

# Forecasting Portfolio Value-at-Risk for International Stocks, Bonds and Foreign Exchange

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## EXTENDED ABSTRACT

The usefulness of economic and financial models is often evaluated by their abilities to provide accurate forecast. In the volatility literature, the forecast performances of conditional volatility models are often measured by the accuracy of their Value-at-Risk (VaR) forecasts. This paper evaluates the VaR forecast performances for four different multivariate conditional volatility models using six different portfolios of assets involving a total of four different countries, with each portfolio includes two bonds, two stocks and two exchange rates from two different countries.

The paper aims to find out whether incorporating volatility spillovers in the model provides better VaR forecast. This is investigated by estimating the BEKK model of Engle and Kroner (1995). The paper also aims to find out whether considering conditional correlation is important in achieving better VaR forecast. This is investigated by estimating the DCC model of Engle (2002). Two other models will also be estimated to make the comparisons, namely the CCC model of Bollerslev (1990) and the Diagonal VECH (DVECH) model of Bollerslev et al. (1988).

The data of bond, stock, and foreign exchange rate are obtained from DataStream database services, from 5/5/1997 to 1/5/2007. The countries of interest are Australia, Japan, New Zealand, and Singapore. Having the coefficients for the estimated models, the models are used to forecast one-day-ahead VaR thresholds. In order to strike a balance between efficiency in estimation and a viable number of forecasts, the sample size used for estimation is from 5/5/1997 to 31/12/2004 with 2000 observations, and the forecasting period is from 3/1/2001 to 1/5/2007 with 607 observations.

The VaR forecasts will be evaluated by using four different statistical tests. The first test is the Unconditional Coverage test of Christoffersen

(1998) to see if the actual number of violation matches the theoretical expectation. The second test is the Independence test to see if the violations are independent from each other. Joining the two test statistics forms the third test, which is the Conditional Coverage test. The paper also applies Test Until the First Failure of Kupiec (1995).

The test results suggest the importance of incorporating volatility spillovers in forecasting VaR. The results also suggest that incorporating dynamic conditional correlation is of little helps in improving VaR forecasts. The results also find that the use of  $t$  distribution, as an alternative of normal distribution, generally leads to too few violations.

## 1. INTRODUCTION

VaR can be viewed as the latest step in the evolution of risk-management tools. It can summarize the worst portfolio loss related to the trading of financial assets over a given time period with a given level of confidence. Even though VaR should be viewed as a necessary but not sufficient condition procedure for controlling risk, it has been used as a standard tool for risk managers (see Jorion (2000) for further discussion about VaR).

The development of GARCH-family models has led to the development of conditional VaR models. The paper intends to estimate various multivariate GARCH-type models to forecast VaR. Three advantages of estimating multivariate GARCH models are the possibility of estimating the conditional covariances between assets, incorporating the interaction across those assets, and considering the conditional correlations across those assets. These might improve the VaR forecasts accuracy of a portfolio comprising these components.

Three assets are to be considered, namely stock, bond and foreign exchange. Bonds are important

in portfolio construction for several reasons. First, long-term government bond returns can explain the cross-sectional variation in portfolio risk premia (see Chen, Roll, and Ross (1986)). Second, many instrumental variables, such as short-term T-bill yields, can forecast stock and bond returns very well (see, for example, Campbell (1987), and Fama and French (1989)).

The issue of the currency risk associated with foreign bonds is addressed in Odier and Solnik (1993). They show that the contribution of exchange rates to the riskiness of bonds is much larger than for stocks. This result arises from the negative correlation between the stock price and currency value, and the positive correlation between bond price and currency value.

Four multivariate GARCH-type models to be considered are the DVECH models of Bollerslev et al. (1988), the BEKK model of Engle and Kroner (1995), the CCC model of Bollerslev (1990), and the DCC model of Engle (2002). The paper considers various tests of Unconditional Coverage (CC), Independence (IND), and Conditional Coverage (CC) of Christoffersen (1998), and Test Until First Failure (TUFF) of Kupiec (1995) regarding the violation of VaR resulted by the models.

