

Optimal Management of an Annual Weed: A Stochastic Dynamic Programming Approach

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EXTENDED ABSTRACT

Weeds are ecologically and economically disastrous. Invasive species in general are considered the greatest cause of global biodiversity loss after habitat destruction, and agricultural weeds cost the Australian economy \$4 billion annually on average.

Weed eradication is difficult. Managers must operate with limited budgets and search costs can increase as weed density decreases. Even if all plants are successfully removed, a seedbank may persist, leading to future outbreaks. Thus complete eradication is often an unrealistic target and the question becomes one of how much control is enough. The decision of what level of management resources to invest must not rely solely on management costs, but also take into account plant population dynamics in the context of a stochastic environment.

Here we examine ongoing management of a contained annual weed with an established seedbank, which as a “sleeper” weed may yet escape and cause harm. Using stochastic dynamic programming, we find the optimal management effort of controlling the weed population, trading off expected costs of escape versus costs of searching and removal.

The optimal removal effort increases non-linearly with the density of emerging plants until management becomes futile at high population densities. Most of the state space nevertheless recommends complete removal of emergent plants. The solution is most sensitive to population growth rate, the escape probability function and the relative costs of escape and management.

Our simple model leads to valuable insights for the ongoing control of a sleeper weed. In our study we

have assumed that immediate eradication is elusive. Nonetheless, continued long-term management may gradually deplete the seedbank and allow the possibility of eradication; thus the management time horizon (i.e., the length of the control program) can influence the optimal strategy. The net costs of management versus escape are also important whereas population dynamics – in this case the relentless growth of a sleeper weed’s seedbank – become more important over the long-term.

It is therefore important to include both economics (costs of management and escape) and biology (population dynamics and escape probability) in seeking optimal weed control strategies.

1. INTRODUCTION

Invasive species constitute one of the greatest threats to global biodiversity (Sala *et al.* 2000), impacting ecosystem structure and function and altering disturbance regimes (Mack and d'Antonio 1998). Invasive plants reduce biodiversity and ecosystem productivity as well as having detrimental effects on ecosystem services to herbivores and humans (DiTomaso 2000).

The economic costs of invasive species are similarly grave. Although the rising number of biotic invasions can be linked to increased global trade (Westphal *et al.* 2007), the worldwide costs of invasive plants, animals and microbes have been estimated at US\$1.4 trillion, or 5% of global GDP (Pimentel *et al.* 2001). The economic cost of weeds on natural and agricultural systems within Australia has been estimated at AU\$3.5-4.5 billion (Sinden *et al.* 2004). This creates an economic as well as environmental imperative to develop sound weed management strategies.

Conservation biology and applied ecology can benefit enormously from the application of decision theory, which facilitates the integration of biology, costs and strategy options within a transparent optimisation framework (Possingham *et al.* 2001). For example, conservation actions such as weed control often operate with limited budgets. Management costs are often overlooked in analyses directing conservation strategies; however, explicit consideration of costs within a decision-theoretical framework can have dramatic changes on which management strategies are recommended (Baxter *et al.* 2006).

Dynamic optimisation studies of invasive plant systems have yielded valuable insights into management. Wu (2001) show that expected crop yields are higher when dynamic rather than static optimisation of agricultural weed control is used; and sequential tactical decisions taking stochasticity into account can maximise the economic benefits of integrated weed management (Jones *et al.* 2006). Optimisations of agricultural weed control often use economically important measures such as increased crop yields (e.g., Bosnic and Swanton 1997; Wu 2001) or discounted profits (Pandey and Medd 1991). Profits can be difficult to define for non-agricultural weeds, however, so that approaches which minimise expected costs (Sells 1995, Regan *et al.* 2006), or maximise some conservation measure within a fixed budget constraint (Taylor and Hastings 2004), may be more adaptable to both economically and ecologically important weeds.

In this paper we consider the optimal management of an annual weed. We assume the weed has been detected and confined within a single location, but is a “sleeper” weed with the potential to spread elsewhere with severe ecological or economic impacts (Cunningham and Brown 2006). Removal of every plant may be prohibitively expensive whereas failing to control the infestation would increase the likelihood of its escape. Given a certain density of the weed, managers must therefore decide on the appropriate level to remove. Our objective was to investigate how the optimal levels of weed removal may be affected by differences in costs, management time horizon, population dynamics and in the relationship between density and escape probability. While the components of our model are relatively simple, integrating them in an optimisation framework can yield novel insights into weed management. In doing so we demonstrate an approach that is uncomplicated yet revealing.

