Genetic Algorithms and Stochastic Forest Property Rights Enforcement

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Abstract This paper analyses the impact of corruption in a timber industry by comparing stochastic and deterministic approximations of a ‘log poaching-enforcement’ difference game. In the paper it is argued that corruption leads to certainty about the effectiveness of enforcement efforts, a situation best represented by a deterministic game. By adding uncertainty, therefore, some light is shed on the impact of corruption. The model is hard to solve analytically, so results are demonstrated by developing heuristic genetic algorithms. The results show that increasing uncertainty induces a government to spend more on enforcement and an illegal logger to harvest less.

Keywords: Deforestation; Genetic algorithms; Illegal logging.

1. INTRODUCTION

This paper extends the use of genetic algorithms (GA) to examine difference games with stochastic elements. This paper then analyses the impact of corruption in a timber industry by comparing stochastic and deterministic behaviour of an illegal logger and government enforcer.

In many developing countries, even though forest property rights are officially held by the government, forests are effectively utilised as an open-access resource (Clarke et al., 1993). Even if a government chooses to enforce its property rights, corruption associated with the forest industries of such countries makes enforcing property rights difficult (Dudley et al., 1995). Often the rhetoric of imposing fines on illegal loggers is replaced by a system where loggers circumvent fines by paying bribes. The certainty of the bribe payment, as distinct from the uncertainty of being caught and fined, means that an illegal logger can plan its harvesting decisions in a deterministic framework. This justification has previously been proposed for the use of a deterministic approach in examining deforestation (McAllister et al., 2001). Therefore, assuming that the deterministic framework represents a system where enforcement is not genuine, and that additional enforcement uncertainty simulates the impact of a more genuine system of property rights enforcement, then a comparison of results should help examine the impact of corruption.

2. MODEL

The model presented here employs a two-player difference game played over an infinite time horizon with annual time steps The logger represents an agent which makes profits by illegal logging. The government represents a central government which makes profits from non-timber forest benefits $x^\gamma$, and royalties $r$ from legal logging $h$ in the region. The government owns the property rights on the remaining area of forest stock $x$. The government enforces its property rights by spending money to detect illegal loggers $k$ and then applying a fine $\vartheta$ (Table 1).

Despite the enforcement efforts of the government, the logger may have an incentive to illegally harvest. This incentive may occur for two reasons. First, particularly in developing countries, enforcement may not be genuine. Instead bribes may be paid to circumvent the rhetoric of fines. Second, even where enforcement is genuine, it is unlikely to be optimal for a government to spend at such levels such to completely enforce its property rights (Zhang, 2001; McAllister et al., 2001).
The objective functions $L_h$, for the logger, and $G_k$, for the government are given below. In these functions all players discount future profits by $\frac{1}{(1+\delta)^t}$ in order to account for their time preference of money.

$$L_h = \sum_{t=0}^{\infty} \left( \frac{pb_t - \phi(x_t, h_t, k_t)\theta}{(1+\delta)^t} \right)$$

$$G_k = \sum_{t=0}^{\infty} \left( \frac{Q(x_t)r + x^2 + \phi(x_t, h_t, k_t)\theta - k_t}{(1+\delta)^t} \right)$$

subject to,

$$\Delta x_t = F(x_t) - h - Q(x_t),$$

$$F(x_t) = \alpha x_t \left( 1 - \frac{x_t}{100} \right),$$

$$Q(x_t) = \frac{x_t F(x_t)}{T},$$

$$\phi(x_t, h_t, k_t) = 1 - \exp \left( \frac{-k_t h_t \beta_t}{x_t - h_t} \right)$$

where, $\beta_t = \beta_0 + \mathcal{N}(0, \sigma \beta_0)$ and where $\mathcal{N}(0, \sigma \beta_0)$ is a normally distributed random variable with a mean of zero and a standard deviation of $\sigma \beta_0$, subject to $\beta_t > 0$.

