A Hybrid Clough-Tocher Radial Basis Function Method for Modelling Leaf Surfaces

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Keywords: *Interpolation, finite elements methods, virtual plants, radial basis functions*

EXTENDED ABSTRACT

We present a novel hybrid approach for leaf surface fitting that combines Clough-Tocher (CT) and radial basis function (RBF) methods to achieve a surface with a continuously turning normal. The hybrid CT-RBF method is shown to give good representations of a Frangipani leaf and an Anthurium leaf, see figure 1.

The development of the algorithm has been made to facilitate the understanding of leaf surface properties. By identifying and quantifying the response of plants to the inputs via their leaves information will be obtained for application to practical and theoretical issues of scientific and sociological importance. The use of pesticides to assist agricultural production has ecological effects; avoidance of the overuse of water is of critical importance and a measured use of resources is of economic importance.

An understanding of the mechanisms of the development of a plant will, generally, include the an understanding of the role played by its leaves. This subject has attracted considerable interest over the last decade as summarised in the introduction (Room *et al* 1996, Prusinkiewicz 1998) . Their shape, size, and position are important in several ways. For example energy uptake is assumed to be a function of light interception. This influences plants both individually and collectively, the latter through competition for resources. Similarly, the amount of precipitation, nutrients or pesticide can be better quantified if a detailed model of a leaf is accessible. Thus important aspects of leaf modelling can be facilitated with accurate knowledge of the leaf surface. This can be obtained from a surface fit to a set of measurements made by a data collection device such as a laser scanner or a sonic digitiser (Loch 2004).

This work will form the basis for a theoretical study of pathways of water droplets on leaves. The initial investigation will assume that the leaf is smooth and the droplet experiences, at most, gravitational, surface tension and viscous forces. It will be necessary to produce a surface fit with a continuously varying gradient. This is assured by interpolation of data values and gradient values on a triangulation of the data points using piecewise bivariate cubics (Clough 1965). Derivative values are obtained by computing the gradient of an RBF which interpolates the data values (Powell 1991).

The issues reported here include: **The selection of points from the data set** The choice of a subset of the data which avoids undesirably shaped triangles was aided by the use of *EasyMesh* a software package which generates Delaunay triangulations.
Choice of RBF and suitable width parameter c. Hardy's multiquadrics were selected in conjunction with the use of Rippa's algorithm to determine the width parameter.
The use of local and global RBF interpolates Numerical experiments investigated the use of local, less costly RBF interpolates compared with global, more expensive and more robust RBF counterparts. The results favoured the former approach.
The method reported is generally applicable to scattered data and has the potential for application to the numerical solution of partial differential equations.

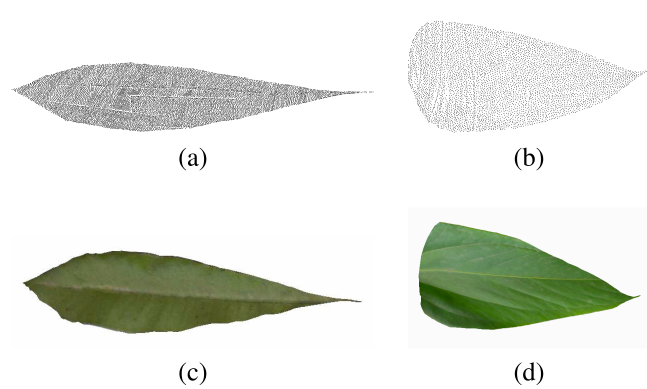


Figure 1. Photos of the scanned (a) Frangipani and (b) Anthurium leaves and corresponding (c) Frangipani and (d) Anthurium leaf surface models for these point sets.

