Toward Making the Complex VRSAP Model Suitable For Controller Development: Application of Linearization and Adaptive Feedback Control of the Complex MIMO System

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EXTENDED ABSTRACT

The VRSAP (Vietnam River System and Plains) model is popular for modeling the hydrology of the Mekong Delta, Vietnam. This is due to its highly complex internal dynamics that have been refined and built upon over several decades to reflect a unique and very complicated hydrological system. Annually updated data is often used to calibrate the model parameters to keep up with the changes in the Delta. As the result, the VRSAP is a highly non-linear and high-ordered system making the development of any control algorithm very difficult. This paper discusses the framework for analyzing the VRSAP's governing dynamics to make the model suitable for supporting land-use decisions. This process can be applied toward similarly complex simulation model that requires simultaneous auto-calibrations to its parameters. Specifically, the paper reports on the following three analysis:

1. A linearization of the governing equations, the St. Venant system of equations for open channel flow, for an inner loop SISO (single-input single-output) controller development. The linearization combines external inputs into the canal segment and roughly cast as lateral inflow.

$$\begin{aligned}
&\frac{\partial Q'}{\partial x} + B_c sZ' = \frac{\partial R}{\partial \theta} Z' \\
&(1 - Fr_0)\frac{\partial Z'}{\partial x} + \frac{\alpha_0}{g w_0} sQ' + \frac{\alpha}{g}\frac{Q_0}{w_0^2}\frac{\partial Q'}{\partial x} \\
&-\frac{\alpha_0}{g}\frac{Q_0 B}{w_0^2} sZ' = -\left(\frac{\partial K}{\partial Q}Q' + \frac{\partial K}{\partial Z}Z'\right)
\end{aligned} \qquad \text{(r1.a)}$$

Dynamic property is given to the lateral inflow parameter *R*. Practical conditions imposed on the new parameter ensures well-posedness of the hyperbolic non-linear partial differential equations.

$$\begin{aligned}
&\theta := Z(x) \otimes Z(t) \otimes Q(x) \otimes Q(t) \\
&R := \{r_1, r_2, r_3, ..., r_i\} \subseteq \theta
\end{aligned} \qquad \text{(r1.b)}$$

The linear temporal logic statement *r1.c* expresses the rule for implementing *r1.b* to maintain well-posedness of *r1.a*.

$$\begin{aligned}
&(\forall i : R_i(r = \theta_i) U (\forall j \neq i : R_j(r = \theta_j)) \\
&G R(\theta)
\end{aligned} \qquad \text{(r1.c)}$$

2. An adaptive control scheme is presented to auto-calibrate model parameters using an outer loop MIMO (multi-input multi-output) controller. The controller is developed through an eigenvalue analysis technique. Result *r2.a* shows that flow characteristic is effected by the matrix eigenvalues. A sample adaptive law is developed, (*r2.b*), to prove as well as demonstrate feasibility of this technique.

$$\phi_{11}(t) = c_3\delta(t + xc_2) - c_4\delta(t + xc_1) \qquad \text{(r2.a)}$$

$$\frac{d\phi}{dt} = -\gamma \frac{k_0}{k_{\mathrm{mod}}} y_m e \qquad \text{(r2.b)}$$

3. An algorithm to recognize and eliminate almost-uncontrollable/unobservable modes of the model. This is achieved by introducing auxiliary input for balanced truncation.

The author suggests an iterative algorithm to arrive at desired truncated result.

1. INTRODUCTION

The Mekong Delta – The Mekong Delta of Southern Vietnam is integral to the country's economy and politics. Almost all land-use in the Delta is for agriculture and aquaculture. In addition, the Delta is home to many rare species of birds, fish, mammals and complex ecological systems that are essential to the global environment. These different factors impose multiple demands on the river system. Therefore, effective hydrology management in the Mekong Delta is one of the top priorities for the Vietnamese government today.

1.1. The VRSAP Model

Since its development several decades ago, the VRSAP has been continuously updated to better reflect the ever-changing conditions in the delta. The VRSAP is popular and is used by governmental agencies [1]. Though widely used, the model has several shortfalls that reduce its effectiveness such as using previous year's data to make management decisions on the current year. Furthermore, parameter adjustments to the model (necessary due to the delta river system constantly changing) based on previous year's data may not reflect the current delta conditions.