The rest of the paper will be organized as follows. Chapter two provides a concise literature review on volatility forecasting. Chapter three discusses the various multivariate GARCH models to be used in this paper, as well as the methods for comparing the forecasting accuracy of these models. Chapter four provides the background on the empirical data, and Chapter five reports the empirical results. Concluding remarks can be found in Chapter six.

## 2. LITERATURE REVIEW

Research on modeling and forecasting VaR can be viewed from two points of views. First is the development of conditional volatility models. This will include the discussion of aggregation level, namely aggregated (portfolio level) and disaggregated (asset level), or the use of high and low frequency data and the associated issue of parametric vs. nonparametric volatility measurement (see Andersen et al. (2005)). Second is the development of testing the VaR forecasts. To date, two hypothesis-testing methods for evaluating VaR forecasts have been proposed: the binomial method, currently the quantitative standard embodied in the Market Risk Amendment (MRA), and the interval forecast method proposed by Christoffersen (1998).

Modelling and forecasting VaR of portfolio consists of stock, bond and foreign exchange has been undertaken by several papers. Wong et al. (2003) investigate the VaR forecast of Australia's All Ordinary Index (AOI) using univariate ARCH and GARCH models. They find that those models fail to pass the Basle's backtesting criteria.

McAleer and da Veiga (2004) propose a new model, Portfolio Spillover GARCH (PS-GARCH) model, to forecast VaR. They compared the forecast performance of the PS-GARCH model with two competing alternatives, namely, VARMA-GARCH model of Ling and McAleer (2002) and the CCC model of Bollerslev (1990). They showed that the inclusion of spillover did not improve the forecast accuracy of VaR significantly, even when the spillover effects are statistically significant.

Sadorsky (2005) provides VaR forecast of stock, bond, foreign exchange and oil price using stochastic volatility model. The forecasts are evaluated using various test statistics such as the unconditional coverage, independence and conditional coverage. The result does not reject the independence of violations, but does reject both the unconditional and conditional coverage. This reflects the difficulty in forecasting VaR accurately.

Asai and McAleer (2007) propose Portfolio Index GARCH (PI-GARCH) model of conditional volatility. Conducting Monte Carlo experiment, they find that PI-GARCH outperforms the DCC model in forecasting VaR thresholds.

Angelidis et al. (2007) analyses parametric models (GARCH-type models), semi-parametric models (Filtered Historical Simulation and Extreme Value Theory), and non-parametric model (Historical Simulation) by examining their ability to forecast VaR for stock indices. Using unconditional coverage test and some loss function to test the forecasts accuracy, they find that the semi parametric model outperforms the other models.

This paper investigates the potential contributions of multivariate GARCH models to VaR forecast using different portfolios consist of stocks, bonds and exchange rates.

## 3. METHODS

VaR at level  $\alpha$  for returns  $y_t$  is the corresponding empirical quantile at  $(1-\alpha)$ . Because quantiles are direct functions of the

variance in parametric models, GARCH-class models immediately translate into conditional VaR models.

For random variable  $y_t$  with the conditional variance follow univariate GARCH specification,

$$y_t = E(y_t|F_{t-1}) + \varepsilon_t \quad (1)$$

$$\varepsilon_t = \eta_t \sqrt{h_t}$$

$$h_{it} = \omega_i + \sum_{l=1}^r \alpha_l \varepsilon_{i,t-l}^2 + \sum_{l=1}^s \beta_l h_{i,t-l}, \quad (2)$$

the VaR threshold for  $y_t$  can be calculated as:

$$VaR_t = E(y_t|F_{t-1}) - z\sqrt{h_t}, \quad (3)$$

where  $z$  is the critical value from the distribution of  $\varepsilon_t$  to obtain the appropriate confidence level. Alternatively,  $h_t$  can be replaced by estimates of various GARCH-family models to obtain an appropriate VaR.

To investigate whether accommodating comovement among and interactions across assets in the conditional variance can improve the forecasts of VaR, four multivariate GARCH models will be estimated. The models are the DVECH models of Bollerslev et al. (1988), the BEKK model of Engle and Kroner (1995), the CCC model of Bollerslev (1990), and the DCC model of Engle (2002).