1 INTRODUCTION

There are many situations in science for which surface observations of a biological system are made. Surface data can often be collected at a discrete set of points and a key problem is to reconstruct the surface, or perhaps capture important features of the surface from a discrete set of measurements. The modelling of plant architecture has been researched extensively over the last decades (Room *et al.* 1996, Prusinkiewicz 1998) and models of leaf surfaces have generally not been generated with great accuracy or level of detail, until recently when (Loch 2004) presented two methods to accurately model leaf surfaces. Leaves play an important role in the development of a plant, and therefore some adequate representation of the leaf is required. This representation may be used for visualization purposes only (Loch 2004) or may be used to study biological processes such as photosynthesis (Sinoquet *et al.* 1998) and canopy light environments (Espana *et al.* 1999).

Virtual plants are developmental plant models that combine geometrical and topological information that can be used to produce a visualization (Room *et al.* 1996). Few of the past leaf models were based on accurate measurements until 3D digitizers and faster computers with improved graphic capabilities became available. Virtual leaf models may be displayed in an abstract way, where the leaf is represented by a disk (Smith 1984), polygons and texture maps (Foley *et al.* 1982) or, more realistically, by a surface model that captures the surface shape and boundary (Prusinkiewicz *et al.* 1990). Hammel *et al.* (1992) used branching skeletons for compound leaves and boundary algorithms were applied by Mundermann *et al.* (2003) for modelling lobed leaves. Maddonni *et al.* (2001) used piecewise linear triangles to represent the leaf surface, where vertices along the boundary are estimated by allometric relationships. Espana *et al.* (1999) modeled the undulations of the boundary. Finally, (Frey 1987), based his approach on splines and texture maps.

Two methods have been presented (Loch *et al.* 2005, Loch 2004) based on finite elements methods (piecewise linear triangular and piecewise cubic Clough-Tocher triangular) to model accurate leaf surfaces in three dimensions. Here a large number of data points sampled by a laser scanner extracted from the real leaf surface were used in an incremental algorithm to reduce the size of the set of data points.

The research presented in this paper introduces a new surface fitting method based on hybrid strategies that combine Clough-Tocher with radial basis techniques for modelling the leaf surface, which is based on a large number of three-dimensional data points captured from the real leaf surface.

This paper consists of four sections. In this section we briefly review surface fitting methods, including the Clough-Tocher and the radial basis function method. In section 2 a new surface fitting method is presented that combines the CT and RBF methods for modelling leaf surfaces. The application of the new method to a Frangipani leaf and Anthurium leaf is presented in section 3, where a processing methodology is detailed. Future work and further applications of the model are discussed in section 4.

1.1 Clough-Tocher finite element method

The Clough-Tocher method (CTM) is an interpolating finite element method that was introduced originally by Clough and Tocher (Clough 1965). This method is used to minimize the degree of the polynomial interpolant fitted across the triangular elements without losing the continuity of the gradient over the whole domain.

The CTM is a seamed element approach, whereby each triangle is treated as a macro-element that is split into subtriangles, which are called micro-elements. The CTM, has the advantage that it results in a smooth surface over the whole domain. It approximates the surface as an interpolating cubic polynomial constructed on each subtriangle which enables a bivariate piecewise cubic interpolant to be devised over the entire triangle that is continuously differentiable. The key result is that only twelve degrees of freedom are required for the CTM, namely the function values and the gradient at each vertex, as well as the normal derivative along the edges.

In the context of leaf surface fitting, the function value is assigned at the triangle vertices. However, the derivative information at the vertices and at the midpoints of each side is unavailable and needs to be estimated. The vertex gradient estimates are generated from neighbouring data information and thereafter the edge normal derivatives are determined as the mean of the normal derivatives estimated at the two vertices associated with the edge. This approximation is based on the assumption that the normal slope along the sides of the triangle changes linearly (Lancaster *et al.* 1986). A more detailed description of CTM including the list of cardinal basis functions for the standard triangular element can be found in (Lancaster *et al.* 1986, Loch *et al.* 2005, Ritchie 1978).