2. METHODS

Our optimisation depends on three components. First we develop a model of weed abundance dynamics and control, which determine the transition probabilities between abundance states. Next we describe plausible escape probability functions based on abundance levels. The objective of the optimisation is to minimise the expected overall costs (rather than to achieve eradication). Therefore, we also include management costs, and the costs of the weed escaping beyond the contained zone. We now describe these components, followed by details of the optimisation itself.

2.1. Population Dynamics

We used a simple linear model for annual weed dynamics by considering the seedbank (S_t) and emergent plants (“adults”, A_t) at time t :

$$S_{t+1} = b(1-g)S_t + (1-m)\rho abA_t f(r_t), \quad (1)$$

$$A_{t+1} = gS_{t+1}. \quad (2)$$

where b is over-winter survival in the seedbank; g is the emergence rate of new plants; ρ is the return rate of seeds to soil; a is the adult survival rate between emergence and seed production; and $f(r_t)$ is a seed production function that depends on r_t , a random environmental signal in year t . We assume that seed production responds linearly to environmental conditions, i.e., $f(r_t) = sr_t$, where s is the number of seeds produced per adult plant under average environmental conditions. The decision variable, m , is the proportion of the adult

plants removed. As we consider both agricultural and non-agricultural weeds we deliberately leave the details of removal vague, ignoring more sophisticated or integrated approaches commonly available to farmers (seedbank fumigation, crop rotation etc.). Note that equation (1) assumes 100% detection of adult plants; however, it may be very costly to approach this perfect level of detection; see 2.3 below. Combining equations (1) and (2) allows us to eliminate the seedbank term, expressing the dynamics in terms of the adult plants only:

$$A_{t+1} = b[1 - g + (1 - m)g\rho asr_t]A_t = \lambda A_t. \quad (3)$$

Thus the population model, while allowing for stochasticity, maintains a simple linear form (for example ignoring density-dependence) which readily allows analysis of its deterministic analogue (e.g. Caswell 2001).

2.2. Escape Probabilities

We assume that the escape probability P_e depends on seed production of $v = asr_t A_t$, ($= asA_t$ under mean conditions) i.e., $P_e = P_e(v)$, and consider four alternative functional forms:

P1. $P_e = 1 - (1 - p_1)^v$ (4)
(derived from the probability of a single seed escaping, p_1);

P2. $P_e = v/(v + k)$ (5)
(representing a density-dependent escape probability; $P_e = 0.5$ when $v = k$);

P3. $P_e = v^\zeta/(v^\zeta + k^\zeta)$ (6)
(sigmoidal density-dependent escape probability assuming successful escape requires some minimum propagule density; again $P_e = 0.5$ when $v = k$; we set the shape parameter $\zeta = 4$);

P4. $P_e = 1 - (1 - p_p)^{\sqrt{4v/\pi}}$ (7)
(assuming escape only from the perimeter of a circular infestation of constant density, where p_p is the probability of a single seed escaping in this scenario).

To make the different probability forms comparable we first fixed $k = 100000$ (i.e., the seed density at which $P_e = 0.5$ in expressions P2 and P3), and then solved expressions P1 and P4 for $P_e(k) = 0.5$, to find the values of p_1 and p_p .

2.3. Costs

We consider three different costs: an escape cost, and fixed and efficiency-dependent removal costs. We ignore economic discounting by assuming that the costs will increase annually at the same rate. We assume that the escape cost C_e is fixed, representing for example the loss of weed-free trade status if containment of an agricultural weed has failed, or some impact valuation of an ecological weed (e.g. cost of setting up a national control program).

For weed removal, we consider two cost components: a fixed removal cost per individual removed κ , and also an ‘‘efficiency cost’’, C_{eff} , which reflects the extra effort involved in removing the last few weeds from an infestation. Extra search costs at low densities may have a pronounced effect on management success (Baxter *et al.* in press). We assume that this cost increases exponentially to some maximum amount (χ) required to remove the last individual (if desired; recall that managers decide what proportion m to remove). This gives a total efficiency cost of

$$C_{\text{eff}} = \chi \int_{x=A-mA}^A e^{-\gamma x} dx = \frac{\chi}{\gamma} (e^{-(1-m)A\gamma} - e^{-A\gamma}), \quad (8)$$

for removing mA out of A weeds. We set the exponent $\gamma = \ln 2$, implying a doubling of search costs with each successive plant removed as eradication is neared. The decision to remove mA plants therefore results in overall expected costs of

$$C_e P_e([1-m]v) + \kappa mA + \chi (e^{-(1-m)A\gamma} - e^{-A\gamma})/\gamma. \quad (9)$$

2.4. Optimisation

We used stochastic dynamic programming (SDP) to find the optimal proportion of plants to remove in order to minimise the total expected costs (9) incurred over a fixed management time horizon (length of control program). SDP provides a means of finding optimal strategies which take into account the states of a system over time (for ecological examples see Mangel and Clark 1988 and McCarthy *et al.* 1999).

