Visualization of Call Options and Implied Volatility

Cornelis A. Los, Ph.D.
Associate Professor of Banking & Finance
Nanyang Technological University
Singapore 639798

Abstract Until recently the visualization of options has been largely limited to two-dimensional representations of market data. Following the example of multicoloured meteorological diagrams, we have created multicoloured 3D and 4D visualizations of financial instruments. This paper compares the empirical with the theoretical implied volatility surfaces of the Hang Seng Index and the S&P500 Index, European style call options. Substantial discrepancies between the theoretical Black-Scholes pricing and the actual daily market pricing of options were discovered, raising the issue of possible lack of empirical corroboration of the Black-Scholes model. Furthermore, we compared two ways of computing implied volatilities, the conventional open-ended Black-Scholes "trial-and-error" method and the new closed-form solution method of Bharadia et al. [1995]. We empirically falsify Bharadia's claim that his at-the-money formula produces a more moderate implied volatility than the conventional trial-and-error method and therefore leads to a more efficient empirical fit. First, Bharadia's formula produces too much curvature in the volatility smile, so that his volatility is quickly too large for relatively low strike prices. Second, on some days, Bharadia's volatility smile can be shown to be empirically larger or lower than the trial-and-error volatility at all available strike prices. This unpredictable inconsistency between the two methods could lead to serious mis-pricing of options.

1. INTRODUCTION

Visualization in finance needs to make perceptible the most critical aspects of market data. Until recently, this has been limited to 2D graphics. This is insufficient since prices of financial instruments have more than two or three variables. For example, bonds have at least four variables: price, yield, maturity and quality grade. More sophisticated instruments, like options, have up to six variables, e.g., as defined in the widely used Black-Scholes (B-S) model: call price, asset price, exercise price, interest rate, maturity, and volatility.

In contrast to financial engineering, visualization in other areas of science has improved dramatically in recent years. Motivated by brilliant research efforts and strong funding, multidimensional modelling and analysis is no longer a novelty in geography, astronomy, physics and biological studies [Hall, 1992; Wolff and Yaeger, 1993]. As financial engineering increases its market scope, more visualization products are embraced by industry professionals to aid in their pricing, trading and financial analysis. (For a beautiful example, see Price, 1996].

In this paper multicoloured MATLAB graphics are used to visualize the interrelationships inherent in call options, in particular, to visualize the empirical and theoretical implied volatilities. This analysis required at least 4D imaging. Of the three software packages, which allow 4D imaging - Maple, Mathematica and MATLAB -, we found MATLAB most user-friendly. Other software, like BMBP/Diamond, IBM Visual Data Explorer, IDL, MathCad 6.0 Plus, Microsoft EXCEL 7.0 and Tecplot, allows only 3D imaging. A dedicated MATLAB application is used to generate coloured 3D images, using daily market data on the Hong Kong Hang Seng Index and the American S&P500 Index for the period October - December 1996.

Substantial systematic differences are observed between the empirical and theoretical implied volatility surfaces of the options, suggesting that the B-S model inadequately captures the systematic data variation. For the computation of both the actual and the theoretical implied volatility, the B-S trial-and-error method is used. A comparison with a closed form solution for the computation of the implied volatility proposed by Bharadia [1995] showed the closed form to be severely biased and incapable of capturing the observed discrepancies.

2. BLACK-SCHOLES MODEL AND DATA

2.1 Black-Scholes Options Pricing Model

Standardized option contracts were first listed and traded on a regulated market open to all when the Chicago Board Options Exchange began operation in April 1973 [Hull. 1995, pp.138]. Just before the CBOE opened its doors for business, two Chicago professors, Fischer Black and Myron Scholes, had made a major breakthrough in options pricing. when they unveiled the significance of time value in their famous (B-S) options pricing model [Black and Scholes. 1973]. The B-S options pricing model for a European call option on non-dividend paying stock is

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

$$\text{with } d_1 = \frac{\ln(S/X) + (r + s^2/2)T}{\sigma\sqrt{T}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{T},$$

where C is the call option price, S the spot price of the underlying asset, X the strike price, r the cash rate of interest, T time maturity, and s the standard deviation of the spot price of the underlying asset. $N(x)$ is the cumulative normal distribution function evaluated at x. Notice that this model has six variables, of which five are directly observable. The *implied volatility* (σ) can only be measured indirectly (= not directly observable) and is thus the typical fudging "hidden variable," similar to what is often used in quantum mechanics. Figure 1 provides the conventional 2D payoff graph of a call option, relating the call price C to the spot price S, for given X, r, T, and s. The curved line is the time value of the option.