1.2 Radial basis functions

A Radial Basis Function (RBF) approximation to f is a function S of the form:

$$S(x) = \sum_{i=1}^{n} a_i \Phi_i(x) \quad x \in \mathbb{R}^2 \tag{1}$$

where $\Phi_i(x) = R(\|x - x_i\|)$, $R(r)$ is a non-negative real-valued function with non-negative argument r and $\|.\|$ denotes the Euclidean norm. The points $\{x_i\}$ belonging to $\mathbb{R}^2$ are called the centres of the RBF approximation. The expansion coefficients $\{a_i\}$ are determined by satisfying some approximation criterion; in this application by interpolation (see equation 3).

In order to obtain a smooth surface representation to estimate the function values at points other than data points, radial basis function schemes have found applications in areas such as geodesy (Junkins *et al.* 1971) and medical imaging (Carr *et al.* 1997). A major problem of the radial basis function method concerns large sets of data points where the computational costs involved in fitting and evaluating the RBF can become time-consuming. A review of the theory of RBF approximation is given by Powell (1991).

Well known examples of radial basis function methods include Hardy's multiquadric RBF which is adopted in this paper:

$$R(\|x - x_i\|) = \sqrt{c^2 + \|x - x_i\|^2}. \tag{2}$$

The parameter c must be specified by the user; it is related to the spread of the function around its centers. The accuracy of the multiquadric interpolant depends heavily on the choice of c (Franke 1982).

Thus, we face the problem of how to select a good value for the parameter c. Many methods for selecting c for the multiquadric interpolants in two-dimensions have been introduced in the literature. Franke (1982) used $c = 1.25 \frac{D}{\sqrt{n}}$ where D is the diameter of the minimal circle enclosing all data points. Hardy (1971) suggested a value of $c = 0.815d$ where $d = \frac{\sum_{j=1}^{n} d_j}{n}$ and d_j is the distance between the j^{th} data point and its closest neighbour.

Rippa (1999) studied the influence of the parameter c on the quality of the approximation of the multiquadric interpolant and concluded that the accuracy depends on the choice of the parameter c. Rippa considered two sets of data points and nine different test functions defined on the unit square. A data vector $f = (f_1, f_2, ..., f_n)^T$ was constructed by evaluating each test function over the set of data points so that

$$S(x_j) = f_j, j = 1, 2, ..., n. \tag{3}$$

Rippa (1999) suggests an algorithm for selecting a good value for the parameter c based on minimizing a cost function that represents the error between the interpolating radial basis function and the unknown function (RMS), see equation 6. This cost function is defined as follows:

Let the error vector $E = (E_1, ..., E_n)^T$ where $E_k = f_k - S^k(x_k) = \frac{a_k}{x_k^{[k]}}, \quad k = 1, ..., n$ and $S^k(x) = \sum_{i=1, i\neq k}^{n} a_i^k R(\|x - x_i\|)$, and then

$$c_{good} = arg \min_{c\in\mathbb{R}} \|E(c)\|_1 . \tag{4}$$

Here, S^k is the interpolant to a reduced data set obtained by removing the point x_k and the corresponding data value f_k from the original data set and E_k is a function of c since it requires translates of a basis function that depends on c. For more details see (Rippa 1999).

2 HYBRID METHOD

We propose a new hybrid approach for surface fitting based on the CTM that uses a multiquadric RBF to estimate the gradient at the vertices and mid-points of the Clough-Tocher triangle. The multiquadric RBF interpolant $S(x)$ is given by equation 1. The gradient of S is then given by

$$\nabla S(x) = \sum_{i=1}^{n} a_i \nabla \Phi_i(x), \tag{5}$$

where $\nabla \Phi_i(x) = \nabla R(\|x - x_i\|) = \frac{x - x_i}{\|x - x_i\|} R'(\|x - x_i\|)$ (R' denotes the derivative of $R(r)$).