1.2. The Control Problem

To address the issue of system efficiency and accuracy outlined above, the author proposes applying a real-time feedback control to the VRSAP. The feedback control loop is implemented with two specific goals: (1) control system actuators at real-time in response to current data, (2) auto-calibrate system parameters to adapt the model to changes in the system. Since it was designed as a simulation model, the VRSAP currently exhibits many limitations that render it unsuitable for the application of automatic control. The limitations can be divided into two main issues: (a) The model is governed by complicated non-linear dynamics convoluted data types, and (b) lack of clear algorithm for parameter calibration.

This paper is organized as follows: *Section 1* motivates and introduces the problem, *Section 2* outlines a linearization of the governing St. Venant system of equations with input consideration, *Section 3* discusses an outer loop MIMO adaptive control scheme, and *Section 4* discusses a scheme to identify and eliminate superfluous parameters, and *Section 5* concludes the paper.

2. INNER SISO LOOP SETUP AND ANALYSIS BASED ON STEADY STATE LINARIZATION WITH DYNAMIC INPUT

The VRSAP model, as it is being used, produces a prediction of the behavior of the hydrology through a schematization scheme (**Figure** 2). That is, the model predicts water flow and level of the area under study – though level is considered to be the more essential control variable. Consider the single feedback loop in **Figure 1** below. The controller, derived from a linearized model, automatically generates a schedule for the river sluice gates based on minimizing the error difference with the controlling variable.

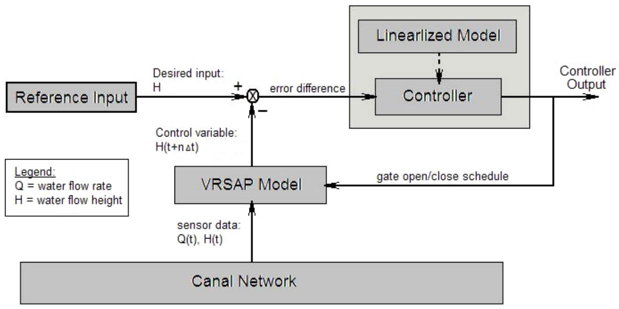

Figure 1 – Inner SISO control loop.

The most common approach to develop a controller for the model is to linearize the governing dynamics at a certain operating condition. Current literature reports effectiveness in linearization of the VRSAP at steady state and steady flow [4], [5], [6]. However, past linearization of the dynamics has been done without consideration for dynamic input.

Dynamic input – Equations (1),(2) below has been revised from a similarly formed system of equations used by the VRSAP model [7] to include dynamic input parameter ('*R'*).

$$\frac{\partial Q}{\partial x} + B\frac{\partial Z}{\partial t} = R \qquad (1)$$

$$\frac{\partial Z}{\partial x} + \frac{\alpha_0}{g}\frac{\partial (Q/w)}{\partial t} + \frac{\alpha}{g}\frac{Q}{w}\frac{\partial (Q/w)}{\partial x} = K \qquad (2)$$

Where:
Q = water flow (m^3/s)
Z = water level (m)
B = canal width at free surface (m)
B_c = canal width averaged over segment (m)
α_0, α = adjustment factor (see [2])

$K = \frac{-|Q|/Q}{(wC\sqrt{R})^2}$ represents the friction term with Chezy parameter.

and

$R = f(\theta)$

R represents the input parameter (such as lateral inflow). Practical conditions, described by c1, are imposed on *R* and *θ* to maintain simplicity to the problem.

$$\begin{aligned} &\theta := Z(x) \otimes Z(t) \otimes Q(x) \otimes Q(t) \\ &R := \{r_1, r_2, r_3, \ldots, r_i\} \subseteq \theta \end{aligned} \qquad \text{(c1)}$$

That is, all inputs to a canal segment, cast as *θ,* can only be single dimensional (w.r.t. *x* or *t*) and only one of two already existing parameter (*Q* or *Z*) – all are "or" conditions are exclusive. *R* can be a vector, all elements of *R* must be of the same type.