### 3. METHOD

Derivation of analytical results from the above model is not possible. A GA approach is therefore used to numerically demonstrate theoretical aspects of the model. Özyildirim (1996) was first to use parallel GAs to solve deterministic dynamic games. In this paper the aim is to apply this deterministic approach, then attempt to introduce uncertainty into the analysis. The algorithm proposed by Özyildirim involves a population of possible strategies for each control variable ($k$ and $h$ in this case). The populations are co-evolved by the GA. Initially, each player’s population of strategies is generated randomly. The fitness of each individual strategy is then determined by playing it against a randomly chosen strategy from the other player’s population. These fitnesses are then used for selecting parents for the next generation. To approximate an infinite time horizon, the game is played to some finite period $T$, with the approximated steady states (assumed stable) then used to approximate fitnesses to perpetuity (McAllister and Bulmer, 2002). Note that the number of generations $G$ used is unrelated to the number of discrete time periods $T$ used in the model, except in that by using more time periods, the size of the problem increases exponentially which requires more generations of evolution to achieve convergence of the algorithm.

Uncertainty enters the system because neither the government nor logger can be sure of just how effective enforcement expenditure will be in the future. As discussed above, the parameter $\beta$ is therefore replaced with the memoryless stochastic parameter $\beta_t$. For each generation of the GA, a sequence of normally distributed random numbers is generated using the Box-Muller transformation (Box and Muller, 1958). This sequence is used to generate $\beta_t$ for both players’ population of strategies in a given generation of the GA. In subsequent generations, the stochastic sequence is regenerated. The algorithm can be summarised by the following steps (see McAllister and Bulmer 2000 for further details):

1. Initialise a random population of strategy vectors, and update a shared memory with a random strategy vector;

### Table 1: Notation and assumed parameter values

<table>
<thead>
<tr>
<th>Control variables</th>
<th>h</th>
<th>Logger’s harvest rate (area)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Government’s enforcement expenditure</td>
<td></td>
</tr>
<tr>
<td>State variables</td>
<td>x</td>
<td>Stock of remaining forest (area)</td>
</tr>
<tr>
<td>Functions</td>
<td>$Q(t)$</td>
<td>Government logging quotas (area)</td>
</tr>
<tr>
<td></td>
<td>$F(t)$</td>
<td>Forest regrowth (area)</td>
</tr>
<tr>
<td>Coefficients</td>
<td>$\beta_0$</td>
<td>Effectiveness of enforcement expenditure</td>
</tr>
<tr>
<td></td>
<td>$\theta$</td>
<td>‘Fine’ imposed for illegal logging</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>Government logging royalties (per area)</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>Intrinsic rate of forest growth</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>Non-timber forest benefits coefficient</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>Quota setting coefficient</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>Players’ discount rate</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>Timber harvesting revenue (per area)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notation</th>
<th>Assumed parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi(x_t, h_t, k_t)$</td>
<td>$1 - \exp \left( \frac{-k_t h_t \beta_t}{x_t - h_t} \right)$</td>
</tr>
<tr>
<td>$\mathcal{N}(0, \sigma \beta_0)$</td>
<td>Normally distributed random variable with mean 0 and standard deviation $\sigma \beta_0$</td>
</tr>
<tr>
<td>$\beta_0 + \mathcal{N}(0, \sigma \beta_0)$</td>
<td>Stochastic parameter $\beta_t$</td>
</tr>
<tr>
<td>$[0, 1]$</td>
<td>Parameter range for $\gamma$</td>
</tr>
<tr>
<td>$x_t, h_t \geq 0$</td>
<td>Constraints for $x_t$ and $h_t$</td>
</tr>
</tbody>
</table>

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1. Initialise a random population of strategy vectors, and update a shared memory with a random strategy vector;
2. (Re)generate a stochastic process for $\beta$ over $T$ time periods, setting the final term $\beta_T$ in the process as zero (as zero is the expected stochastic mean to perpetuity);

3. Calculate the fitness of each strategy vector against the strategies of other controls held in the shared memory, using the same generated stochastic process;

4. Perform cross-over and mutation to create a new population;

5. Wait until all other parallel GAs have completed their respective fitness calculations;

6. Update the shared memory with the fittest strategy (the champion);

7. If the maximum number of generations $G$ is reached then stop, otherwise return to step 2.

Greater uncertainty in $\beta_t$ means that each player maintains a broader range of strategies in their population. Initially one may expect this range to contain the deterministic solution. However, if tending to one side of the deterministic solution exposes a player to the risk of incurring greater costs than the corresponding “risk” of reduced costs on the alternate side, then the strategy populations will ultimately converge towards the side that minimises the exposure to unfavourable outcomes. The GA employed here scales down mutation (Michalewicz, 1999), so towards the end of the evolution of the population of strategies each player consolidates what they have learned through $G$ generations of different stochastic processes.

