Modeling Flexibility with Spline Approximations for Fast VR Visualizations

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Abstract:
This paper presents modeling of bending objects using Splines for fast calculations suitable for virtual reality (VR) visualizations in a CAD tool. In Micro-Electro-Mechanical Systems (MEMS), some structures that are involved in the electro-mechanical interaction are subject to flexible deformation, e.g. flicking and bending cantilevers, springs, valves, or membranes for pumps. Few MEMS simulators display animations of moving structures at all. Flexible deformations are calculated with Finite Elements Modeling (FEM), whose slow data processing is unsuitable for interactive VR. We are trying to overcome this shortage by building a MEMS simulator that displays VR visualizations as simulated functioning of the device being designed. For 2D screen visualizations, the information of the internal 3D structure of a body obtained with FEM is often not relevant. It is therefore we are finding methods for fast approximations that are suitable for VR, yet physically truthful. We calibrate our models with FEM, and find the families of splines corresponding to bending specific shapes for different materials and thicknesses. The obtained models can then be used in our CAD tool for fast VR visualizations.

1. INTRODUCTION
In this paper, we are presenting a new method for the efficient modeling and simulation of elasticity and plasticity in MEMS using splines. By efficient, we mean visualization algorithms that give accurate results in short time rather than calculating using Finite Element, which takes much longer.

The main objective of our research is to develop simulation and visualization techniques to facilitate object deformation in a virtual environment with natural and real-time interaction similar to sculpting in the real world (Sitte, 2001). Simulation is an important step in the design of MEMS, and the use of CAD tools help in reducing the overall costs and time between conception and prototyping up to 80%. Many techniques and systems have been developed for object modeling and deformation.

For a visualization on a 2D computer screen, it is often not necessary to know the details of ongoing physical details. With the increase of CAD visualizations for industrial product development, a range of shortcuts has emerged for fast approximations of the visible details, for example surfaces. However, more research is needed to fill the demand for shortcuts for faster physically truthful visualizations.

This paper describes relevant studies of the deflection of a cantilever as a case study. Kurmann and Engeli (1996) proposed a spatial approach for interactive modeling to support architectural design in a VR environment. This enables the user to formulate design ideas in 3D space. However, it is not suitable for artistic free-form deformation. An attempt was made to allow the user to move sample points on the object model using direct manipulation of Free-Form Deformations (FDD). Hsu et al. (1992) describe the results of this modeling.

1.1. Micro-Electro-Mechanical Systems
In recent years, the field of Micro-Electro-Mechanical systems (MEMS) has grown rapidly and has entered into many defense and communication applications. Much of this activity has been driven by the ability of MEMS to miniaturize, reduce the cost, and improve the performance of transducers and actuators previously fabricated by hybrid techniques (Huang et al., 2001).

The commercialization of MEMS technologies require the evaluation of the mechanical properties of the thin films used as structural materials in order to guarantee the reliability of the device (Ando et al., 2001). Thin films play an indispensable role as structural materials in MEMS. In order to guarantee the long-term operation of MEMS according to their
specifications, it is highly desirable to know mechanical and reliability-related properties of the thin films at the design stage. This may help to avoid catastrophic failure due to fracture, which is particularly drastic since it terminates the operation of a microdevice (Yang and Paul, 2002).

Over the past decade, there has been a substantial thrust to reduce the size of many electronic and electromechanical systems to the micron and sub-micron scale by fabricating devices out of thin film materials. In these applications, successful device development requires a thorough understanding of thin film mechanical properties. At this scale, specimen geometry and dimensions are similar in size to the microstructural features. Therefore, tests capable of accurately measuring the effect of microstructure on mechanical properties need to be used (Espinosa et al., 2002).

There is a distinct lack of appropriate CAD tools to aid in the efficient visualization of MEMS deformation. There are deficiencies in the present deformation algorithms i.e., they are too expensive and too slow. We want to develop a new technique that enables fast visualization for membrane and cantilever deflections.

