Simulating the Response of Shallow Foundations using Finite Element Modelling

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Abstract: In 1994, the Federal Highway Administration held a symposium to recognise the various techniques for predicting settlements of shallow foundations. Thirty one predictions were received for five spread footings, sixteen of these estimates were made by academics and consultants gave the remaining fifteen. Each of the participants were allowed access to a vast amount of site test data and were required to give predictions for the loads which would result in the footing settling 25 mm and 150 mm. It was noted that most participants preferred to use traditional semi-empirical methods to predict the load for 25 mm settlement. However the load corresponding to 150 mm proved to be difficult, as traditional methods do not compensate for the highly plastic strain region of the load-displacement behaviour. Eight of the participants chose to use finite element modelling to simulate the constitutive behaviour of the soil profile. All eight models devised by the estimators consisted of eight noded axisymmetric elements. Even though all meshes were constructed in a similar fashion all participants gave varying results ranging from 0.97 to 12.63 times the true load measured in the field. One of the main reasons behind the varying results was due to the input parameters placed into the models. This paper addresses the effect on the simulations accuracy due to the constitutive behaviour chosen of the soil, and whether the simplification of the model from 3D space into 2D was reasonable. Conclusions from the research showed that the model was highly dependent on the correlations used to determine soil properties, the simulations geometry and the constitutive models that define elastic-plastic behaviour.

Keywords: *ABAQUS; Shallow foundation, Finite element model; Settlement*

1. INTRODUCTION

Shallow foundations are the integral part of a structure that transmits the load directly to the underlying soil. Generally, foundations are considered to be shallow when the depth is less than approximately three metres, or less than the breadth of the footing. When designing shallow foundations two things must be considered: the bearing capacity of the soil and the total settlement.

Estimating settlements of shallow foundations in granular soils is prone to many uncertainties (Sivakugan and Johnson 2002). Numerous methods have been developed to predict the settlement of shallow foundations in granular soils. One common assumption in each method is that the sand is represented as having elastic strains only, and plastic deformations are not taken directly into account. With expanding technology and development of sophisticated numerical methods such as finite element method (FEM), a debate into the constitutive behaviour of sand has come to the forefront. The constitutive behaviour governs the response of sand under the foundation and therefore influences the estimates of bearing capacity and settlement.

The Federal Highway Administration in 1994 held a predicting symposium to recognise the various techniques for predicting settlements of shallow foundations. It was felt this symposium would be the best way to evaluate current industry and academic procedures into design of spread footings. The participants were required to predict the load corresponding to 25 mm and 150 mm settlement of five shallow foundations. A complete set of site test data obtained from several testing devices was available for the predictors.

The work reported herein utilises the finite element computer package ABAQUS. A 3D model was constructed to obtain a load-deflection curve for each of the five footings. It is shown through the numerical modelling that the empirical correlations used to obtain constitutive parameters, and constitutive models themselves have a substantial effect on the model results.

2. SYMPOSIUM INSITU PROPERTIES

The five square footings constructed on a sandy site ranged in size from 1 m to 3 m and were loaded vertically at their centres. Each of the participants of the symposium received the results from a detailed geotechnical investigation that was undertaken at the site. However, the load-displacement results from the footings themselves were not available until the required estimates were submitted to the prediction committee. A general soil profile for the entire site, along with the tests carried out on site and the footing geometry have been summarised in Table 1.

The shallow foundations were constructed with ample distance between them to ensure minimum overlap of pressure bulbs under each footing. Soil testing was completed by several geotechnical firms and took place at various stages before and after the load tests were completed.

It was noted by several predictors that the load tests on the footings were conducted in the middle of a dry season, while the some of the soil tests were carried out in a relatively wet season. This led to some guesswork on behalf of the predictor when trying to estimate some of the constitutive parameters. The soil properties induced by the wet season tests may lead to weaker sand properties, than those under the footing at the time of the load test.

Table 1: Footing sizes, general soil profile and soil test.