It is appropriate at this point to reaffirm well-posedness of the system of equations. Conditions imposed by (c1) guarantees that no new variable is introduced to the system of equations. However, in order to maintain well-posedness, R, it is necessary to implement operational rules described by the linear temporal logic statement below. *Note*: The VRSAP simulates the canal network successively segment-by-segment. It is not dissimilar to a spanning tree in graph theory; therefore, it is appropriate to use temporal logic to describe rules for the VRSAP operation.

$$\begin{aligned} &(\forall i : R_i(r = \theta_i) U (\forall j \neq i : R_j(r = \theta_j)) \\ &G\, R(\theta) \end{aligned} \qquad \text{(c2)}$$

c2 stipulates that: All elements of R must be of the same type for a given segment with the option of changing in a different segment. *R*, *θ,* and their relationship must be globally true.

Linearization – There exists several options of linearization based on available data and intended purposes – i.e. around steady state or at steady flow. Since boundary conditions are determined empirically and should be updated as often as possible, steady state linearization is chosen.

Replacing *Q = Q*$_0$ *+ Q', and Z = Z*$_0$ *+ Z'* into equations (1), (2) with approximation *w*= *B*$_c$*Z* and apply the partial derivative:

$$\frac{\partial Q'}{\partial x} + B_c \frac{\partial Z'}{\partial t} = \frac{\partial R}{\partial Z} Z' \qquad (3)$$

$$\begin{aligned} &(1 - Fr_0)\frac{\partial Z'}{\partial x} + \frac{\alpha_0}{gw_0}\frac{\partial Q'}{\partial t} + \frac{\alpha}{g}\frac{Q_0}{w_0^2}\frac{\partial Q'}{\partial x} - \\ &\frac{\alpha_0}{g}\frac{Q_0 B}{w_0^2}\frac{\partial Z'}{\partial t} = -\left(\frac{\partial K}{\partial Q}Q' + \frac{\partial K}{\partial Z}Z'\right) \end{aligned} \qquad (4)$$

Where the partial derivatives with respect to (*Z*) and (*Q*) are evaluated at reference point.

Fr = Froud number, a parameter describing flow condition, defined as $\frac{\alpha B Q^2}{g w^3}$

Taking the Laplace transformation of (3), and (4) yields:

$$\frac{\partial Q'}{\partial x} + B_c s Z' = \frac{\partial R}{\partial Z} Z' \qquad (5)$$

$$\begin{aligned} &(1 - Fr_0)\frac{\partial Z'}{\partial x} + \frac{\alpha_0}{gw_0} sQ' + \frac{\alpha}{g}\frac{Q_0}{w_0^2}\frac{\partial Q'}{\partial x} \\ &-\frac{\alpha_0}{g}\frac{Q_0 B}{w_0^2} sZ' = -\left(\frac{\partial K}{\partial Q}Q' + \frac{\partial K}{\partial Z}Z'\right) \end{aligned} \qquad (6)$$

Which can be reformulated as:

$$\begin{pmatrix} \frac{\partial Q'}{\partial x} \\ \frac{\partial Z'}{\partial x} \end{pmatrix} = A \begin{pmatrix} Q' \\ Z' \end{pmatrix} \qquad (7)$$

The equation is now in the control state space form with:

$$a_{11} = 0$$

$$a_{12} = \frac{\partial R}{\partial Z} - B_c s$$

$$a_{21} = -\frac{1}{1 - Fr_0}\left(\frac{\partial R}{\partial Z} + \frac{\alpha_0 s}{gw_0}\right)$$

$$a_{22} = \frac{\alpha Q_0 (B_c s - \partial R/\partial Z)}{(1 - Fr_0) g w_0^2} + \frac{1}{1 - Fr_0}\left(\frac{\alpha_0 B Q_0 s}{g w_0^2} - \frac{\partial K}{\partial Z}\right)$$

The eigenvalues of (A) can be calculated as:

$$\lambda_{1,2} = \frac{1}{2}\left[a_{22} \pm \sqrt{4a_{12}a_{21} + a_{22}{}^{2}}\right] \quad (8)$$

Since (A) is scalar in (x), the matrix exponential solution to (7) can be calculated from:

$$\begin{pmatrix} Q' \\ Z' \end{pmatrix}_x = P\begin{bmatrix} \exp(\lambda_1 x) & 0 \\ 0 & \exp(\lambda_2 x) \end{bmatrix} P^{-1} \begin{pmatrix} Q' \\ Z' \end{pmatrix}_{x=0}$$

Where $P(.)P^{-1}$ diagonalize A.