The hybrid method is essentially an interpolating finite element method. We outline this procedure in the following steps.

Step 1: *Given n data points $\{x_i, i = 1, ..., n\}$ and a data vector $\{f_i, i = 1, ..., n\}$, choose a subset of m data points from the n data points for the purpose of a triangulation of the leaf surface.*

Step 2: *Find c using Rippa's method (section 1.2).*

Step 3: *A global multiquadric RBF interpolant that uses the triangulation points is then constructed and used to estimate the gradients for all triangles.*
OR
A local multiquadric RBF interpolant that uses a local set of points constructed on each triangle is used to estimate the gradients for a particular Clough-Tocher triangle.

Step 4: *In both methods, global and local RBF, the truncated singular value decomposition TSVD (Tony* *et al. 1990) is applied to solve the linear system (3) for the coefficients* $\{a_i\}$.

Step 5: *The CTM is applied to construct the leaf surface.*

3 APPLICATION OF THE HYBRID METHOD FOR THE FRANGIPANI AND ANTHURIUM LEAVES

Reconstruction of the shape of a leaf using surface fitting techniques requires a set of representative data points sampled from the surface. The process of sampling data points from the leaf surface using a measuring device is called digitizing such that the visible exterior data points of the leaf are enough to capture the surface of the leaf. Loch *et al.* (2005) collected data points for different types of leaves (such as, Frangipani, Anthurium, Flame and Elephant's Ear) using a laser scanner. The boundary points were selected by hand from the complete set of points using the PointPicker, software written by McAleer (Hanan *et al.* 2004).

3.1 Data from laser scanner

In this research the hybrid Clough-Tocher Radial basis function interpolation method was applied to the laser scanned Frangipani and Anthurium leaf data taken from (Loch *et al.* 2005) to construct the surface of those two leaves. The Frangipani leaf data set contains two subsets of data. The first set consists of 3,388 points, which represents the entire leaf surface scanned points; while the second set consists of 17 points representing the boundary points of the Frangipani leaf surface. The Anthurium leaf data set consists of a set containing 4,688 points, which represent the entire leaf surface points and a second set containing 79 points representing the boundary points of the Anthurium leaf surface. These point sets are displayed in Figures 1 (a) and (b).

3.2 Leaf reference plane

The coordinate system used by the scanner, which returns the coordinates of points on the leaf, may not be suitable for interpolation due to the possibility of multivalued and vertical surfaces. A solution is to use a reference plane that is a least squares fit to these data points. We construct a reference plane by making a linear least squares fit to the data and rotating the coordinate system so that the reference plane becomes the $xy-$plane. This rotation can be achieved by rotating the normal vector of the reference plane about the $x-$axis into the $xz-$plane and then about the $y-$axis into the $yz-$plane (Oqielat *et al.* 2007). This procedure is successful if the vertical height of the data points is single valued in the transformed coordinate system.

3.3 Triangulation method

In order to apply the hybrid method to the leaf data sets a triangulation of the leaf surface needs to be constructed. Since the number of data points that represent the surface is large, the computational expense is reduced by selecting only a subset of this set to generate a triangulation of the leaf. In this work the triangulation of the leaf is constructed using the *EasyMesh* generator, software written in the C language by Bojan Niceno (2002). *EasyMesh* generates two-dimensional *Delaunay* and constrained *Delaunay* triangulations in general domains. We will explain the triangulation process for only the Frangipani leaf because the process is the same for the Anthurium leaf.

An input file that must be provided to *EasyMesh* is one that contains the 17 boundary points (nodes) and the desired length of the triangle sides. *EasyMesh* returns a good triangulation if the domain is convex. However, because the piecewise linear boundary defined by the 17 chosen points do not enclose a convex set, e.g see Figure 2 (a), *EasyMesh* was unable to produce a triangulation with the required properties. To overcome this problem, an algorithm was used to generate a convex hull from the entire set of leaf data points. This process provided a total of 27 points, and the next closest points to the given 17 boundary points from these points were found using the *Matlab* command *dsearch*. This process resulted in 11 boundary points being identified as defining the convex domain exhibited in Figure 2 (c).