We discretised the state-space into adult abundance ‘‘bins’’ increasing geometrically by a factor of 1.05. The edges of the bins were [0.976, 1.025, 1.076, ..., 968227, 1016638], so that mean abundances were given by the 284 geometric midpoints [1, 1.05, 1.11, ..., 992137]. We then assigned transition probabilities between states, depending on the chosen removal level, m . We encapsulated population dynamic stochasticity as

the response of seed production to environmental fluctuations. Standardising the environmental variability as $r_t \sim N(1, c^2)$, where c is the environmental coefficient of variation, gives

$$A_{t+1} \sim N(\mu, \sigma^2), \quad \text{where} \quad (10)$$

$$\mu = (1-m)bgpasA_t + bA_t(1-g), \quad \text{and} \quad (11)$$

$$\sigma = (1-m)bgpasA_t c. \quad (12)$$

Therefore the probability of moving from abundance state A to Z , given removal level m , is

$$\Pr(A \rightarrow Z | m) = \Phi(Z_{(+)}) - \Phi(Z_{(-)}), \quad (13)$$

where $\Phi(x)$ is the cumulative probability of x on a $N(\mu, \sigma^2)$ distribution; $Z_{(+)}$ and $Z_{(-)}$ define the upper and lower bounds of the abundance bin containing Z ; and μ and σ are defined as above (11, 12). The optimal management effort m^* in state A , is the value of m that minimizes the expected future costs J of reaching each subsequent state:

$$\begin{aligned} J_m(A, t) = & \kappa mA + C_{\text{eff}}(\chi, m, A) \\ & + C_e P_e((1-m)Aas) \\ & + \sum_Z J^*(Z, t+1) \Pr(A \rightarrow Z | m) \end{aligned} \quad (14)$$

and

$$J^*(A, t) = \min_m [J_m(A, t)]. \quad (15)$$

This algorithm also requires setting a final cost for each state A , which we set equal to $C_e P_e(sA)$.

2.5. Implementation and analysis

As we were interested in general results, we chose biologically plausible parameters and adjusted seed production s to produce desired population growth rates λ (in general we assume $\lambda = 1$ for a sleeper weed), rather than parameterise the model to a particular case study. We found optimal management effort levels (choosing from $m = 0, 0.05, \dots, 1$) as a function of weed abundance for the four escape probability functions. We tested the sensitivity of these results to changes in population parameters, management costs and management time horizon.

3. RESULTS AND DISCUSSION

3.1. General results

For all escape probability functions, the optimal strategy over most of the range of abundances is to

attempt 100% removal of adult plants (Fig. 1). In general, the optimal management effort increases with adult density, reaching 100% removal at moderate densities (about 500). At very high densities (in the order of 10^4 - 10^5), however, the cost of removal exceeds the expected escape cost and so total control becomes economically futile. Noticeably, at this “giving-up density”, the optimal effort drops sharply from 100% to 0% (mainly due to the discretisation yielding large abundance bins at that scale). There is therefore a range of abundances over which complete removal is recommended. This abundance range expands on both sides as the time horizon increases (Fig. 1), because the transition to an abundance state with higher escape probability becomes more likely over time.

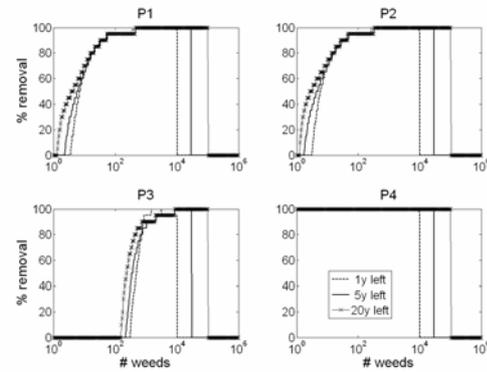


Figure 1. Optimal management strategies for different escape probability curves (eqns 4-7), expressed as optimal removal effort m^* for each level of adult abundance. Results for three different management time horizons are shown: one (—), five (---) and twenty (— · —) years.