For the purpose of Engineering and CAD design visualizations, the goal is to produce simulations of the devices and equipment being designed when functioning, with animated VR visualizations.

While flow, friction and timing are some of the key issues in industrial virtual prototyping, elasticity is an increasingly important issue, because it relates to deformations, desired or not, but also to wear and tear, fatigue and reliability of the product. Deformation modeling can become quite complex.

2. SIMULATION EXPERIMENT

Solids are in general modeled with the particle approach. The total energy of their particles is rather low, the particles do not move freely. The preferred model for visualizations of solids is the spring model, derived from the Lennard-Jones model, as a special case approximation. For our purpose however, we are more interested with the truthful shape of the object bending and shaping under the influence of a force. In our experiment we use Finite Element Analysis to simulate the deformation of a micro-cantilever. We then fit a second order polynomial and derive a spline model to predict the shape of the bending device.

2.1. Specimen Design

All experiments were conducted on ultrananocrystalline diamond UNCD cantilevers. Four types of specimens were used in this study. The four specimens are freestanding, thin-film cantilever structures made of UNCD with film thickness ranging from 0.3 µm to 0.6 µm. The dimensions of the cantilevers that we used in the simulation are defined in Table 1.
Table 1 Values of cantilever dimensions

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Length (Microns)</th>
<th>Width (Microns)</th>
<th>Thickness (Microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88.2</td>
<td>20</td>
<td>0.6</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>20</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>20</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>88.2</td>
<td>20</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The cantilevers were modelled as rectangular blocks. In order to account for the rectangular geometry the cantilevers, finite element analyses (FEA) were performed using ANSYS 5.7 to obtain beam stiffness. The structure was meshed with SHELL63 elements. A Young’s modulus of 1000 GPa and a Poisson’s ratio of 0.07 were used as input parameters accounting for material properties. The simulated concentrated load was applied at the free-end at a node with 3 different forces, 0.5 mN, 1.0 mN, 2.0 mN.

Since the cantilever in ANSYS is modeled in 3-D, a large number of nodes was obtained. For the sake of simplicity and due to the symmetry of the cantilever block, only the surface nodes for the deflection values were chosen.

Moreover, due to symmetry of the cantilever, nodes found on the edge of the surface along the length, i.e., with coordinates: \( y = 0; \ z = -10 \, \mu m \), were chosen. Twelve nodes were chosen, one where there is zero deflection at the fixed end of the cantilever, one where is maximum deflection at the free end of the cantilever, and ten in between. For each node, its X-Coordinate on the cantilever surface and the corresponding Y-Displacement for three positions of the cantilever deflection was simulated in ANSYS.

2.2. Data Reduction

The values obtained from ANSYS are then imported into MATLAB where splines and quadratic polynomials are fitted to them. Then the equations describing the curves are obtained as well as the coefficients and errors of the structures.

To get the 3 positions of the cantilever deflection, three different forces were applied to the cantilever (0.5 mN, 1.0 mN, 2.0 mN).

Figure 1 Smoothing spline fit of third position of cantilever deflection for sample 1.

For the case of Sample 1, Position 3 is achieved by applying a force of 2.0 mN, and it results with a maximum deflection of \(-63.0 \, \mu m\). Any other position of the cantilever deflection will have a displacement less than \(-63.0 \, \mu m\) in magnitude. If we choose a state of cantilever deflection with displacement \(-15.7 \, \mu m\) (Position 1), this corresponds to the state of maximum deflection for a force of 0.5 mN. Similarly, another state with displacement \(-31.5 \, \mu m\) (Position 2) corresponds to the maximum deflection for a force of 1.0 mN. In this manner we can deduce several intermediate positions for a cantilever deflection with a force of 2.0 mN. By just applying a force less than 2.0 mN, we get an intermediate position.