Footing			*Soil Test Data Available*
1	3∗ 3m	**0 m** *Medium Dense Silty Fine Sand*	• Borehole shear Test • Cross Hole Wave Test • Piezo-Cone Penetration Test
2	1.5∗ 1.5 m	**3. 5 m** *Medium Dense Silty Sand W/Clay and Gravel* ▽ **4.9 m**	• Dilation Test • Dilatometer Test w/Thrust Measurement
3	3∗ 3 m	**7 m** *Medium Dense Silty Sand to Sandy Clay with Gravel*	• Pressure Meter Test • Step Blade Test • Standard Penetration Test
4	2.5∗ 2.5 m	**11 m**	• Water Content & Unit Weight • Atterberg Limits
5	1∗ 1 m	*Very Hard Dark Clay* **33 m**	• Relative Density • Triaxial Test • Resonant Column Test

3. NUMERICAL MODEL

The shallow foundations were square in shape and due to this geometry modelled in a 3D space. To reduce the problem size lines of symmetry (LOS) were utilised as shown in Figure 1.

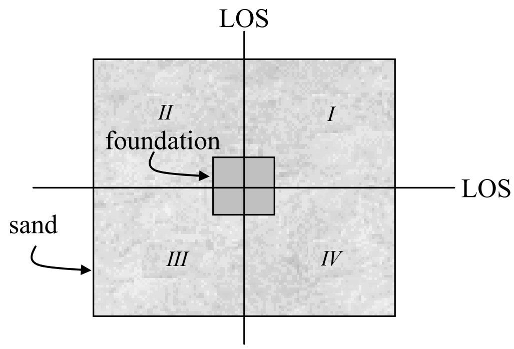

Figure 1: Lines of symmetry for square footing.

The lines of symmetry allow the full 3D problem to be reduced to quarter of its original size. On either side of the LOS the sand has an equal response. Movement normal to the LOS is equal and opposite resulting in zero displacement. However movement tangent to the LOS is present and equal on both sides. Through the use of boundary restraints these symmetry conditions can be imposed onto the model allowing a reduction in problem size.

3.1 Mesh Construction

Three rectangular meshes were constructed with varying node densities. An example of one of the meshes is shown below in Figure 2.

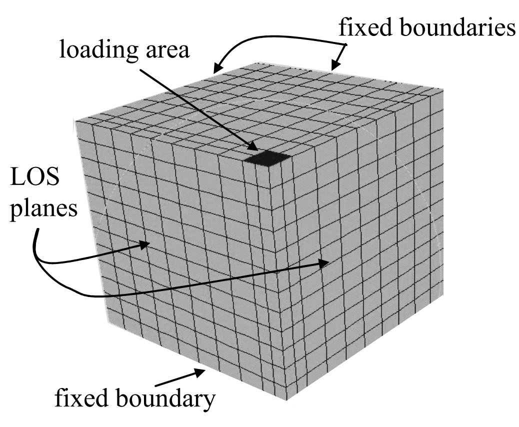

Figure 2. Footing mesh using ABAQUS.

The general characteristics of the meshes are as follows:

- *Element Type:* due to the footing shape the problem was discretized using 3-D finite elements. Twenty noded reduced rectangular prisms were also utilised. These rectangular prisms have parabolic shape functions to approximate the displacement pattern between nodes.
- *Boundary Conditions:* two separate boundary conditions were imposed onto the models. Due to symmetry the problem space could be reduced into a quarter of its original size. The nodes at the base of the mesh and far bounds are fixed against displacement. The nodes on the plane of symmetry are restrained from moving normal to the plane, but can move along the plane in horizontal and vertical directions, as seen in Figure 2.
- *Input Data:* the footing was assumed to be a rigid element used only as a means of delivering the load into the sand. Therefore, the model could then be simplified such that only the body of sand needed to be modelled. Displacement constraints were used to simulate the behaviour of the sand at the footing and soil interface. The sand contact nodes were restrained to allow for the rigid characteristics of the concrete footing. The interface nodes were permitted to move vertically but not horizontally. Differential settlements under the footings were assumed to be insignificant due to the rigidity of the foundation. Therefore, the contact nodes were restrained so that the vertical displacement was identical under a given load.
- It was assumed that the sand behaved linear elastically until a plastic failure envelope was reached. Once the stress state at any location reaches the failure surface it will undergo plastic deformation. These constitutive characteristics and an initial stress state were imposed on the model before the analysis was started. The initial stress accounted for the insitu stresses due to self-weight of the system before the load is applied.