4. RESULTS

Even though the game is solved over infinite time, only the first time period is analysed here. The analysis has been limited in order to be concise. The first period is considered the most relevant because the game solutions relate to planned behavioural decisions (i.e. open-loop equilibria). The gap between what is planned and what eventuates is assumed to widen as $t$ increases. Therefore, by analysing the first period, actual rather than planned behaviour can more closely be examined. In seeking to analyse the impact of corruption, the choice variables analysed are the level of enforcement expenditure $k$ and illegal log harvest $h$. Total payoffs of both the logger (equation 1) and the government (equation 2) are also considered. Because the model is demonstrated using numerical approximations, parameters must be assigned values. Sensible but arbitrary parameters used are presented in Table 1 above.

Table 2 summarises the results. The deterministic results are considered first. Because the numerical approximations are evolved through random mutation, the algorithm is unlikely to approximate the same solution twice. Figure 1 shows a histogram based on 100 runs of the GA. While subsequent runs of the GA approximate differently, the results are very similar, the 95 confidence intervals being extremely tight.

The stochastic approximations appear far more broadly distributed than the deterministic equivalents. This is to be expected because here each run of the GA is faced with unique stochastic properties. Table 2 summaries the distribution of approximations from 100 runs of the GA. Table 2 shows that when uncertainty contributes to how the each player’s strategy set evolves, 95 percent of approximations for $h_0$ and $k_0$ lay within 0.443908 and 0.444635, and 0.180482 and 0.180872 respectively when $\sigma = 0.1$. As $\sigma$ increases the distribution of the approximations increases, while the mean of $h_0$ and $k_0$ decreases and increases respectively.

Simulation results are graphically presented in Figure 2. This figure demonstrates how in time period zero increased uncertainty reduces the harvest decision and increases the government’s enforcement expenditure decision.

![Figure 1: Distribution of deterministic approximations $t = 0$](image-url)
Table 2: Deterministic and stochastic numerical open-loop approximations for the first discrete time period

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>standard deviation (s)</th>
<th>lower-95%a</th>
<th>upper-95%a</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deterministic (σ = 0)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>· Harvest (h₀)</td>
<td>0.452644</td>
<td>0.000000</td>
<td>0.452644</td>
<td>0.452644</td>
<td>0.000000</td>
</tr>
<tr>
<td>· Enforcement (k₀)</td>
<td>0.169530</td>
<td>0.000000</td>
<td>0.169530</td>
<td>0.169530</td>
<td>0.000000</td>
</tr>
<tr>
<td><strong>Stochastic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ = 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>· Harvest (h₀)</td>
<td>0.444310</td>
<td>0.001619</td>
<td>0.439087</td>
<td>0.44635</td>
<td>0.000727</td>
</tr>
<tr>
<td>· Enforcement (k₀)</td>
<td>0.180683</td>
<td>0.001005</td>
<td>0.180482</td>
<td>0.180872</td>
<td>0.000390</td>
</tr>
<tr>
<td>σ = 0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>· Harvest (h₀)</td>
<td>0.439607</td>
<td>0.002185</td>
<td>0.439087</td>
<td>0.440052</td>
<td>0.000965</td>
</tr>
<tr>
<td>· Enforcement (k₀)</td>
<td>0.190553</td>
<td>0.001685</td>
<td>0.190239</td>
<td>0.190917</td>
<td>0.000679</td>
</tr>
<tr>
<td>σ = 0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>· Harvest (h₀)</td>
<td>0.433757</td>
<td>0.002952</td>
<td>0.433039</td>
<td>0.434455</td>
<td>0.001416</td>
</tr>
<tr>
<td>· Enforcement (k₀)</td>
<td>0.198703</td>
<td>0.002296</td>
<td>0.198155</td>
<td>0.199275</td>
<td>0.001120</td>
</tr>
<tr>
<td>σ = 0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>· Harvest (h₀)</td>
<td>0.427539</td>
<td>0.003840</td>
<td>0.426900</td>
<td>0.428105</td>
<td>0.001205</td>
</tr>
<tr>
<td>· Enforcement (k₀)</td>
<td>0.209120</td>
<td>0.003030</td>
<td>0.208473</td>
<td>0.209832</td>
<td>0.001359</td>
</tr>
</tbody>
</table>

a Confidence intervals constructed by applying t-percentile method bootstraps using 10,000 re-samples.