In the interior of the convex hull (leaf surface) we can define either a horizontal, or vertical, line in the domain to enable *EasyMesh* to produce fewer and better shaped triangles. For the Frangipani and Anthurium leaves (Oqielat *et al.* 2007) it appears that the vertical line produces a more suitable triangulation than the horizontal line, see for example Figure 2(c).

In summary, we applied the following steps to construct the triangulation of the Frangipani leaf using *EasyMesh*:

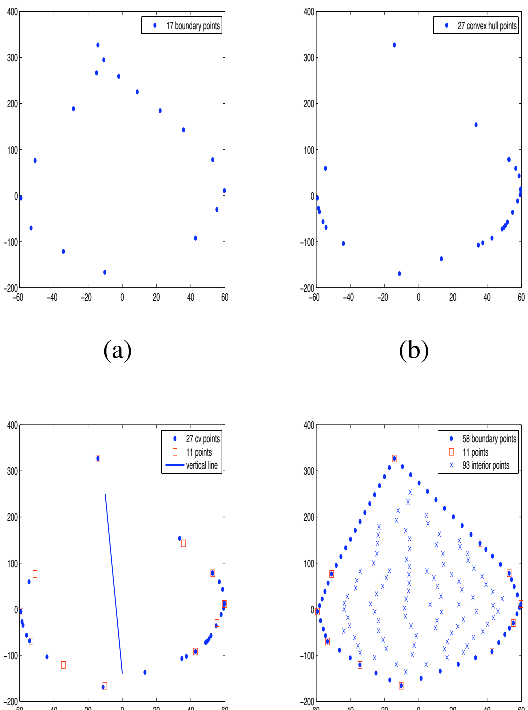
Step 1: *EasyMesh* was provided with an input file that contains the 11 boundary points, the vertical line and the desired triangle edge length. *EasyMesh* returned the node file that contained the same boundary points, together with additional boundary points (58 point) and a set of points distributed inside the leaf (93 internal points). These represented the triangle vertices of the mesh structure, see Figure 2 (d).

Step 2: Import the node file to *Matlab* and then locate the closest points in the leaf data set from the internal points generated in Step 1 using *dsearch*. These resulting points represent the triangle vertices of the leaf surface mesh structure.

Step 3: To obtain the boundary points of the leaf for which we do not have surface values, we find the closest points from the leaf data set to the *EasyMesh* boundary points and use their surface values.

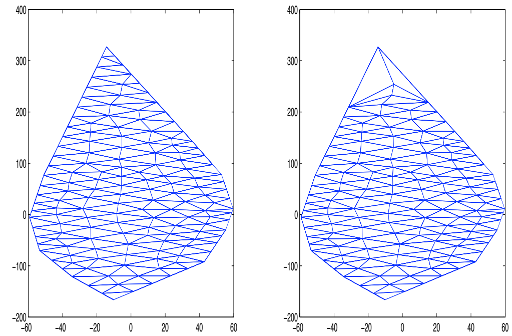
Step 4: Use the *Matlab* command *Delaunay* to triangulate the leaf points obtained from step 2 and 3.

This process gives the final triangulation for the leaf surface illustrated in Figure 3 (a). After the triangulation of the leaf surface is constructed the hybrid Clough-Tocher Radial Basis Function method is applied to construct the leaf surface. The **local hybrid approach** applied here is based on choosing the set of 5 nearest neighbours to each vertex and to the center of the triangle. Next, a local radial basis function is built from the 20 points for each triangle, which is then used to estimate the directional derivative at the triangle vertices and midpoints. A **global hybrid approach** is also applied, which is based on building one single global RBF from the triangulation points and then using it to evaluate the gradients at the vertices and midpoints of all triangles. The parameter c in both cases was estimated globally using the triangulation points following the Rippa framework (Rippa 1999).



