature in the Murrumbidgee Valley is the Lowbidgee area. This area features inorganic farming using opportunistic (non-regulated) water supply available during supplementary (high flow) events. The Lowbidgee along with the adjoining Great Combung Swamp also makes up one of the most significant wetland habitats for water birds in Eastern Australia.

The following section describes an exercise in implementing a trade model loosely using some Murrumbidgee IQQM model calibration data for the water year 1993/94. The exercise was basically about "a proof of concept" and should not be taken as representing 1993/94 behaviour or constraints.

## 2.3 IQQM and WRAM implementation for the Murrumbidgee Valley

49 nodes were used to represent all the irrigation areas in IQQM for the Murrumbidgee. Each irrigation node contains a number of properties and usually grows a mixture of crops. Based on data for 1993, the average cropping area was 8175 ha, ranging from 143 to 51 077 ha among the 49 nodes. The average amount of water available in 1993 was 61 700 ML/node, varying from 3716 to 408 700 ML. The level of surface water allocation was 120% in 1993, and

this was assumed to be potentially supplemented with significant amount of groundwater extraction. 11 distinct crops were modelled in IQQM for the Murrumbidgee. 46 separate crops were considered in the optimisation model for water allocation and trade because of the regional differences in water requirement and production costs. For WRAM, we estimated crop yield and price, water charge and other variable costs for these crops from farm budget information made available through NSW Agriculture (Elton, 1997; NSW Ag., 2003). For this version of WRAM for the Murrumbidgee, there are 419 decision variables, and 1670 non-zero cells. Computational time is of the order of 0.1 sec per LP solution on a Toshiba Satellite Pro 6100. If two planting decisions are called for per year for summer and winter crops, we estimate an additional computational cost of 3-4 min when WRAM is invoked by IQQM for long-term (about 100 years) simulations.

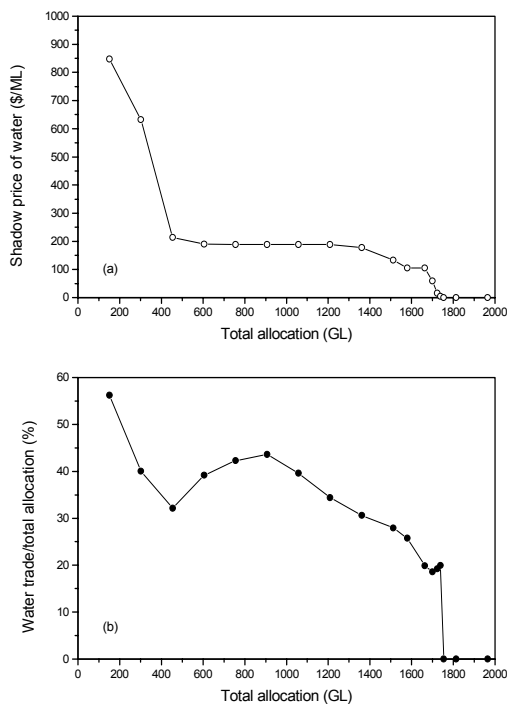


Fig. 1 Simulated shadow price of water and level of trade in relation to water allocation for the Murrumbidgee.

### 3. SIMULATION RESULTS

Whilst the Murrumbidgee may have what Randall (1981) called a 'mature water economy', a mature water market is yet to be established for water trading in the basin. As a result, the opportunity to validate the water

reallocation model is limited. For this paper, we present some simulation results on water trading for the Murrumbidgee. Fig. 1.a shows the shadow price of water as a function of total allocation. As the water availability decreases, water becomes a limiting resource and the marginal benefit of water increases, hence the shadow price. When water allocation is adequate for crop production for all the nodes, there is no incentive to trade. As water becomes scarce, both the shadow water price and the volume of trade relative to total amount of allocation increase (Fig. 1.b). The minimum level of trade is about 20% of total allocation for the Murrumbidgee from Fig. 1.b. While the simulated trend in the shadow price of water and the level of water trade is entirely consistent with what is expected from an economic perspective. The trade volume relative to total allocation is much higher than the level of trade over the past few years in the Murrumbidgee basin. Historical intra-valley trade is no more than 10% of total allocation at its peak for the 1997-1998 water year (Fig. 2). This shows that the current version of WRAM over-estimates the amount of water trade considerably. One of the main reasons for this discrepancy is the absence of a perfectly competitive market to allow full participation and realise maximum net return.

