A Generalized Capital Asset Pricing Model

C.T. Lin

International Graduate School of Management, University of South Australia, City West Campus, Adelaide SA 5000 Australia (chienting.lin@unisa.edu.au)

Abstract: In light of the inadequacy of Sharpe's one-period Capital Asset Pricing Model (CAPM) in explaining stock returns, this paper develops a multi-period CAPM that captures more variation in average stock returns. Specifically, the derivation of the generalized model captures growth in earnings as an additional factor besides beta. This suggests that Sharpe's CAPM may be misspecified due to the omission of the earnings growth variable. In addition, it may explain why size and book-to-market effect found by Fama and French are significant when they are regressed along with beta since earnings growth and the two anomalies are highly correlated.

Keywords: *Asset Pricing, beta, size, and book-to-market equity, efficient market.*

1. INTRODUCTION

Current efforts in the asset pricing literature have been focused on finding the underlying factors behind size and BE/ME effect. Fama and French (1995), in an attempt to lay down an economic foundation for the empirical relationship between size and book-to-market equity (BE/ME) and expected stock returns that are observed in their earlier studies, analyze how these two variables are related to stock earnings and profitability. They reason that if stocks are priced rationally, not only must size and BE/ME proxy for sensitivity to common risk factors in returns, but they also must be driven by common factors in shocks to expected earnings that are related to size and BE/ME. Consistent with the prior, they found that growth in earnings variable, a proxy for shocks to expected earnings, is statistically significant in explaining portfolio returns and is also significant in relating to the market and size factor. Other study like Harris and Marston (1994), also provide indirect evidence linking growth variable to the anomalies and ultimately, to stock returns. These reports may suggest that, in the context of econometric, misspecification of CAPM due to omission of growth variable may lead to the poor estimation and insignificance of true beta, and size and book-to-market equity could in part capture the portion of the multi-factor model that is missed by the one-period CAPM.

Such empirical evidence motivates a direct examination from the theoretical perspective that the single period, two-parameter CAPM is over simplified and that growth in earnings may be the missing factor that should be incorporated into a multi-factor asset pricing model. The objectives of this paper are therefore twofold: first, to develop a generalized CAPM that not only includes Sharpe's CAPM but also finds earnings growth, an additional factor in a multi-period setting; second, to explain the empirical findings using the multi-period model. Unlike many of the recent research that fails to address the theoretical foundation in the literature, this paper theorizes and pinpoints growth factor as an underlying factor that may prove to be the missing link in the asset pricing model.

The structure of the study is as follows: Section 2 provides a brief literature review and pays more attention to the recent attempts by those market efficiency believers of finding a multi-factor pricing model that can explain and absorb size and book-to-value market effect. Section 3 develops and presents a multi-period CAPM which identifies the growth in earnings variable as a missing factor in explaining stock returns. Last section concludes the paper.

2. LITERATURE REVIEW

The anomalies in the stock returns literature have stirred voluminous empirical studies since the CAPM has been put into question. The evidence supporting that stock returns are predictable by variables besides beta has become overwhelming. What differs among researchers is the interpretation to their findings. One school of thought attributes the anomalies to the errors of measuring beta or market portfolio. The idea here is that since beta and market portfolio are unobservable and the CAPM leaves no guidance as to how to measure them, improper measurements may cause errors-in-variables problems and statistical artifacts that are associated with the empirical regularities found in the studies. For instance, beta estimates are found to be heavily dependent on the return intervals that are used to compute them (Kothari, Shaken, and Sloan (1995)). The stationarity of beta may also be problematic over a long sample period. Failure to consider these issues may seriously invalidate the significant role of beta in rational market pricing (Ball and Kothari (1989)).