3.2. Effect of escape probability functions

The general behaviour is repeated broadly for all probability curves, albeit with some differences (Fig. 1). Whereas there is little appreciable difference between the individual-based (P1) and density-based (P2) escape-probability curves, differences are more marked for P3 and P4.

The sigmoid probability curve P3 reflects a situation where a critical propagule size may be necessary for establishment elsewhere, and so optimal management is lighter at low densities. In contrast, the perimeter-based escape curve P4 (which recall has been standardised so that the same abundance gives 50% escape probability) has relatively higher escape probability at low densities because every plant is nearer the perimeter; therefore complete removal is optimal even at low densities.

Thus the escape probability function may greatly affect the optimal management strategy, especially at low densities. In contrast, there is little difference in the giving-up densities (Fig. 1). Because all escape probabilities are close to one at these very high densities, the expected costs are close to C_e , and the management decision becomes dominated by the relationship between C_e and κA . At low weed densities, management decisions are primarily influenced by whether the escape probability is negligible (P1-P3) and therefore the relative values of expected escape cost and search inefficiency costs (C_{eff}) become important. At higher abundances, we may expect other cost factors and population dynamics to play a considerable role on the optimal management strategy (see 3.4 and 3.5 below).

3.3. Sensitivity to management time horizon

We examined the effect of time horizon on the optimal strategies under escape probability function P1, summarising the results by focussing on the giving-up density at high abundances and the (lower) density at which 50% removal first becomes optimal (Fig. 2). At low densities (e.g., seven plants), the probability of escape is low over a short time-frame and the increased expense of removal is relatively uneconomical. With a longer time-frame (e.g. 20 years), however, the chances for population recovery and eventual escape increase so the incentive to control is stronger and the 50%-removal recommendation begins at weed densities of less than four.

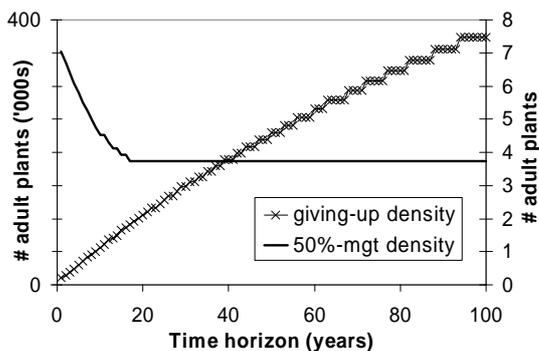


Figure 2. Sensitivity of optimisation results to the management time horizon, assuming escape probability curve P1. Lowest abundances at which management is futile due to weed over-abundance (“giving up density”; left axis) and 50% removal becomes optimal (“50%-mgt density”; right axis) are shown.

The giving-up density, at which managers abandon removal due to excessive removal costs, increases with management time horizon (Fig. 2). At these high densities, the escape probability is close to

one so the expected cost of escape is close to C_e ; but the cost of removal is also very high so management becomes futile. For any given high density however, if no removal occurs, the probability (and thus expected cost) of escape increases over time so that managers with longer time horizons should choose to remove weeds, even if escape is unlikely within one year.

3.4. Effect of population dynamics

In our population model we assumed that the weed population is static on average (i.e., $\lambda = 1$) but that any population change and thus the risk of eventual escape is linked to environmental conditions. The population parameter which invokes most sensitivity in the model (while constraining $\lambda = 1$) is seedbank survival b . This sensitivity can be confirmed by rephrasing the population dynamics in a matrix model; perturbation analysis (Caswell 2001) shows that seedbank survival gives the highest elasticity value. Whereas this sensitivity may suggest it as a good population parameter to address directly (but see Baxter *et al.* 2006), we assume here that only adult plants can be targeted and therefore changes in b of $\pm 10\%$ yield little difference in the optimal management strategy (Fig. 3, left panels). At low weed densities, lower seedbank survival requires slightly more adult plant removal effort; because we fixed $\lambda = 1$ the reduction in b is offset by greater seed production s which increases the risk of escape.