Finally, Figure 3 shows the impact of uncertainty on the payoffs yielded by both players (equations 1 and 2). Figure 3 shows that as uncertainty increases, the changes the respective players’ choice variables lead (Figures 3) to a decrease in both logger and government payoffs. This may be counter-intuitive because one may expect that if illegal harvest activity decreases, then government profits would increase. However, in this complex system it seems that added uncertainty induces additional expenditure on property rights enforcement. Caution must also be used in interpreting the level of the results because the parameter values used are not calibrated.

5. COMPUTATIONAL SUMMARY

The GA was implemented using the computer programming language C. The compiled code was run on a multi-user SGI Origin 2000 computer with 64 MIPS R10000 CPU’s and 16 Gigabytes of memory. For each level of σ, the GA was run 100 times using two million generations. On average this code took 24 hours to complete, depending largely on the number of other batch jobs running. The program allocated no more than 3 Megabytes of memory. All confidence intervals were constructed by applying t-percentile method bootstraps using 10000 re-samples. The bootstraps were performed in Matlab.

Figure 2: Deterministic and stochastic approximations represented for various σ.
6. GAs AND UNCERTAINTY

Uncertainty enters the model presented above in two manners. The parameter $\beta_t$ is stochastic, but the GA itself is also stochastic, since random mutation drives the evolution towards its approximate solutions. In some applications of GAs in economics, random mutation is maintained throughout all stages of the GA in order to analyse the adaptive nature of learning (Ari-fovic, 1995). However here, the random mutation is used only to facilitate solution evolution, and its influence is scaled down such to eventually minimise its influence on the final approximation. Any remaining influence of the random mutation is GA error. The GA used here uses two million generations to allow the algorithm to minimise the GA error. It is useful to consider whether this number of generations $G$ is sufficient to minimise the GA error. To do this the model was run using $\sigma = 0.1$ and various $G$ of the GA. Figure 4 shows the confidence intervals of the standard error $s$ of GA approximations for $k_0$ and $h_0$.

Since the intervals bound a standard error from a stochastic model, they remain positive in all cases. The standard error however contains two parts. One part, which the GA does not necessarily seek to minimise, is the error introduced by the stochastic parameter $\beta_t$. The other part is the GA error, which is minimised by the algorithm given enough generations. Figure 4 shows that as $G$ increases, the standard errors drop sharply at first, then stabilise. One may conclude that the initial drop in the standard error signifies a reduction in the GA error. The level at which the standard error stabilises may then represent the error introduced by the stochastic parameter $\beta_t$. Based on these observations, it seems that two million generations appears sufficient to accurately represent the stochastic approximations.

Figure 3: Average approximations and payoffs for various $\sigma$.

Figure 4: Confidence intervals of the standard error against the number GA generations.

7. POLICY IMPLICATIONS AND CONCLUSIONS

Despite the arbitrary nature of the parameter values used in the above model, the model results demonstrate the theory contained in the model. The results from the model indicate that when the effectiveness of enforcement expenditure is uncertain (i.e. stochastic), illegal loggers will harvest less compared to when the effectiveness is certain (i.e. deterministic). Generally, when enforcement is considered, the difference between the uncertain and deterministic cases may be that the case of uncertainty better reflects reality. In many developing countries, however, a government’s
efforts to enforce its property rights is related more to rhetoric than to reality. Consequently the uncertainty inherent in enforcement may be replaced by certainty in bribe payments.

Corruption within the forest industries in developing countries is secretive (Lang, 2001) and little is known about the level of bribe payments, which themselves may take many forms (Walker, 1999, p.164). Therefore further modelling efforts are required before strong conclusions can be drawn, particularly with regards to the impact on payoffs. Nevertheless, by arguing that deterministic enforcement represents the certainty of bribe payments while the uncertain counterpart represents genuine efforts to enforce forest property rights, this paper has demonstrated that corruption may lead to higher rates of forest extraction. This paper has also demonstrated an approach to capturing uncertainty when GAs are used to approximate non-cooperative open-loop difference games played over infinite time.

8. REFERENCES