(a) (b) (c) (d)

Figure 2. (a) The 17 Frangipani leaf boundary points. (b) The 27 points generated from the convex hull algorithm. (c) The square points represent the final 11 boundary points. (d) The vertices of the mesh structure generated using Easymesh. The square points represent the 11 boundary points that are given to Easymesh; the dot points represent the 58 extra points added by Easymesh, while the x points represent the 93 internal points.



(a) (b)

Figure 3. (a) Triangulation of 151 points of Frangipani leaf surface generated using *EasyMesh*. (b) Triangulation of 141 points of Frangipani leaf surface.

One problem that arose when applying the local RBF method to the Frangipani leaf concerned the poor interpolant values arising at the "tail" of the leaf located near the stem. The reason for the poor interpolant values occurred because there was insufficient data to construct these interpolants. To overcome this problem we needed to delete some of the smaller triangles from the mesh at the leaf tail (by deleting 10 points from the boundary points added from *EasyMesh* at the tail) to form larger triangles that contained enough data to proceed with the hybrid method. Triangulations determined from this construction process are illustrated in Figure 3 (b). This problem did not arise for the Anthurium leaf.

3.4 Numerical experiments

In this section we present the results of applying the hybrid method to the Frangipani and Anthurium leaf data. After the triangulation points were selected, the rest of the m data points (denoted by $f_k = f(x_k), k = 1, ..., m$) from the leaf data set were used to measure the quality of the approximation of the hybrid method. We noted that some of the m data points occurred outside of the virtual leaf mesh and these points were ignored in the quality analysis. We then applied the hybrid method to estimate the surface values for the data points occurring inside the triangulation to construct the leaf surface, see Figure 1 (c) and (d).

Table 1. RMS computed using hybrid local and global RBF for the Frangipani leaf data points as well as the maximum error associated with the surface fit.

	Hybrid Local RBF	Hybrid global RBF
Scaled RMS	0.0086	0.0139
Scaled maximum error	0.0700	0.0655
boundary points	48	48
points tested	3155	3155
Triangulation points	141	141
outside points	104	104
No. of Triangles	257	257

Table 2. RMS computed using hybrid local and global RBF for the Anthurium leaf data points as well as the maximum error associated with the surface fit.

	Hybrid Local RBF	Hybrid global RBF
Scaled RMS	0.0043	0.0068
Scaled maximum error	0.0537	0.0435
boundary points	66	66
points tested	4460	4460
Triangulation points	212	212
outside points	59	59
No. of Triangles	387	387

The error metric we used was the root mean square error RMS, given by:

$$\text{RMS} = \sqrt{\frac{\sum_{k=1}^{k=m}[S(x_k) - f_k]^2}{m}}. \qquad (6)$$

$S(x_k)$ represents the CT estimated value at the m data points and f_k represents the given function values at the same data points. The second error metric measured the quality in terms of the maximum error associated with the surface fit in relation to the maximum variation in f.

$$\textbf{scaled max error} = \frac{\max(|S(x_k) - f_k|)}{\max(f_k) - \min(f_k)},$$

with $k = 1, 2, ..., m$.

Tables 1 and 2 show the scaled maximum errors and the scaled RMS $= \frac{\text{RMS}}{\max(f_k)-\min(f_k)}$ for the Frangipani and the Anthurium leaf data sets respectively using the local and global hybrid method. For the Frangipani leaf there were a total of 3,155 data points used to assess the accuracy of the surface. Note the *EasyMesh* triangulation comprised 141 vertices. There were more than 100 points ignored in the analysis because these points were deemed to lie outside the leaf mesh structure.