Bollerslev et al. (1988) propose VECH model to model the covariance matrix of a multivariate GARCH model. The VECH model suffers from a common problem associated with multivariate GARCH models, namely the curse of dimensionality. The model also requires further parametric restrictions to ensure the positive definiteness of the estimated covariance matrix. To reduce the number of parameters, Bollerslev et al. (1988) suggest the DVECH model. However, the model does not incorporate the spillovers across assets. The BEKK model of Engle and Kroner (1995) resolves the positive definiteness issue and incorporates spillover effects; it did not resolve the problem associated with the curse of dimensionality.

These multivariate GARCH models focus on the dynamic of the conditional covariance matrix, whereas models such as the CCC model of Bollerslev (1990) and the DCC model of Engle (2002) focus on the dynamic of the conditional variances and the conditional correlation matrix.

The specification of the VECH model is:

$$y_t = E(y_t|F_{t-1}) + \varepsilon_t \quad (4)$$

$$\varepsilon_t = D_t \eta_t \quad (5)$$

$$vech(H_t) = C + \sum_{i=1}^q A_i vech(\eta_{t-1} \eta_{t-1}') + \sum_{j=1}^p B_j vech(H_{t-j}) \quad (6)$$

where  $y_t = (y_{1t}, \dots, y_{mt})'$ ,  $\eta_t = (\eta_{1t}, \dots, \eta_{mt})'$  is a sequence of identically and independently (i.i.d) random vectors,  $F_t$  is the past information available to time  $t$ ,  $D_t = \text{diag}(h_{1t}^{1/2}, \dots, h_{mt}^{1/2})$ ,  $m$  is the number of returns, and  $t=1, \dots, n$ ,  $H_t = (h_{1t}, \dots, h_{mt})$ ,  $vech(\cdot)$  denotes the column stacking operator of the lower portion of a symmetric matrix,  $C$  is a  $\frac{1}{2}N(N+1) \times 1$  vector,  $A_i, i=1, \dots, q$ , and  $B_j, j=1, \dots, p$ , are  $\frac{1}{2}N(N+1) \times \frac{1}{2}N(N+1)$  matrices. The DVECH model is obtain by taking the main diagonal of matrices  $A$  and  $B$  in (6).

The CCC model of Bollerslev (1990) assumes that the conditional variance for each return,  $h_{it}, i=1, \dots, m$ , follows a univariate GARCH process, namely

$$h_{it} = \omega_i + \sum_{j=1}^r \alpha_{ij} \varepsilon_{i,t-j} + \sum_{j=1}^s \beta_{ij} h_{i,t-j}, \quad (7)$$

where  $\alpha_{ij}$  represents the ARCH effect, or the short-run persistence of shocks to return  $i$ , and  $\beta_{ij}$  represents the GARCH effect, of the contribution of shocks to return  $i$  to long-run persistence, namely

$$\sum_{j=1}^r \alpha_{ij} + \sum_{j=1}^s \beta_{ij} < 1. \quad (8)$$

The conditional correlation matrix of CCC is  $\Gamma = E(\eta_t \eta_t' | F_{t-1}) = E(\eta_t \eta_t')$ , where  $\Gamma = \{\rho_{ij}\}$  for  $i, j=1, \dots, m$ . From (5),  $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_t' D_t$ ,  $D_t = (\text{diag} Q_t)^{1/2}$ , and  $E(\varepsilon_t \varepsilon_t' | F_{t-1}) = Q_t = D_t \Gamma D_t$  where  $Q_t$  is the conditional covariance matrix. The conditional correlation matrix is defined as  $\Gamma = D_t^{-1} Q_t D_t^{-1}$ , and each conditional correlation coefficient is estimated from the standardized residual in (4) and (7).