Others have pursued along the line of market inefficiency. In their interpretation, the market is inefficient because systematic excess returns can be achieved by forming portfolios that mimic size, book-to-market equity and other effects. Investors under this hypothesis are assumed to behave irrationally and always overreact to new information and hence, the strategy of buying losers and selling winners always yields abnormal returns. Market overreaction, information asymmetry or transaction costs, which are often viewed as consistent with market inefficiency, are used to explain the predictability in variation of stock returns. (Lakonishok, Shleifer and Visney, (1994)) argue that naive investors tend to extrapolate past earnings growths too far into the future, assume a trend in stock prices, overreact to good or bad news or invest in a well-run company regardless of price. All these actions cause "glamour" stocks to be overpriced and out of favor to "value" stocks to be underpriced. Hence, contrarian investors who exploit the mistakes of naive investors by overinvesting in underpriced stocks and underinvesting in overpriced stocks outperform the market.

Finally, there are those who take the route of CAPM misspecification while assuming that the market is efficient. The argument here is that if stocks are priced rationally in the long run (assuming investors are rational), then systematic differences in expected returns must be due to differences in risk. Hence, such anomalous effects must proxy other dimensions of risks that are not captured by beta. In an attempt to identify systematic "state variables" that might affect stock returns, Chan, Roll and Ross (1986) argue that macroeconomic variables should affect stock prices through changes in the discount rate and expected cash flow. They identify four factors that might affect the discount rate: (1) the level of rates, (2) term spread (spreads across different maturities), (3) default spread (risk premium), and (4) real consumption changes. As for expected cash flow, changes in the expected rate of inflation, unanticipated price-level changes and changes in the expected level of real production should all influence current real value of cash flows. They found that only industrial production, unexpected inflation, the term spread and the default spread are significantly related to expected stock returns.

Under the assumption that the CAPM theory developed by Sharpe, Lintner and Mossin is misspecified, much of the effort has been directed toward finding firm-specific variables that can account for other systematic risk which beta fails. Fama and French (1993-1996) have found a three-factor model that captures much of the variation in average stock returns. The model says that the excess expected return of a portfolio is explained by three factors: (1) the excess return on a market portfolio $r_m - r_f$; (2) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB); and (3) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks (HML). The authors interpret that this equilibrium pricing model is consistent with Ross's (1976) arbitrage pricing theory (APT). However, the problem here is much like APT where we don't know what the state variables are that associate with these factors. Subsequently, a generalized asset pricing model capable of identifying firm-specific factors is needed. In the next section, I take the microeconomic approach by developing and extending an alternative asset pricing model that incorporates growth in earnings in the framework of CAPM.

3. MULTI-PERIOD CAPITAL ASSET PRICING MODEL

3.1 Additional Assumptions

In order to extend Sharpe's one-period CAPM into a more generalized multi-period one, additional assumption on how investors form their cash flow expectation in the subsequent periods is needed. Myers and Turnbull (MT,1977) assume that investors follow a simple adaptive expectation model. The forecasted value of the future expected cash flows are to be generated by the simple adaptive expectations model:

$$E(D_{t+1}|\phi_t) = a_1 D_t + a_2 D_{t-1} + a_3 D_{t-2} + ... \quad (1)$$

where D_t , D_{t-1} , …. are the actual cash flow at time t, t-1, …., the weights $a_1, a_2, ...$ are constants declining geometrically and which sum to unity, $\sum_{t=1}^{T+1} a_t = 1$, where T here is the terminal point, and ϕ_t is the all information available at time t . MT choose this particular earnings expectation model for its simplicity and for the intuitively attractive formulas it leads to. The qualitative properties and results derived later also do not depend on the specific expectation model used. This particular adaptive expectation model has also been supported by previous studies such as Griffin (1977) and Foster (1977), Brown and Rozeff (1979), and Brown, Griffin, Hagerman and Zmijewski [1987]. Incorporating constant growth trend in the expected cash flow,[1] equation 1 becomes

$$\begin{aligned} E(D_{t+1}|\phi_t) &= (1+g)(a_1 D_t + a_2 D_{t-1} + ...) \\ &= E(D_t|\phi_{t-1})(1+g)(1+\eta\varepsilon_t) \end{aligned} \quad (2)$$

Where η is the same constant as a_1 and ε_t is a proportional random disturbance term representing the difference between the actual cash flow and its expected value based on the information set ϕ_t .