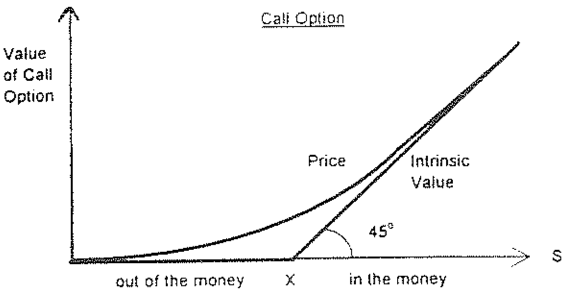

Figure 1: 2D- Payoff Graph of a Call Option

2.2 Implied Volatility and Methodology

The subject of calculating implied volatility has attracted much discussion from both industry and academia. Chance [1992] claims that since the B-S model is a fairly complex non-linear function, the implied volatility s cannot generally be obtained from the directly measurable variables C, S, X, r and T by rearranging the formula. Brenner and Subrahmanyam [1988] and Feinstein [1988] published formulas that are accurate for at-the-money options only. Tomkins (1995, pp.142-146) stated that there are three basic methods for determining implied volatility: (1) using a volatility/price graph, (2) applying a Newton-Raphson algorithm, or, (3) employing a method of bisection. However, all three methods use elements of trial-and-error.

With the "what-if" capabilities of an EXCEL spreadsheet currently widely used by market makers, we determine the implied B-S volatility of both the Hang Seng Index and S&P 500 Index Futures call options. In addition, we tested a new closed-form formula of Bharadia et al. [1995] for calculating implied volatility on the S&P 500 Index Futures December call option contracts. We compared the volatility smiles of Bharadia's method with that of the conventional trial-and-error method.

2.3 Data

The data are recent trading data of the Hang Seng Index (HSI) November 96 Call Option for reported trading dates from 23 September through 30 October 1996 and the S&P 500 Index December 96 Call Option for reported traded days from 25 October through 7 November 1996. Both sets of data are from *The Asian Wall Street Journal* and the *Bloomberg System*. Both call options are European-style options and hence the B-S model is applicable. Both contracts expire in the last week of the maturity month. Tables 1 and 2 provide samples of the Hang Seng Index Futures November 1996 Call Options and the S&P500 Index Futures December Call Options with the ensuing computations. The US T-Bill rate was adopted as the relevant risk-free interest rate also in Hong Kong because of the (Currency Board) fixed exchange rate between the US dollar and the HK dollar.

3. VISUALIZATION IN 3D AND 4D

3.1 Visualization of Hang Seng Index and S&P 500 Index Futures Call Options

Figure 2 is a 4-D illustration of the empirical value of the Hang Seng (HSI) November Call Option on 3 October 1996. Two of the six variables, S and r, are fixed for that day. The relationship between C, X and T is represented by the 3D value surface. The colour scale indicates different ranges of *implied volatility* σ, as computed by iteratively solving the B-S model for σ. The implied volatility typically varies daily and differs across strike prices. As implied by the theoretical B-S model, the call option's value is higher when the strike price is lower. However, less clear is the relationship between call price and time to maturity, which initially increases and then declines.