The result matrix exponential solution to (7) is denoted as Φ ,

$$\begin{pmatrix} Q' \\ Z' \end{pmatrix}_x = \Phi \begin{pmatrix} Q' \\ Z' \end{pmatrix}_{x=0} \quad (9)$$

Where $\phi_{11} = \dfrac{\lambda_1 \exp(\lambda_2 x) - \lambda_2 \exp(\lambda_1 x)}{(\lambda_1 - \lambda_2)}$ and so on for ϕ_{12} , ϕ_{21} , ϕ_{22} where calculation is similar and straight forward.

With the result of (9), one can see that elements of Φ yield the direct transfer functions between upstream (subscript x) and downstream (subscript o) water level and flow.

Remark: Different engineers manipulate the structure of Φ in (9) to better reflect their choice of input/output parameters (for example, Baume et. al. [6] reports the effect of upstream water release on downstream discharge and the influence of downstream water level on upstream water level).

3. ADAPTIVE CONTROL OUTER FEEDBACK LOOP FOR AUTO-CALIBRATION OF MODEL PARAMETERS

3.1. Eigenvalue Analysis

Over many years of being used and built upon, the VRSAP model includes many extraneous parameters that make the computation heavy and extensive. Furthermore, the ambiguous relationship between these parameters and the model's behavior make any calibration difficult. Before any adaptive control rule can be applied, the relationship between the model parameters (roughly cast as input) and the model's states (output) must be clearly established.

Analysis is performed on the homogenous form of the system of equations (natural/unforced response).

Consider:

$$\frac{\partial Q'}{\partial x} + B_c \frac{\partial Z'}{\partial t} = 0 \quad (10)$$

$$(1 - Fr_0)\frac{\partial Z'}{\partial x} + \frac{\alpha_0}{g w_0}\frac{\partial Q'}{\partial t} + \frac{\alpha}{g}\frac{Q_0}{w_0^2}\frac{\partial Q'}{\partial x} - \frac{\alpha_0}{g}\frac{Q_0 B}{w_0^2}\frac{\partial Z'}{\partial t} = -\left(\frac{\partial K}{\partial Q}Q' + \frac{\partial K}{\partial Z}Z'\right) \quad (11)$$

Applying the same treatment outlined in previous section, a similar result as (9) is obtained.

Let us now apply further analysis of the transfer function,

$$\phi_{11} = \frac{\lambda_1 \exp(\lambda_2 x) - \lambda_2 \exp(\lambda_1 x)}{(\lambda_1 - \lambda_2)} \quad (12)$$

Which captures the effect of upstream water flow on downstream flow.

Examination of the formulation of $a_{12,21,22}$ shows that the resulting eigenvalues of A, formulated by (8), takes the form:

$$\lambda_{1,2} = c_{1,2}s \quad (13)$$

Where $c_{1,2}$ are constant coefficients determined by system parameters.

Subsequently, ϕ_{11} in (12) takes the form:

$$\phi_{11}(s) = c_3 e^{xc_2 s} - c_4 e^{xc_1 s} \quad (14)$$

Taking the inverse LaPlace transform yields:

$$\phi_{11}(t) = c_3\delta(t + xc_2) - c_4\delta(t + xc_1) \quad (15)$$

Note: in the flow regime known as *sub-critical* flow, the sign of c_1 is negative and c_2 is positive.

It is easy to see from the time delays of the time-domain transfer function that flow at a given point is effected by its upstream (positive time delay) flow and downstream (negative time delay) flow –

characteristic of a *sub-critical* flow regime. Intuitively, the delay is characterized by the eigenvalues of A and augmented by the canal length, x.

The formulation of result (15) shows the relationship between each parameter and flow characteristics at a given point. Reversing this relationship, the engineer can wisely choose effective candidate parameters for model calibration to match flow condition at hand.