3.2 Verification

To verify the reliability and sensitivity of the FEM shallow foundation meshes, load-deflection curves from the models were compared against Terzaghi and Peck's (1948) pressure curves. In a controlled experiment Terzaghi and Peck load tested 300 mm square plates. Load-displacement curves were established for the square plates in three separate sand densities. The sand densities corresponded to a constant penetration blow counts (N) of 10, 30 and 50. Where the magnitude of the blow count represents the sand's strength (i.e. increased blow count gives higher stiffness).

Using the standard penetration blow count data and existing empirical correlations, the friction angle (Peck et al., 1974) and Young's modulus (Das, 1999) of the sand could be estimated. The sand was assumed to be homogenous layer with the sand properties given in Table 2.

Table 2: Sand properties.

Blow Count	Friction Angle (°)	Young's Modulus (MPa)	Poisson's Ratio
10	30	8	0.3
30	36	24	0.3
50	41	40	0.3

A basic Mohr-Coulomb plastic failure envelope was used to define the constitutive behaviour during the verification process. This failure envelope is shown in Figure 3.

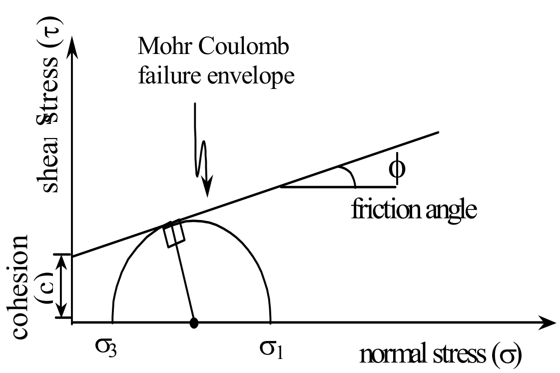

Figure 3. Mohr Coulomb's failure surface.

The failure envelope is only dependent on the major and minor principle stresses (σ_1, σ_3), and is independent of intermediate principle stress (σ_2) (Chen and Saleeb, 1983)

Three mesh densities and fixed boundary locations were trailed until a converged numerical solution was achieved. The sensitivity plot for the N=50 case is shown in Figure 4. The plot shows the displacement versus number of nodes for both the numerical model and the experimental 300 mm plate.

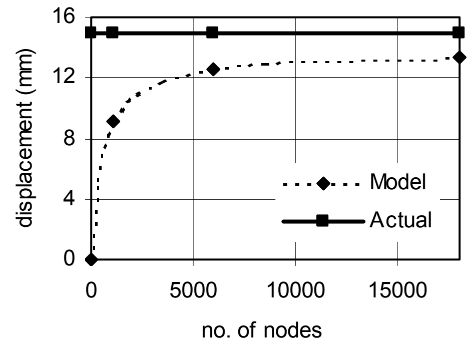

Figure 4. Sensitivity plot for N=50.

The final converged solutions for the load-deflection curve for the N=50 for each of the mesh densities are shown Figure 5.

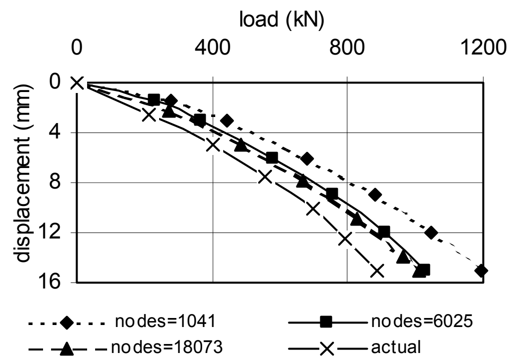

Figure 5. Load-deflection curve for N=50.