One observes for the Frangipani leaf that using the local hybrid RBF method produced slightly more accurate RMS value than using the global hybrid RBF method while it is the converse for the maximum error. The trends depicted in Table 1 for the Frangipani leaf appear consistent with observations from Table 2 for the Anthurium leaf.

4 CONCLUSIONS AND FUTURE RESEARCH

The work presented in this paper describes a new mathematical surface fitting technique for modelling the leaf surface. It allows the user to construct an accurate leaf surface based on three-dimensional data points.It provides a basis on which future research can be built. Surface representations can be extended to generate not only realistic images of leaves but also be applied to models determining a droplet path on the leaf surface. Knowing this path is important for many application; for example, in the simulation of a pesticide application to plant surfaces (Hanan *et al.* 2003,Reichard *et al.* 1998) Knowledge of this behaviour may be used to determine the effectiveness of a treatment, and then to develop certain pesticides that have the ability to protect leaves for longer periods of time. Similar models may treat moisture precipitation and energy uptake through photosynthesis enabled by ray tracing techniques.

At present projections of the image boundaries in the reference plane are piecewise linear. Work on genuinely curved boundaries is in progress.

An advantage of the leaf models described in this paper is that they may be used in different plant modelling environments such as AMAP (Godin *et al.* 1997), xfrog (Lintermann *et al.* 1999) or LStudio (Prusinkiewicz *et al.* 2000).

5 ACKNOWLEDGMENTS

This paper was carried out thanks to funding from the School of Mathematical Sciences. Many thanks to Dr. Joseph Young and Mr. Mark Dwyer from the Queensland University of Technology HPC centre for the leaf visualizations.

6 REFERENCES

Bojan Niceno (2002), *www.dinma.univ.trieste.it/nirftc/research/easymesh*.

Carr, J.C., Fright, W.R. and Beatson, R.K. (1997), Surface interpolation with radial basis functions for medical imaging, *IEEE Transactions on Medical Imaging*, 16(1), 96-107.

Clough, R.W. and Tocher, J.L. (1965), Finite element stiffness matrices for analysis of plate bending, *In Proceedings of the Conference on Matrix Methods in Structural Mechanics. Wright-Patterson A.F.B., Ohio*, 515-545.

Espana, M., Baret, F., Andrieu, B. and Chelle, M. (1999), Radiative transfer sensitivity to the accuracy of canopy structure description, *The case of a maize canopy, Agronomie*, 19, 241-254.

Foley, J.D. and van Dam, A. (1982), Fundamentals of interactive computer graphics, *RadialSoft Corp*.

Foley, T. (1994), Near optimal parameter selection for multiquadric interpolation, *Journal of applied science and computation*, 1, 54-69.

Franke, R. (1982), Scattered data interpolation: tests of some methods, *Mathematics of Computation*, 38(157).

Frey, W.H. (1987), Selective refinement: a new strategy for automatic node placement in graded triangular meshes, *Int. J. Numer. Meth. Eng.*, 24, 2183-2200.

Godin, C., Guedin, Y., Costes, E. and Caraglio, Y. (1997), Measuring and analysing plants with the AMAPmod software, *Plants to ecosystems, Michalewicz, M. (ed.), CSIRO Australia*, 53-84.

Hammel, M.S., Prusinkiewicz, P. and Wyvill, B. (1992), Modelling compound leaves using implicit contours, *In T.L. Kunii, editor, Visual computing: Integrating computer graphics with computer vision, Proceedings of Computer Graphics International '92,Tokyo, Japan*, 119-212, 22-26 June.

Hanan, J., Loch, B. and McAleer, T. (2004), Processing laser scanner plant data to extract structural information, *Proceedings of the 4th International Workshop on Functional-Structural Plant Models, 7-11 June 2004, Montpellier, France, pages 912, 2004.*

Hanan, J., Renton, M. and Yorston, E. (2003), Simulating and visualising spray deposition in plant canopies, *ACM GRAPHITE 2003, Melbourne, Australia*, 259-260, 11-14 February.