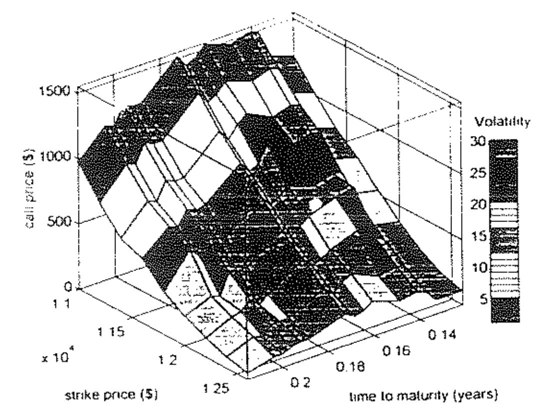

Figure 2: 4D Hang Seng Index Nov. Call Option, 3 Oct 96

A complete set of sequential pricing surfaces - empirical, theoretical and their difference - with each day its own index spot and interest, has been produced for consecutive days and is available for use, say, in the classroom, with MATLAB.

At the close of each trading day, there is only one underlying spot price S. Hence, if only one particular contract is considered at a time, there would be one point on the diagram per day. The result would become an uninformative single line linking all the various points over the term to maturity T for the option. Therefore, in our empirical diagram, we plot the observed strike prices X for the same option contract S. Thus, the "strike price" X axis is preferred over the "spot price" S axis for empirical imaging of options.

For the theoretical B-S option value surface we derive the theoretical call prices from smoothed observed *historical* volatility. The 10-day moving standard deviation of the logarithmic returns of the HSI is used to calculate the call premia. Figure 3 shows that the theoretical surface in the right panel is considerably smoother than the surface plotted with the implied volatility in the left panel. This result is certainly in line with many findings that the smoothed asset price volatility differs from the implied volatility. Notice that the empirical left panel shows some "hot spots" for implied volatility.

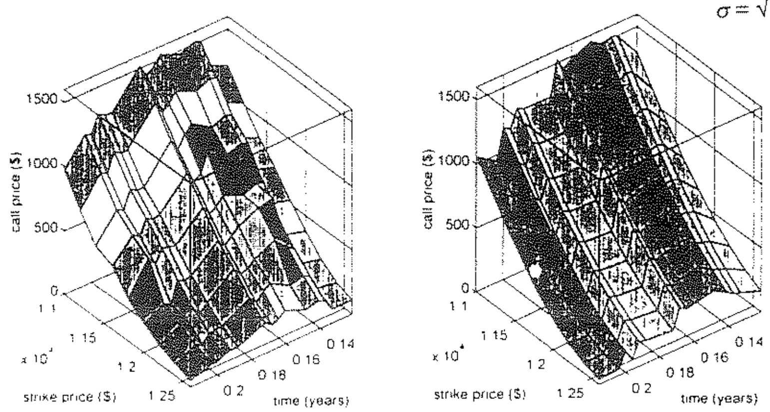

Figure 3: Comparison between actual call price (with implied volatility) and theoretical call price (with smoothed volatility) of the Hang Seng Index Call Option, 3 Oct 96

In the right panel of Figure 3, the historical volatility *is constant* across various strike prices (= B-S' stationarity of volatility assumption). In the left panel of Figure 3 the computed implied volatility differs across different strike prices and the plotted surface is more textured. The colour scale shows that the smoothed empirical volatility is higher than implied volatility, presumably because it contains epistemic uncertainty in addition to the systematic volatility. The smoothed volatility is smoothed from the spot price data. Because it is larger, it produces a higher theoretical call price. Thus the theoretical price implicitly values also the epistemic uncertainty, or "B-S' modeling error." Similar images were computed for the S&P500 Index with similar conclusions.

A considerable number of call option contracts (especially the lower strike contracts) were hardly traded during this period, hence not many transaction prices were observed. To still create a surface, these "prices" are calculated by using the implied volatility of the closest preceding traded price for each strike[1]. Therefore, there are strings of similar implied volatility for certain days, as projected in the uniform colour across maturity in the left panel of Figure 3. Thus, even in the most "empirical" pricing surface visualizations, empirical practice "dictates" some "smoothing" of the data. Certainly, the propriety of such assumptions should be questioned when the option has not been traded for some time.