We are now ready to develop the adaptive control law to auto-calibrate the model.

3.2. Adaptive Law Formulation

To show that the eigenvalue analysis approach presented above can be used to effectively formulate an adaptive control law, an example formulation is provided as proof.

Consider the feedback diagram in **Figure 3** below.

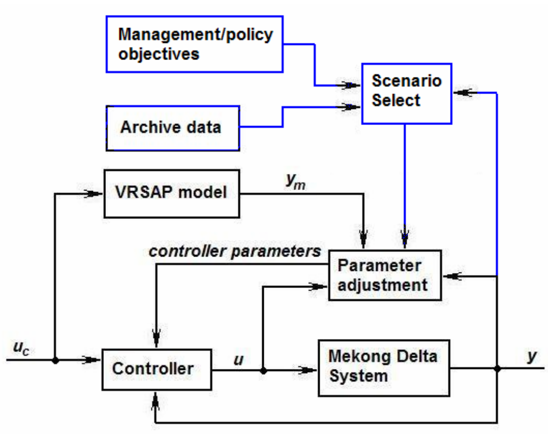

Figure 3 – Implementation of the popular model-reference adaptive system (MRAS)

For demonstration purposes, the author opted for the model-reference adaptive system technique. This is also known as a direct method because the controller parameters are directly adjusted to minimize error between the actual system output and the reference model output. The difficulty is, of course, in selecting the right controller parameter for adjustment. *A priori* information from the engineer's experience aids in this parameter selection.

Remark: It is important to recognize that the outer adaptive auto-calibration feedback loop can not be implemented concurrently with the inner feedback loop while it is implementing control on the physical system.

Let us revisit the earlier formulation of equation (7):

$$\begin{pmatrix} \dfrac{\partial Q'}{\partial x} \\ \dfrac{\partial Z'}{\partial x} \end{pmatrix} = A \begin{pmatrix} Q' \\ Z' \end{pmatrix} \qquad (16)$$

Where the A matrix includes the lateral inflow parameter $R = f(Z_l)$

Let $\phi = R$ denote the lateral inflow parameter available for adjustment.

Let $J(\phi)$ be a cost function defined as

$$J(\phi) = \tfrac{1}{2} e^2$$

Where $e = y - y_m$ is the error difference between the actual system output and the reference model output.

In minimizing $J(\phi)$, the well-known MIT negative gradient rule can be used:

$$\frac{d\phi}{dt} = -\gamma \frac{dJ}{d\phi} = -\gamma e \frac{de}{d\phi} \qquad (17)$$

Where $\dfrac{de}{d\phi}$ is also known as the *sensitivity derivative.*

A priori knowledge indicates that the primary control actuator for the system is the opening and closing of sluice gates on the river network. As the gates are opened, at steady state the system operates normally. The effect of the controller at steady state can then be modeled as a constant gain ($k_0 \approx 1$). Hence, the system transfer function and reference model transfer function are, respectively,

$$G_{sys} = k_0 G(s), \textit{ and } G_{\mathrm{mod}} = k_m$$

$$\textit{where } k_0 \neq k_m \approx 1$$

The system control input is now:

$$u = u_c \phi G_{controller} = u_c \phi k_s \cong u_c \phi \, . \qquad (18)$$

The transfer function between y_c and u_c is:

$$\frac{y}{u_c} = \phi G_{sys} = \phi k_0 G(s) \qquad (19)$$

In setting up the lateral inflow parameter, one can choose to set it up in the form:

$$\phi = \frac{k_{\text{mod}}}{k_0}(\xi_z)$$

Where ξ_z is a scalar value of lateral input water level (Z) at the time of evaluation.

The error and sensitivity terms are (respectively):

$$e = y - y_m = \phi k_0 G(s) u_c - k_m G(s) u_c \qquad (20)$$

And,

$$\frac{\partial e}{\partial \phi} = k_0 G(s) u_c = \frac{k_0 y_m}{k_{\text{mod}}} \qquad (21)$$

Applying the MIT negative gradient rule, we arrive at the adaptation law:

$$\boxed{\frac{d\phi}{dt} = -\gamma \frac{k_0}{k_{\text{mod}}} y_m e} \qquad (22)$$

This adaptation law is straight forward and intuitively sensible. The parameter ϕ should be adapted in proportion with the error, the estimated model output and scaled by the efficiency of the model vs. actual system.