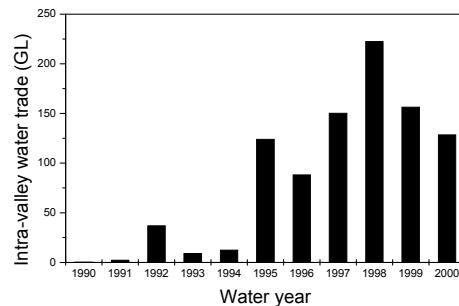


Fig. 2 Water trade within the Murrumbidgee for the water years 1989/90 – 1999/00.

Table 1 shows, as an example, the simulated crop pattern for one of the 49 irrigation nodes for the Murrumbidgee. With water trade, the optimal crop pattern can be quite different from the existing crop pattern for the node. Optimal solutions in this instance suggest that crops with high water requirements can be forgone. The notional farmer representing the node can be adequately compensated by either higher returns using more valuable crops or through attractive water price or both. Fig. 3 gives an example of

the magnitude of water trade among all nodes at a particular level of allocation of 1700 GL. The amount of spatial variation is considerable with up to 63GL traded. At this level of trade, there are 21 ‘buyers’ out of these 49 nodes and the rest are ‘sellers’. The average trade volume is 15 GL/node for the year.

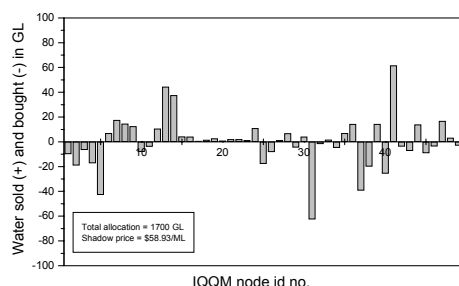


Fig. 3 Simulated trade volume among 49 irrigation nodes in the Murrumbidgee valley given a total allocation of 1700 GL.

When addressing the simulation results, we must consider the assumptions underlying this water reallocation model. A perfectly competitive water market is assumed to exist, and water traders have perfect information to make rational decisions. Water requirement and allocation are deterministic. There is negligible transaction costs, and the short-term net benefit can represent social welfare. When a well-developed water market does not exist, water trade occurs at suboptimal levels, resulting in the historical water trade being considerably less than the optimal level of trade for the Murrumbidgee. Sub-optimal reallocation outcomes have been explored using trading experiments in a research project on an evaluation of the temporary water market (Tisdell *et al.*, 2002).

Table 1. Sample output for node ‘YAN2B’ in the Murrumbidgee. Total irrigation land use remains unchanged at 51 077 ha for the node.

	Pre-trade	Post-trade	Post-trade
Allocation for the node (GL)	352	198	352
Water requirement (GL)	352	137	138
Shadow price (\$/ML)	-	58.93	0.00
<b>Fractional crop pattern</b>			
Other grains	0.2969	0.1446	0.1159
Fallow and miscellaneous	0.0982	0.0000	0.0000
Fodder	0.0218	0.1231	0.1231
Lucerne	0.0058	0.0277	0.0277
Summer oil seed	0.0057	0.0000	0.0000
Winter oil seed	0.0019	0.0183	0.0183
Orchards	0.0447	0.0000	0.0000
Winter pasture	0.2725	0.6000	0.6000
Rice	0.2297	0.0000	0.0000
Vegetables	0.0087	0.0000	0.0000
Vines	0.0142	0.0863	0.1150

#### 4. CONCLUDING REMARKS AND FUTURE DIRECTIONS

This paper reports the first genuine attempt to dynamically link hydrologic water allocation models such as IQQM with economic models for optimal water allocation. Data required to specify water demand and other hydrologic constraints for each crop at each irrigation node were sourced entirely from IQQM for the Murrumbidgee basin. Simulated water reallocation and crop pattern as a result of incentive-based water trade can be fed back into IQQM in a seamless manner when needed. In

the absence of a well-established water market, the simulated volume of trade was found to be considerably greater than the actual intra-valley water trade in the Murrumbidgee basin.

WRAM is currently being refined to introduce additional features to improve its modelling capabilities. Hydrologic constraints due to channel or pumping capacities consistent with the IQQM implementation for the Murrumbidgee will be added. Model users will be able to select an arbitrary collection of irrigation nodes to participate in water trading to simulate trade volume. Shadow price of water can be determined for any combination of

irrigation nodes among which water trading is allowed. We believe that a spatially limited trading block is necessary in order to reduce the amount of trade to realistic levels. It may also be necessary to limit the extent to which the existing crop pattern can be adjusted because of the persistence of cropping patterns at the nodal level in the basin.

## 5. ACKNOWLEDGMENTS

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