3.2 Alternative Implied Volatility Computations

As an alternative to the conventional open-ended "trial-and-error method for computing implied volatilities, we implement the explicit closed-form solution of Bharadia et al. [1995], which is exact (only) at-the-money:

$$\sigma = \sqrt{\{2\pi/T[(C-\delta)/(Xe^{-rT}+\delta)]\}}, \text{ where } \delta = (S- Xe^{-rT})/2$$

To focus our research, we compared 3D volatility surfaces, or "volatility smiles." In these cases, the spot price S, the interest rate r *and* the call price C are the same for each day. The colour in the following graphs has thus no additional meaning, but is just an indication of the magnitude of the volatility, already measured on the vertical axis. For some discussion of option smiles, see Corrado [1996], Derman et al. [1996], and Dupire [1994].

We mapped a surface of the implied volatility of S&P 500 Index Dec 96 call options in Figure 4.

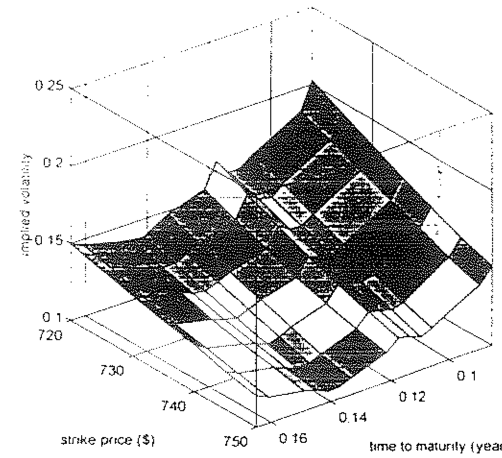

Figure 4: Implied volatility surface of S&P500 Index Futures Options, using B-S model, 7 Nov 96

[1]Practiced by Bloomberg, which claims that " the best guide to estimating the next trade's implied volatility is the previous trade's implied volatility. Thus using the 'Last -Trade' implied volatility to generate an estimated price according to the current underlying futures price should be a good indication of the price at which the option would trade at this moment." [Bloomberg, 1997, pp.7-8]

Our data set is complete for the trading period in question and the result is a smooth surface without gapping. The plotted surface shows that the implied volatility is higher for deep in-the-money contracts with lower strike prices, and rises as the contract edged towards maturity. This confirms the empirical observation of Macbeth and Merville [1979, 1980] and Derman et al. [1996], when they produced volatility surfaces of the S&P 500 Index Options.

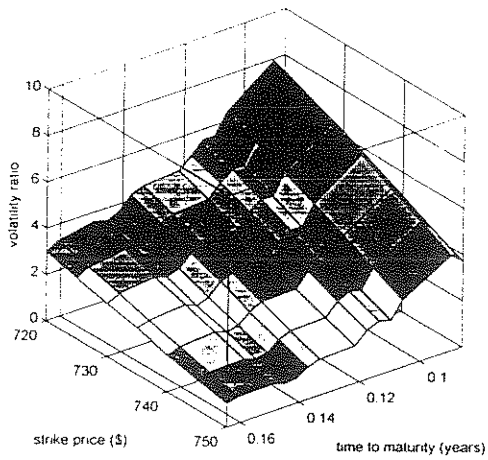

Figure 5: Ratio of Bharadia et al. [1995] implied volatility and B-S trial-and-error implied volatility of S&P500 Futures Options, 7 Nov 96

However, Figure 5 shows that the range of implied volatility computed by the closed-form formula of Bharadia et al. [1995] is wider than that of the trial-and-error method, although it produces a similarly shaped surface. The ratio of the implied volatility of Bharadia to that by the trial-and-error method shows that this ratio increases to more than eight times when the option is extremely in-the-money and close to maturity. When the option is relatively further from maturity, the ratio decreases to around 2-4 times for close-to-the-money contracts (contracts with strike prices 720 and 730). The two volatilities are very near to each other when the options are increasingly out-of-the-money (e.g. those with strike price =750).

This observation is more focused when we compare the Bharadia et al. [1995] versus the conventional B-S trial-and-error implied volatility in the implied volatility/time to maturity (σ/T) plane in Figure 6.