The same procedure is followed for the other three samples, and in all cases, Position 3 corresponds to the maximum deflection of the cantilever.

2.3. Data Fitting

When the data from ANSYS is imported into MATLAB, the equations of the curves can be obtained as well as the errors and coefficients of the structures. For each sample, a spline and a quadratic polynomial are fitted into the three positions of cantilever deflection. Moreover, a spline is fitted to every 3 data points of each position and the corresponding coefficients and errors are deduced. For same coefficients, a quadratic polynomial is fitted and its coefficient and errors deduced.
### 2.4. Transition

The transition that a cantilever undergoes from Position 1 to position 2 and to position 3 is deduced. Moreover, by varying the thickness of the cantilever, the transition from sample 1 to sample 4 for the first position of cantilever deflection is deduced. Using these transitions, the equations of a cantilever of different thickness can be predicted and compared to the equation obtained from MATLAB. In this way, a technique for faster visualisation and modelling is developed.

### 3. RESULTS AND DISCUSSION

#### 3.1. Transition Matrix

The equations obtained from MATLAB for sample 1 are as follows:

**Position 1:**

\[ y = -0.0015x^2 - 0.05x + 0.2 \]  \hspace{1cm} (1)

**Position 2:**

\[ y = -0.003x^2 - 0.1x + 0.4 \]  \hspace{1cm} (2)

**Position 3:**

\[ y = -0.006x^2 - 0.2x + 0.8 \]  \hspace{1cm} (3)

The Transition matrix from position 1 to position 2 is found to be

\[
C = \begin{bmatrix}
0 & 0 & -0.0150 \\
0 & 0 & -0.5000 \\
0 & 0 & 2.0000
\end{bmatrix}
\]  \hspace{1cm} (4)

When the Transition Matrix, (4), is multiplied to position 2, (2), it gives (3). Therefore the Transition matrix (4) correctly describes the transition from position 1 to position 2, and from position 2 to position 3. Moreover, by multiplying equation (1) by 2 we get equation (2), which in turn gives equation (3) when multiplied by 2.

The Transition matrix from position 1 to position 3 is:

\[
D = \begin{bmatrix}
0 & 0 & -0.0300 \\
0 & 0 & -1.0000 \\
0 & 0 & 4.0000
\end{bmatrix}
\]  \hspace{1cm} (5)

We can see that \( D = 2C \), and by multiplying equation (1) by 4 we get equation (3). This means, once we know the equation describing the first position of the cantilever deflection, we can predict the equation of the next two positions without fitting the data into MATLAB.

#### 3.2. Coefficients and Errors

**Table 2. Coefficient and Errors from spline fitting for position 1.**

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Spline Fit for position 1 of Cantilever Deflection Errors</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSE</td>
<td>RMSE</td>
</tr>
<tr>
<td>1</td>
<td>0.00075</td>
<td>0.015</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.086</td>
</tr>
<tr>
<td>3</td>
<td>9.61E-06</td>
<td>0.002</td>
</tr>
<tr>
<td>4</td>
<td>0.004</td>
<td>0.035</td>
</tr>
</tbody>
</table>

**Table 3. Coefficient and Errors from spline fitting for position 2**

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Spline Fit for position 2 of Cantilever Deflection Errors</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSE</td>
<td>RMSE</td>
</tr>
<tr>
<td>1</td>
<td>0.004</td>
<td>0.034</td>
</tr>
<tr>
<td>2</td>
<td>0.066</td>
<td>0.14</td>
</tr>
<tr>
<td>3</td>
<td>5.03E-06</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.004</td>
<td>0.033</td>
</tr>
</tbody>
</table>