Hardy, R.L. (1971), Multiquadric equations of topography and other irregular surfaces, *Journal of Geophysical Research*, 76, 1905-1915.

Junkins, J., Miller, G. and Jancaitis, J. (1971), A weighting function approach to modeling of irregular surfaces, *Journal of Geophysical Research*, 78, 1794-1803.

Lancaster, P. and Salkauskas, K. (1986), Curve and Surface Fitting, an introduction, *Academic Press, London, Orlando*.

Lintermann, B. and Deussen, O. (1999), Interactive modeling of plants, *IEEE Computer Graphics and Applications*, 19(1), 56-65.

Loch, B. (2004), Surface fitting for the modelling of plant leaves, *PhD Thesis, University of Queensland, http://www.sci.usq.edu.au/staff/lochb*.

Loch, B., Belward, J. and Hanan, J. (2005), Application of surface fitting techniques for the representation of leaf surfaces. *MODSIM 2005 International Congress on Modelling and SimulatMion*, 12-15 Dec 2005, Melbourne, Australia.

Maddonni, G., Chelle, M., Drouet, J.-L. and Andrieu, B. (2001), Light interception of contrasting azimuth canopies under square and rectangular plant spatial distributions: simulations and crop measurements, *Field Crops Research*, (13), 1–13.

Mundermann, L., MacMurchy, P., Pivovarov, J. and Prusinkiewicz, P. (2003), Modeling lobed leaves, *In Computer Graphics International, Proceedings, Tokyo*, (13), 60–67, 9–11 July 2003.

Oqielat, M.N., Belward, J.A and Turner, I.W. (2007), A Hybrid Clough-Tocher method for surface fitting with application to leaf data, submitted to *Applied Mathematical Modelling*.

Powell, M.J.D. (1991), Advances in numerical analysis, wavelets, subdivision algorithms and radial functions, *W. Light, Oxford University Press, Oxford, UK*, 105–210.

Prusinkiewicz, P. (1998), Modelling of spatial structure and development of plants: a review, *Scientia Horticulturae*, 74, 113-149.

Prusinkiewicz, P., Karwowski, R., Mech, R. and Hanan, J. (2000), L-studio/cpfg: A software system for modelling plants, *Lecture Notes in Computer Science*, 1(1)33–38.

Prusinkiewicz, P., Lindenmayer, A., Hanan, J.S., Fracchia, F.D., Fowler, D.R., de Boer, M.J.M. and Mercer, L. (1990), The algorithmic beauty of plants, *Springer Verlag, New York, Berlin*.

Reichard, D.L., Cooper, J.A., Bukovac, M.J. and Fox, R.D. (1998), Using a videographic system to assess spray droplet impaction and reflection from leaf and artificial surfaces, *Pesticide Science*, 53, 291-200.

Rippa, S. (1999), An algorithm for selecting a good value for the parameter c in radial basis function interpolation, *Advances in Computational Mathematics*, 11, 193-210.

Ritchie, S.I.M. (1978), Surface representation by finite elements, *Master's thesis,University of Calgary, Canada*.

Room, P., Hanan, J. and Prunsinkiewicz, P. (1996), Virtual plants: new perspectives for ecologists, pathologists and agricultural scientist, *Trends in Plant Science*, 1(1), 33-38.

Sinoquet, H., Thanisawanyangkura, S., Mabrouk, H. and Kasemsap, P. (1998), Characterization of the light environment in canopies using 3D digitising and image processing, *Annals of Botany*, 82, 203-212.

Smith, A.R. (1984), Plants, fractals, and formal languages, *Computer Graphics*, 18(3), 1-10.

Tony, F.C. and Hansen, P.C (1990), Computing truncated singular value decomposition least squares solutions by rank revealing QR-factorizations, *SIAM J. Sci. Stat. Comput.*,11(3), 519–530.