According to Myers and Turnbull, the factor η is the weight of this year's cash flows in forecasting next year's earning and is defined as the elasticity of expectations. The value of η normally lies within 0 and 1. The growth trend of expected cash flow defined by MT is the rate of expansion of cash flow used to measure a firm's growth opportunities.

MT also assumed that the actual and expected cash flow differ by the proportional factor ε_t :

$$D_t = E(D_t|\phi_{t-1})(1+\varepsilon_t) \quad (3)$$

3.2 Derivation of the Two Factor Model

One can express one-period CAPM in terms of its security price:[2]

$$P_t = \frac{E((D_{t+1} + P_{t+1})|\phi_t) - \lambda Cov((D_{t+1} + P_{t+1}), R_{m,t+1})}{1 + R_f} \quad (4)$$

Where

$E((D_{t+1} + P_{t+1})|\phi_t)$ represents the investors' expectations of cash flow $D_{t+1,j}$ and price $P_{t+1,j}$ of asset j of the next period given all the available information ϕ_t in this period; $R_f =$ the risk-free rate; $\lambda = \frac{E(R_m) - R_f}{VAR(R_m)}$, is the market price of risk; $Cov((D_{t+1} + P_{t+1}), R_{m,t+1}) =$ covariance between earnings D_{t+1} plus price P_{t+1} of an asset j and the return of market portfolio of the next period.

In order to solve for this period's price P_t in equation 4, investors' expectations of future cash flow must be known. Since investors are assumed to follow simple adaptive model, we can use backwardation to solve for P_0 .

$$P_0 = E(D_1|\phi_0)(1 - \lambda\sigma_{\varepsilon m}) / [R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)] . \quad (5)$$

Where $\sigma_{\varepsilon m}$ is the covariance between the disturbance term and the market return. Equation 5 is the uncertainty equivalent of the Gordon constant growth model. When $\sigma_{\varepsilon m} = 0$, the model reduces $P_0 = D_1|\phi_0 / (R_f - g)$. From equation 5, the expectation of price for asset j at time t+1 given the information set ϕ_t at time t yields

$$E(P_{t+1}|\phi_t) = E(\frac{E(D_{t+2}|\phi_{t+1})(1 - \lambda\sigma_{\varepsilon m})}{R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)}\Bigg|\phi_t) . \quad (6)$$

Similarly, by extending equation 2,

$$E(D_{t+2}|\phi_{t+1}) = E(D_{t+1}|\phi_t)(1+g)(1+\eta\varepsilon_t) \quad (7)$$

And substituting equation 6 into equation 7 yields

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[1] If growth is stochastic, the random part of growth can be incorporated into the uncertain cash flow.

[2] See page 202 on Copeland and Weston (1992) for a similar valuation model based on CAPM.

$$E(P_{t+1}|\phi_t) = E(\frac{E(D_{t+1}|\phi_t)(1+g)(1+\eta\varepsilon_t)(1-\lambda\sigma_{\varepsilon m})}{R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)}) . \quad (8)$$

Since $E(\eta\varepsilon_t) = \eta E(\varepsilon_t) = 0,$

$$E(P_{t+1}|\phi_t) = \frac{E(D_{t+1}|\phi_t)(1+g)(1-\lambda\sigma_{\varepsilon m})}{R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)} . \quad (9)$$

By definition, one can also express the expected return of asset j at time t in this form:

$$E(R_t) = \frac{E(D_{t+1}|\phi_t)}{P_t} + \frac{E(P_{t+1}|\phi_t)}{P_t} - 1 \quad (10)$$

Where $E(D_{t+1}|\phi_t)$ represents the investors' expectations of cash flow D_{t+1} and expectations of price $E(P_{t+1}|\phi_t)$ of asset j of the next period given all the available information ϕ_t in this period.