The conditional covariance of BEKK model can be written as follows:

$$Q_t = QQ' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B'. \quad (9)$$

The DCC model is given by:

$$Z_t = (1 - \theta_1 - \theta_2)\bar{Z} + \theta_1\eta_{t-1}\eta'_{t-1} + \theta_2Q_{t-1} \quad (10)$$

$$\Gamma_t^* = \left\{ (\text{diag}Z_t)^{-1/2} \right\} Z_t \left\{ (\text{diag}Z_t)^{-1/2} \right\}, \quad (11)$$

where  $\theta_1$  and  $\theta_2$  are scalar parameters, and  $Z_t$  is the conditional correlation matrix after it is standardized by (11). For further detail about multivariate GARCH models, see McAleer (2005).

To evaluate the VaR forecasts accuracy, several back tests will be used, namely tests of unconditional coverage (UC), independence (IND), conditional coverage (CC), and test until first failure (TUFF). The UC test was first proposed by Kupiec (1995). The test examine whether the failure rate of a model is statistically different from expectation. Later Christoffersen (1998) derived likelihood ratio (LR) of UC, IND and CC.

In UC test, the probability of observing  $x$  violations in a sample of size  $T$ , is given by:

$$\Pr(x) = C_x^T (f)^x (1-f)^{T-x} \quad (12)$$

where  $f$  is the desired proportion of observations.  $C_x^T = \frac{T!}{x!(T-x)!}$  where ! denotes

the factorial operator such that  $T! = \prod_{i=0}^{T-1} T-i$ . The

null hypothesis is that the empirical failure rate,  $\hat{f}$ , is equal to the confidence level of the VaR,  $\alpha$ . The LR statistic of UC is:

$$LR_{UC} = 2 \log \left[ \frac{(1-\alpha)^{n_0} \alpha^{n_1}}{(1-\hat{f})^{n_0} \hat{f}^{n_1}} \right], \quad (13)$$

where  $\hat{f} = x/T$ ,  $n_0$  is the number of failures and  $n_1$  is the number of success. The statistic is distributed as  $\chi^2$  with 1 degree of freedom.

The weakness of UC test is that it tests only the equality between the VaR violations and the confidence level. However, simply testing for the correct unconditional coverage is insufficient

when dynamics are present in the higher-order moments. Therefore it is also important that the VaR violations are not correlated in time. The LR statistic of Christoffersen (1998) for testing whether the series are independent is:

$$LR_{IND} = -2 \log \left[ \frac{(1-\hat{f})^{n_{00}+n_{10}} \hat{f}^{n_{01}+n_{11}}}{(1-\hat{f}_{01})^{n_{00}} \hat{f}_{01}^{n_{01}} (1-\hat{f}_{11})^{n_{10}} \hat{f}_{11}^{n_{11}}} \right], \quad (14)$$

where  $n_{ij}$  is the number of observation with value  $i$  followed by  $j$ . The statistic is distributed as  $\chi^2$  with 1 degree of freedom.

The joint of unconditional coverage and independence tests are the conditional coverage test, with the following LR statistic:

$$LR_{CC} = LR_{UC} + LR_{IND}. \quad (15)$$

The statistic is distributed as  $\chi^2$  with 2 degree of freedom.

TUFF of Kupiec (1995) is based on the number of observations until a failure is recorded, which is important in a performance-based verification scheme. The null hypothesis is the same as the UC test, namely the empirical failure rate,  $\hat{f}$ , is equal to the confidence level of the VaR,  $\alpha$ . Given  $v$ , the number of days until the first failure occurs, it tests whether the underlying potential loss estimates are consistent with the null. Therefore, the null can be further set to  $H_0 = \alpha = \hat{f} = 1/v$ . The LR statistic, which follows  $\chi^2$  with 1 degree of freedom, is as follows:

$$LR_{TUFF} = -2 \log \left[ \frac{\hat{f}(1-\hat{f})^{v-1}}{\frac{1}{v} \left(1 - \frac{1}{v}\right)^{v-1}} \right]. \quad (16)$$

#### 4. DATA ANALYSIS

The data used in the paper are the daily closing price index of bond, stock, and foreign exchange rates from Australia, Japan, New Zealand, and Singapore. All the data are obtained from the DataStream database services. The sample ranges from 5/5/1997 to 1/5/2007, with 2,607 observations for each asset. The returns of market

$i$  at time  $t$  are calculated as  $R_{i,t} = \log(P_{i,t} / P_{i,t-1})$ , where  $P_{i,t}$  and  $P_{i,t-1}$  are the closing prices of asset  $i$  for days  $t$  and  $t-1$ , respectively. All returns are found to be stationary, based on both ADF and Phillips-Perron tests.