**Table 4. Coefficient and Errors from spline fitting for position 3**

<table>
<thead>
<tr>
<th>Sample #</th>
<th>Spline Fit for position 3 of Cantilever Deflection Errors</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSE</td>
<td>RMSE</td>
</tr>
<tr>
<td>1</td>
<td>0.013</td>
<td>0.062</td>
</tr>
<tr>
<td>2</td>
<td>0.262</td>
<td>0.279</td>
</tr>
<tr>
<td>3</td>
<td>1.75E-05</td>
<td>0.002</td>
</tr>
<tr>
<td>4</td>
<td>0.004</td>
<td>0.034</td>
</tr>
</tbody>
</table>
When splines are fitted to every 3 data points, and the coefficients of each segment obtained, the coefficient of the second and third segment are always the same. This heuristic holds for all 4 samples. When a quadratic polynomial is then fitted to the second and third segments, very low errors in the range E-35 to E-27 are obtained, which implies much better fits are obtained from quadratic polynomials compared to fits from splines.

Splines fitted to every 3 data points of each cantilever position for all 4 samples generate errors with a maximum of 1.186 and a minimum of 9.18E-10 for SSE and a maximum of 1.54 and minimum of 4.63E-05 for RMSE. For all segments, the SSE average from 8.5E-07 to 0.3 while RMSE average from 0.00083 to 0.4.

3.3. Predicting Equation

Using the quadratic equations of the first position of the cantilever deflection (thickness = 0.6 µm and 0.3 µm) we shall try to deduce the equation of the first position of the cantilever deflection with thickness = 0.15 µm. Here we keep length and width fixed, we vary only thickness.

For sample 1 (thickness = 0.6µm), the equation of the first position is given by equation (1).

For sample 2 (thickness = 0.3µm), the equation of the first position is:

\[ y = -0.0025x^2 - 0.1x + 0.1 \]  \hspace{1cm} (6)

The Transition matrix from (1) to (6), i.e., when the thickness is halved is

\[
T = \begin{bmatrix}
0 & 0 & -0.0125 \\
0 & 0 & -0.5000 \\
0 & 0 & 0.5000
\end{bmatrix}
\]  \hspace{1cm} (7)

Using this Transition matrix, we can deduce the equation of the first position of cantilever deflection with thickness = 0.15 µm, and the predicted equation is:

\[
y = \begin{bmatrix} 0 & 0 & -0.0125 \end{bmatrix} \begin{bmatrix} \text{equation (6)} \end{bmatrix} = -0.0013x^2 - 0.05x + 0.05
\]  \hspace{1cm} (8)

From MATLAB, when we fit a quadratic polynomial to the ANSYS data for the 0.15µm, the equation is

\[
Y = -0.0027x^2 - 0.13x + 0.02
\]  \hspace{1cm} (9)

With errors: SSE = 0.17 and RMSE = 0.14.

As seen from figure 6, there is a difference between the predicted curve, calculated from the transition matrix, and the actual curve, deduced from ANSYS. This difference stems from the fact that the data of the actual curve is obtained from ANSYS, which already has 10% error in its calculations. This difference can also be explained by the fact that the plots generated by MATLAB have quite large sum-squared errors (SSE) and root-mean-squared errors (RMSE).

Figure 4. Spline fit for the second 3 data points for sample 1 at position 3

Figure 5. Quadratic fit for the second 3 data points for sample 1 at position 3
A measure of the "goodness" of fit is the residual, the difference between the observed and predicted data is done and illustrated in figure 7. The residuals for the actual curve are more random than the residuals of the predicted curve. This means that we have a better fit with the predicted curve. Moreover, the predicted curve generates much smaller errors (SSE = 0.009; RMSE = 0.03) than the actual curve when fitted into MATLAB.

5. REFERENCES


Hsu, W., J. Hughes, and H. Kaufmann, Direct manipulation of free-form deformation, ACM Computer Graphics, 26(2), 1778-184, 1997


Espinosa, H.D., B.C. Porok, and B. Peng, Plasticity size effects in freestanding submicron FCC metal films subjected to pure tension, Department of Mechanical Engineering, Northwestern University, 2002.