Equation 7 and 9 can now be substituted into the first and second terms of equation 10 respectively. That is,

$$E(R_t) = \frac{R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)}{1-\lambda\sigma_{\varepsilon m}} + \frac{\dfrac{E(D_{t+1}|\phi_t)(1+g)(1-\lambda\sigma_{\varepsilon m})}{R_f - g + \lambda\eta\sigma_{\varepsilon m}}}{E(D_{t+1}|\phi_t)(1-\lambda\sigma_{\varepsilon m})/[R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)]} - 1$$

$$= \frac{R_f - g + \lambda\eta\sigma_{\varepsilon m}(1+g)}{1-\lambda\sigma_{\varepsilon m}} + (1+g) - 1$$

$$= \frac{R_f + \lambda\eta\sigma_{\varepsilon m}(1+g) - \lambda\sigma_{\varepsilon m} g}{1-\lambda\sigma_{\varepsilon m}} . \quad (11)$$

By adding and subtracting $R_f\lambda\sigma_{\varepsilon m}$ in the numerator and simplifying the equation yields

$$E(R_t) = \frac{R_f + \lambda\eta\sigma_{\varepsilon m}(1+g) - \lambda\sigma_{\varepsilon m} g - R_f\lambda\sigma_{\varepsilon m} + R_f\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}}$$

$$= \frac{R_f(1-\lambda\sigma_{\varepsilon m}) - (1-\eta)\lambda\sigma_{\varepsilon m} g + (R_f + \eta)\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}}$$

$$E(R_t) = R_f - \frac{(1-\eta)\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}} g + \frac{(R_f+\eta)\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}} . \quad (12)$$

Equation 12 is the generic form of the two-factor CAPM where the first term is the risk free rate, while the second term is the first factor due to the expected growth rate and the third term is the second factor due to systematic risk. The systematic risk factor can be expressed in the form of β . Since λ is the market price of risk which equals to $\frac{R_m - R_f}{\sigma_m{}^2}$, the third term in equation 12 can be expressed as:

$$E(R_t) = R_f - \frac{(1-\eta)\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}} g + \frac{(R_f+\eta)(R_m - R_f)\sigma_{\varepsilon m}/\sigma_m{}^2}{1-\lambda\sigma_{\varepsilon m}} \quad (13)$$

Equation 13 can also be expressed in terms of β yielding

$$E(R_t) = R_f - \frac{(1-\eta)\lambda\sigma_{\varepsilon m}}{1-\lambda\sigma_{\varepsilon m}} g + (E(R_m) - R_f)\beta . \quad (14)$$

Equation 14 can be viewed as a version of the two factor CAPM which consists of the following two factors (1) the expected growth in earnings variable, g, and (2) the beta, β . Under the one-period framework where η = 1, equation 14 reduces to the Sharpe's CAPM. That is the coefficient of g becomes zero. Therefore, the one-period CAPM is just a special case of the multi-period model derived in equation 14. If the multi-period model is true, then it shows that the one-period CAPM is misspecified in that it omits the expected growth variable.

3.3 Implications of the Multi-Period Model

As in the one-period model, the coefficient of beta in the multi-period model is the market risk premium. However, the model does not postulate that the coefficient of the earnings growth variable will always negative, which is consistent with the size and book-to-market effect. The sign of the coefficient is largely dependent on the size of $\lambda\sigma_{\varepsilon m}$ and whether the forecast error term is positively or negatively correlated with market return. That is, when 0 $> \lambda\sigma_{\varepsilon m}$ > 1, $(1-\eta)\lambda\sigma_{\varepsilon m}/(1-\lambda\sigma_{\varepsilon m})$ stays negative and the slope of the growth factor is positive. On the other hand, when 0 $< \lambda\sigma_{\varepsilon m}$ < 1, the slope of the growth factor is negative.

The interesting implication here is that those who invest in one strategy (eg. glamour or contrarian strategy) cannot indefinitely and consistently outperform the other. As the model suggests, the appropriate winning strategy is determined by how the forecast earnings error and market return interact during the time. In fact, empirical studies have shown that while size effect was significant in the earlier period from mid 1960 to beginning of 1980, it disappears altogether in the past 15 years. Dimson and Marsh (1999) reports that the small-cap premium is 4.1% from 1955 to 1983 but the discount is -7.2% from 1984 to 1997 in the U.S. The U.K. data tells a similar story of reversal of fortune in the small-cap firms during this entire period. The empirical results therefore suggest that the coefficient of growth factor is negative in the earlier period (i.e. $0 < \lambda\sigma_{\varepsilon m} < 1$), while positive in the subsequent period (i.e. $0 > \lambda\sigma_{\varepsilon m} > 1$). Whether the model correctly predicts the direction and the magnitude is an empirical question, and is beyond the scope of this study.