## 5. ESTIMATION AND FORECAST

In this section, the forecasting performance of the various models described in the previous section is compared. For purposes of the empirical analysis, it is assumed that the portfolio weights are equal and constant over time, but this assumption can be relaxed. Exchange risk is controlled by converting all prices to a common currency, namely the US dollar. The models described in Section 3 are used to estimate the conditional variances. All the conditional volatility models are estimated under the assumption of normal and  $t$  distribution.

The estimated models are used to forecast 1-day ahead 99% VaR thresholds. The sample ranges from 5/5/1997 to 1/5/2007, with 2,607 observations for each index and foreign exchange rates. In order to strike a balance between efficiency in estimation and a viable number of forecasts, the sample size used for estimation is from 5/5/1997 to 31/12/2004 with 2000 observations, and the forecasting period is from 3/1/2001 to 1/5/2007 with 607 observations.

From four countries investigated, there are six portfolios to be considered. Each portfolio contains two bonds, two stocks and two foreign exchange rates, since the exchange rates use US dollar as the benchmark.

As there are six portfolios with six variables each, and assuming two type of distribution, the paper will not report all the coefficients due to page number constraints. However, it can be reported that most of the coefficients are statistically significant. There is evidence of volatility spillovers across assets provided by the BEKK models. There is also evidence of varying conditional correlation provided by the DCC model (see Table 1).

With 95% confidence level, the critical value of chi-square for  $LR_{UC}$ ,  $LR_{IND}$  and  $LR_{TUFF}$  are 3.84 (6.64), while that of  $LR_{CC}$  is 5.99.

The results from the UC, IND and CC tests are given in Table 2. For the VaR forecasts under normal distribution, only DVECH model fails the test, namely in two of the six portfolios, the VaR

forecasts lead to excessive violations. As DVECH model does not incorporate the spillovers across assets, the result might indicate the importance of incorporating such spillovers in forecasting VaR. However, the results might not be strong as the DVECH model does not fail the test after considering the independence test, namely test of conditional coverage.

The VaR forecasts calculated using a  $t$  distribution show that, for the UC tests, DVECH model fails in 4 cases, CCC and DCC models fail in 3 cases, and BEKK model fail in 2 cases. The fail of the tests are due to the existence of too few violations. All models do not fail the IND tests, but the results for the CC tests are the same as those of the UC tests. Again, the results suggest that incorporating volatility spillovers might be important in forecasting VaR. Furthermore, since the CCC and DCC models show the same performance in forecasting VaR, despite the significance of dynamic conditional correlation, it seems to suggest that modeling the dynamic conditional correlation does not contribute substantially to VaR forecast accuracy.

The results of TUFF, based on both normal and  $t$  distribution, suggest that all models perform well. This means that the empirically determined probability matches the given probability.

## 6. CONCLUSION

This paper forecasted VaR for six portfolios consist of two pairs of countries, resulted from 4 countries investigated. Each portfolio includes two bonds, two stocks and two exchange rates from the pairs of countries. The paper investigated four different types of multivariate GARCH models, namely the DVECH model of Bollerslev (1998), the BEKK model of Engle and Kroner (1995), the CCC model of Bollerslev (1990), and the DCC model of Engle (2002).

Based on several back tests, namely UC, IND and CC of Christoffersen (1998), the paper found that incorporating Volatility spillovers might help in obtaining better VaR forecast, while considering dynamic conditional correlation is of little help. The result of TUFF suggests that the empirically determined probability matches the given probability.