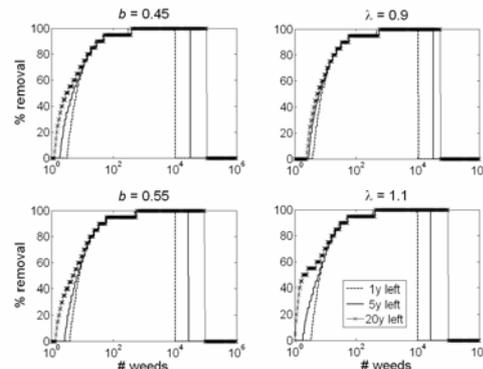


Figure 3. Optimal management strategies for different seedbank survival b (left panels) and overall population growth rate λ (right panels), assuming escape probability function P1 (eqn 4). The parameters b and λ are adjusted to either 90% (upper panels) or 110% (lower panels) of their default values. Results for three different management time horizons are shown: one (—), five (---) and twenty (— · —) years.

The optimal strategy is more sensitive to our assumption that the weed population is not

growing on average ($\lambda = 1$). Unsurprisingly, a higher population growth rate λ demands greater management effort (Fig. 3, right panels). The increase in management is more pronounced for a longer time horizon, as the implications of high population growth are more severe over a longer timeframe. This highlights the importance of considering life-history parameters carefully in choosing management intervention levels. While our default value of $\lambda = 1$ may seem low in the context of invasive plants, it is reasonable for a sleeper weed which may maintain a static population for decades before its eventual escape.

3.5. Effect of costs

Management costs and expected escape costs have a pronounced effect on optimal management strategy. Not surprisingly, more management effort is required at both high and low weed densities if the expected escape cost C_e increases (Fig. 4, left); this is true for all management time horizons. The effects of escape cost and removal cost on the optimal management effort were almost symmetric across much of the abundance range (not shown). For example, a doubling of escape cost prescribes the same management as a halving of the removal cost per plant κ (loosely, the net incentive to remove a weed doubles in both cases). Reducing the cost of removal of low-density plants (χ ; Fig. 4, right) entails increased management at low densities as that action becomes more feasible. The optimal management strategy at upper densities is not affected.

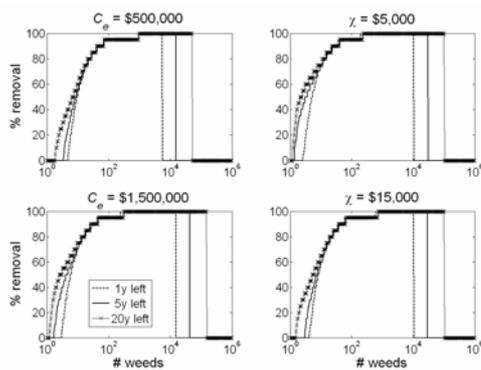


Figure 4. Optimal management strategies for different costs, assuming escape probability function P1 (eqn 4). Costs are economic cost of the weed escaping (C_e ; left panels), and the maximum search-efficiency cost (χ ; right panels), and are either 50% (upper panels) or 150% (lower panels) of their default values. Results for three different management time horizons are shown: one (—), five (---) and twenty (—x—) years.

3.6. Caveats

As stated above, our generalised model presents a much simplified case rather than a specific prescriptive management guide. The model should therefore be re-parameterised (or re-formulated) prior to application to particular plant species or functional types. For example our population dynamics are linear, ignoring density dependence; we assume simple removal (rather than other management actions which may effect smaller or larger changes in parameter values; cf. Fig. 3). Management actions could also be made ‘smarter’ by also responding to other factors in addition to mere abundance (which would be included in the SDP as system states) e.g. environmental condition.

4. CONCLUSIONS

Our optimisation approach, whilst relying on quite simple models, leads to valuable insights for the control of a sleeper weed. The relative costs of management versus escape are central whereas over the long term, population dynamics—especially a relentlessly growing seedbank—may become important. The relationship between plant density and escape probability, which in turn relates to dispersal mode and surrounding land use, can also have considerable effects on when control should be implemented or abandoned. Managers therefore need to include both economics (costs of management and escape) and biology (population dynamics and escape probability) in seeking optimal control strategies. We assumed here that immediate eradication is elusive. Nonetheless, continued management over a long time may gradually deplete the seedbank and allow the possibility of eradication; the management time horizon is therefore important in the choice of optimal strategy. Overall, 100% adult plant removal is optimal for most densities, leading to the general recommendation to remove all sleeper weed plants unless infestation has already exceeded management capacity or the population is too low for efficient removal.

5. ACKNOWLEDGEMENTS

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