3.4 Size and Book-to-Market Anomalies

According to the two-factor model, the misspecification of the one period CAPM may lead to the statistical insignificance of beta found in the Fama and French (1992) study. In their findings, FF report that even when beta is the only independent variable in the cross-sectional regression, it cannot significantly explain average stock returns. However, if the true regression model according to equation 14 is

$$R_{p,t} = \alpha_1 \beta_p + \alpha_2 g_p + u_{p,t} \tag{15}$$

Where
$R_{p,t}$ = excess portfolio returns;
β_p = portfolio beta;
g_p = portfolio growth rate;
and $u_{p,t}$ = error term.

The estimated regression (according to one-period CAPM) is,

$$R_{p,t} = \alpha_1^{'} \beta_p + w_{p,t} \ , \tag{16}$$

Where
$w_{p,t}$ = corresponding error term.

Then the variance of $\alpha_1^{'}$ of the estimated regression 16 is,

$$Var(\alpha_1^{'}) = \frac{E(w_{p,t}^{\,2})}{\sum \beta_p^{\,2}} = \frac{\sigma_w^{\,2}}{\sum \beta_p^{\,2}} \ , \tag{17}$$

While the variance of α_1 of the true regression 15 is

$$\mathrm{var}(\alpha_1) = \frac{\sigma_u^{\,2} \sum g_p^{\,2}}{\sum \beta_p^{\,2} \sum g_p^{\,2} - (\sum \beta_p g_p)^2} \ . \tag{18}$$

But $w_{p,t} = \alpha_2 g_p + u_{p,t}$ by equating equation 15 and equation 16; therefore,

$$\sigma_w^{\,2} = \alpha_2^{\,2}\, \mathrm{var}(g_p) + \sigma_u^{\,2} \ . \tag{19}$$

Hence, the estimate of the variance of $\alpha_1^{'}$ is

$$\mathrm{var}(\alpha_1^{'}) = \frac{\alpha_2^{\,2}\, \mathrm{var}(g_p) + \sigma_u^{\,2}}{\sum \beta_p^{\,2}} \ . \tag{20}$$

Under the traditional estimates of beta where g and β are uncorrelated, the second term in the denominator of equation 18 is zero. Hence,

$$\mathrm{var}(\alpha_1) = \frac{\sigma_u^{\,2} \sum g_p^{\,2}}{\sum \beta_p^{\,2} \sum g_p^{\,2}} = \frac{\sigma_u^{\,2}}{\sum \beta_p^{\,2}} \ . \tag{21}$$

Comparing the estimate variance of α_1 from equation 20 and the true estimate from equation 21, the estimate of variance is upward biased by a factor of $\alpha_2^{\,2}\, \mathrm{var}(g_{p,t}) / \sum \beta_p^{\,2}$. This may explain why beta is statistically insignificant in FF findings because the variance or the standard error of the beta is upward biased for the misspecified regression.

4. CONCLUSIONS

Unlike many recent studies, this paper focuses on the theoretical development of an alternative asset pricing model. The purpose of this study is to bridge the gap between the existing empirical findings and the absence of the theoretical fundamentals. The motivation here is that Sharpe's model may be overly simplified in that only one single period is taken into account in formulating the rate of return and the systematic risk. Such simplicity may not capture other important aspects of the relationship and may cause different proxies and anomalies to explain returns. By incorporating multi-period framework and therefore the generality into the model, I have captured earnings growth as an additional factor in explaining stock returns and hope that in the process, have enhanced further understanding in the asset pricing theory. This two-factor model is capable of explaining the reversal of size and book-to-market effects during the entire period through the correlation between forecast errors and market returns. It also suggests that no particular strategy (eg. value strategy) can persistently outperform others as supported by the empirical findings.

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