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APPENDIX

Table 1. DCC Model: Coefficients of Conditional Correlation Equation

Pairs of Countries	Normal Distribution		<i>t</i> Distribution	
	$\theta_1$	$\theta_2$	$\theta_1$	$\theta_2$
Australia-Japan	<b>0.130</b>	0.000	<b>0.163</b>	<b>0.970</b>
	<b>(26.760)</b>	(1.000)	<b>(5.136)</b>	<b>(132.895)</b>
Australia-NZ	<b>0.015</b>	<b>0.979</b>	<b>0.013</b>	<b>0.983</b>
	<b>(3.552)</b>	<b>(126.294)</b>	<b>(19.565)</b>	<b>(603.835)</b>
Australia-Singapore	<b>0.016</b>	<b>0.975</b>	<b>0.017</b>	<b>0.740</b>
	<b>(4.176)</b>	<b>(126.792)</b>	<b>(5.020)</b>	<b>(160.450)</b>
Japan-NZ	<b>0.024</b>	<b>0.952</b>	<b>0.017</b>	<b>0.967</b>
	<b>(5.506)</b>	<b>(82.231)</b>	<b>(6.107)</b>	<b>(125.885)</b>
Japan-Singapore	<b>0.0135</b>	<b>0.810</b>	<b>0.013</b>	<b>0.982</b>
	<b>(4.870)</b>	<b>(221.23)</b>	<b>(4.581)</b>	<b>(213.617)</b>
NZ-Singapore	0.013	<b>0.981</b>	<b>0.009</b>	<b>0.989</b>
	(1.777)	<b>(74.181)</b>	<b>(6.264)</b>	<b>(410.444)</b>

Entries in **bold** are significant at the 95% level.

Entries in brackets are the corresponding *t* ratios of the coefficients.

Table 2: Tests of VaR Thresholds

Pairs of Countries	Models	Normal Distribution					<i>t</i> Distribution				
		Number of Violations	LR <sub>UC</sub>	LR <sub>IND</sub>	LR <sub>CC</sub>	TUFF	Number of Violations	LR <sub>UC</sub>	LR <sub>IND</sub>	LR <sub>CC</sub>	TUFF
Australia-Japan	BEKK	8	0.56	0.19	0.75	0.08	2	3.73	0.01	3.73	0.05
	DVECH	12	<b>4.56</b>	0.44	5.00	0.09	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	CCC	8	0.56	0.19	0.75	0.08	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	DCC	6	0.00	0.10	0.10	0.26	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
Australia-NZ	BEKK	6	0.00	0.10	0.10	0.28	2	3.73	0.01	3.73	1.73
	DVECH	12	<b>4.56</b>	0.44	5.00	0.09	2	3.73	0.01	3.73	1.73
	CCC	4	0.81	0.04	0.85	0.72	2	3.73	0.01	3.73	1.73
	DCC	6	0.00	0.10	0.10	0.28	2	3.73	0.01	3.73	1.73
Australia-Singapore	BEKK	8	0.56	0.19	0.75	0.08	2	3.73	0.01	3.73	0.03
	DVECH	9	1.24	0.24	1.48	0.33	2	3.73	0.01	3.73	0.03
	CCC	7	0.14	0.14	0.28	0.16	3	1.93	0.02	1.95	1.11
	DCC	6	0.00	0.10	0.10	0.28	3	1.93	0.02	1.95	1.11
Japan-NZ	BEKK	6	0.00	0.10	0.10	0.08	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	DVECH	5	0.20	0.07	0.27	0.46	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	CCC	8	0.56	0.19	0.75	0.08	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	DCC	7	0.14	0.14	0.28	0.16	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
Japan-Singapore	BEKK	7	0.14	0.14	0.28	0.02	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	DVECH	10	2.15	0.30	2.45	0.05	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	CCC	6	0.00	0.10	0.10	0.08	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	DCC	6	0.00	0.10	0.10	0.08	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
NZ-Singapore	BEKK	6	0.00	0.10	0.10	0.28	2	3.73	0.00	3.73	0.03
	DVECH	5	0.20	0.07	0.27	0.44	1	<b>6.58</b>	0.00	<b>6.58</b>	0.00
	CCC	5	0.20	0.07	0.27	0.44	2	3.73	0.01	3.73	0.03
	DCC	4	0.81	0.04	0.85	0.72	2	3.73	0.01	3.73	0.03

Entries in **bold** are significant at the 95% level.