From figures 4 and 5 it can be shown that the relative difference between the output from the meshes with 6025 and 18073 nodes was 6%. However computational time also increased by 50% between these node limits. It was then considered the mesh containing 6025 nodes behaved with reasonable accuracy and acceptable computational time. The 6025 noded mesh was therefore used for the remaining analyses.

4. NUMERICAL PREDICTION FOR FEDERAL HIGHWAY ADMINISTRATION

The Federal Highway Administration (FHA) tested five shallow foundations of various sizes. Unlike Terzaghi and Peck's experiments where the site conditions were controlled the FHA tests were conducted in undisturbed soil. This introduces a realm of uncertainty into the settlement prediction process. Soil is a non-homogenous material that often exhibits anisotropic behaviour. Therefore, assumptions and simplifications must be made to obtain a rational prediction for the foundation's settlement. It is these simplifications that can affect the accuracy of a settlement prediction method.

4.1 Empirical Correlations

One of the major objectives for the FHA study was to predict the load corresponding to 25 mm and 150 mm settlement, given a vast amount of site test data. Using the model developed in section 3 the reliability of some common field tests and empirical correlations could be explored. The numerical model's reliability is dependent on the quality of the field tests, and the empirical correlations used to derive the constitutive parameters of the soil.

From the bore logs it is seen that the test site is predominately sand with traces of clay. The soil was broken into homogenous layers with varying properties. The layering based on standard penetration test (SPT), and cone penetration test (CPT) are given in Table 2 for the two 3 m foundations.

Table 2: Sand properties.

Footing	Blow Count (N_{60})	Cone Resitance (q_c) [MPa]
3m north	21 (4.5)	5.8 (3.95)
	16 (3.9)	3.8 (4.19)
	56 (6.0)	13.4 (2.86)
3m south	28 (5.4)	3.8 (1.22)
	15 (3.0)	0.6 (2.67)
	56 (6.0)	5.1 (2.44)
	N/A	6.4 (4.67)

Note: number in brackets represents the thickness of the layer in meters. The first row of numbers for each foundation is the first layer directly under the footing.

Two empirical correlations relating the blow count to the elastic and Mohr-Coulomb parameters were explored. These correlations are as followed:

- corrected blow count $(N_1)_{60}$ used to obtain friction angle ϕ developed by Hatanaka and Uchida (vide Das, 1999); and
- corrected blow count $(N_1)_{60}$ used to obtain friction angle ϕ based on Tomlinson (1991) charts.

The friction angle controls the slope of the Mohr-Coulomb failure envelope as shown in figure 3. Therefore this has a direct impact on whether the displacement associated with a node is undergoing elastic or plastic strain. The empirical correlations above will utilise the SPT data to determine the friction angle.

The empirical correlation used to determine a value for Young's modulus (E) also plays an important role. Through the use of the elastic modulus the stress can be related to the strain linearly. There are several common empirical correlations to approximate the Young's modulus of sand. The uses of these correlations often depend on the type of soil tests conducted. This paper will investigate only the following two correlations.

- $E = 8^* N_{60}/10$ [MPa] (Das 1999). This correlation relates the blow count (N_{60}) to E and requires SPT data.
- $E = 2.5^*q_c$ (Schmertmann 1978). This correlation relates the cone resistance (q_c) to E and requires CPT data.

Using the correlations previously stated, the load-displacement curves were constructed and plotted against actual test results for the 3m foundations. These plots can be seen in Figure 6 and 7.