Since we defined above $\phi = \frac{k_{\text{mod}}}{k_0}(\xi_z)$, adjusting the parameter ϕ in this case can be conveniently achieved by tuning k_{mod} constant.

So far, only the case of single adjusting parameter is discussed. For multiple parameters, ϕ can be treated as a vector. This investigation is left as future work.

4. BALANCED TRUNCATION OF INPUT PARAMETERS

The final treatment to be applied to the VRSAP concerns input parameters. Though thus far input parameters are treated as a single input, it does not have to be the case. Especially with the VRSAP model, many extraneous data types are considered in its calculation.

Difficulties arise when not all data have the same contribution, or even contribute at all, to the model calculation. Highly ambiguous relationships exists between the model and its parameters making their elimination non-obvious. The author proposes using balanced truncation to eliminate unnecessary data types to make the model less computationally extensive.

The balanced truncation method relies on teasing out the contribution of a certain parameter to the model by looking at its controllability and observability index.

Remark: Every auxiliary input into the model are assumed to effect either the flow parameter (Q) or level parameter (Z).

Denoting inputs with subscript (i), system states (Q and Z) are reformulated as:
$Q_{total} = Q + Q_i$ and $Z_{total} = Z + Z_i$

Equation (7) can be rewritten in full state-space system as:

$$\begin{pmatrix} \frac{\partial Q}{\partial x} \\ \frac{\partial Z}{\partial x} \\ \frac{\partial Q_i}{\partial x} \\ \frac{\partial Z_i}{\partial x} \end{pmatrix} = \begin{bmatrix} [A] & 0 \\ 0 & [A_i] \end{bmatrix} \begin{pmatrix} Q \\ Z \\ Q_i \\ Z_i \end{pmatrix} + B\vec{u} \qquad (23)$$

$$\vec{y} = C \begin{pmatrix} Q \\ Z \\ Q_i \\ Z_i \end{pmatrix} + D\vec{u}$$

This formulation allows the engineer to check controllability and observability of the system using *grammians*.

The procedure may be computationally extensive but conceptually fairly straight forward. Matrices *B* and *C* are of scalar values and is up to the engineer's choice. An algorithm for choosing B and C is outlined in **Figure 4**.

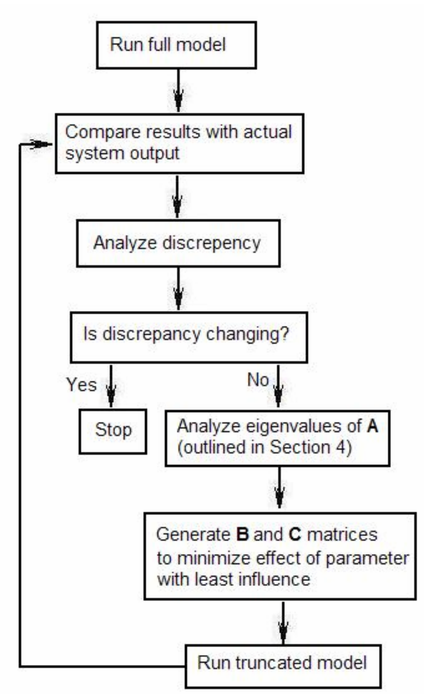

Figure 4 – Balanced truncation implementation.

5. CONCLUSIONS

The VRSAP model, though popular and proven to work as a simulation tool, is not very efficient and is not suitable for the implementation of automatic control. This paper reports the three separate treatments to the system toward this purpose. First, a linearization around steady state is shown. This scheme takes into account the lateral inflow of the system. The result can be generalized for other types of input. Secondly, an adaptive feedback scheme is presented and an example was demonstrated to develop the adaptation law for the controller inflow parameter. Finally, brief discussion addressed using balanced truncation as a method to reduce model complexity. Though the results are preliminary, the method reported should be applicable to other similarly formed models. Future development should yield further interesting properties and insights.

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