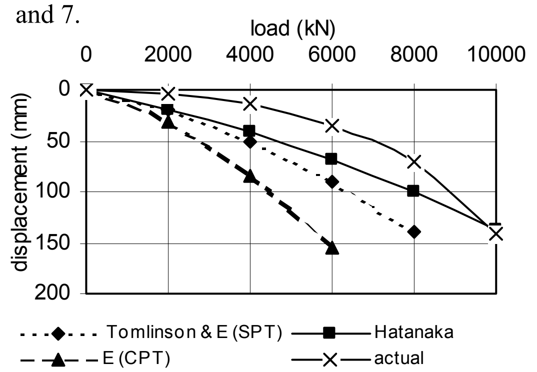

Figure 6. Load-deflection curve for 3 m north footing.

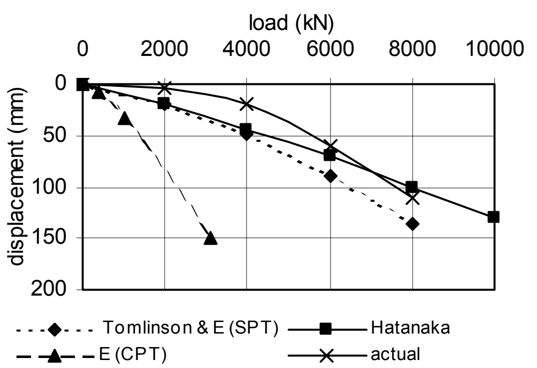

Figure 7. Load-deflection curve for 3 m south footing.

5. DISCUSSION

5.1 Comparison of Empirical Correlations

The shape of the curves in figures 6 and 7 are dependent only on the input soil properties placed into the model. The model geometry was identical for both cases as the foundations were identical in size. Ideally the results produced from each of the different correlations should be equal to the actual test value. However, a degree of scatter can be seen between each of the correlations.

The percentage error between the actual and predicted varied from one foundation to another. For example: the average percentage error for the Tomlinson correlation was 66% for the 3 m North foundation, while 3 m South footing was 43%. The difference in the accuracy of prediction may be attributed to the soil test data. Due to the non-homogenous nature of soil a particular soil test can at best only represent the immediate local area. Therefore, the soil's profile could vary as the distance from the test location is increased.

Figures 6 and 7 indicate that Tomlinson's correlation gives a more conservative solution, which can give a good approximation, provided the test data available is of good quality. Hatanaka and Uchida on the other hand produced results which were conservative for the elastic range but risky as the load increased towards the plastic limit. The correlations involving Young's modulus depended on the quality of the SPT and CPT tests. The 3 m north footing indicated that the SPT and CPT were in agreement with each other. However, for the 3 m south footing there was a large difference between the two correlations. The CPT test conducted did not represent the true soil profile, and this in turn affected the numerical results considerably.

5.2 Comparison of Model to Federal Highway Administration

The Federal Highway Administration found that the 31 predictors used a total of 22 settlement prediction techniques. Of those 22 techniques finite element method (FEM) was the third most popular choice. The 8 predictors who modelled the foundations in FEM all used a 2D axisymetric model with an equivalent load area to that of the square footings. Again the results were scattered varying from 0.97 to 12.63 times the actual value. The average prediction for the five foundations made by the 3D square model developed in this paper was 1.6 times the actual value. The differences between the FEM models stemmed mainly from the constitutive parameters chosen to represent the sands behaviour. From the work completed in section 4 it was found that the choice of parameters could have a major impact on the performance of the model.

6. CONCLUSION

When modelling a shallow foundation in the field it is important to include all the major factors attributing to the footing's behaviour. The constitutive parameters, stress history and the interactions between elements are all key aspects to consider. The work in this paper showed that a numerical prediction is as good as the input data placed into the model. In a controlled environment like the one created by Terzaghi and Peck, numerical models can be accurate. However in the field randomness and uncertainty are factors that must be accounted for and addressed. Numerical models can offer a great deal of information when used correctly, and they can provide a platform for developing design procedures for industry.

7. ACKNOWLEDGMENT

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