Our observations contradict the empirical evidence produced by Bharadia [1995] which asserts that his version of implied volatility consistently more moderate than that of the market's, and much closer to the true volatility compared to those derived from the market as derived by Brenner-Subramanyam [1988], and Feinstein [1988]. From our tabulations and data visualization it is clear that the volatility produced by Bharadia's new formula has more curvature than that of the conventional trial and error method, which is much "flatter." Moreover, Figure 7 shows that Bharadia's implied volatility is not consistently above or below that of the trial-and-error method - in two panels it is consistently above and in two below.

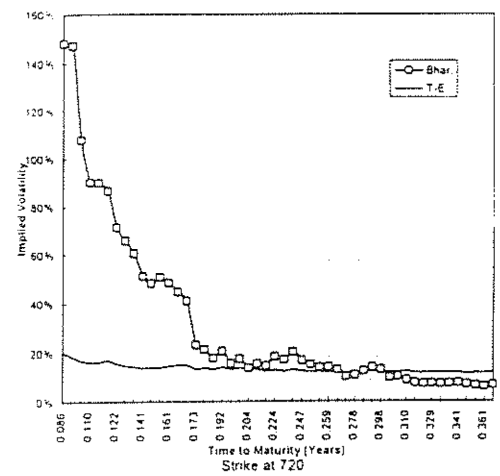

Figure 6: Bharadia et al. [1995] versus trial-and-error B-S implied volatility, with strike X = 720, Dec 96 Call Option

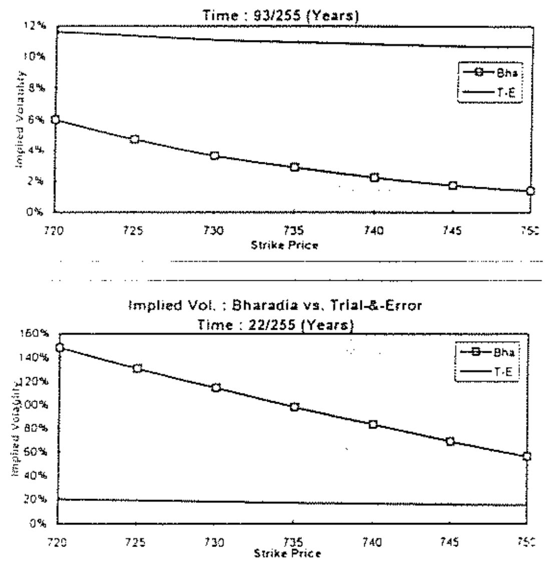

Figure 7: Implied volatility of Bharadia et al. [1995] versus B-S trial-and-error

4. CONCLUSIONS AND RECOMMENDATIONS

3D and 4D visualization shows that the Black-Scholes theoretical option model produces prices, which are different from the actual, traded option prices. Fluctuations in the daily cash rate and possible non-stationarity of the underlying price volatility make the empirical call option price surface not smooth. As there is typically a range of strike prices for a particular type of options, but only one spot price per traded contract, one must include "strike prices" as one of the projection axes for the visualizations. Smoothed price volatility is constant across strike prices, thus the resulting theoretical call price surface is smoother than the call price surface with the implied volatility. The smoothing of the volatility is reflected in the observed smoother surface. But the smoothed volatility is a bit too large since it contains some epistemic uncertainty or "modeling error."

The empirical B-S implied volatility differs across strike prices, and is higher for options that are deeper in-the-money. The range of implied volatility calculated by the new closed form formula of Bharadia et al. [1995] is larger than the B-S trial-and-error implied volatility, because Bharadia's volatility smile shows more curvature. In many circumstances, the two methods produce significantly different implied volatility and Bharadia's new method is not consistently better or worse than the trial-and-error method. Consequently, the issues of how to best determine the implied volatility in the six variable B-S models remains an open question. Moreover, this issue raises the serious question if the B-S model can be called "scientific" if it is allowed to have an "unobservable" variable like "volatility" with an ambiguous and non-unique connection to measurable variables, which biases the call pricing.

4.1 Acknowledgements

With thanks to my students Heng Puay Hiong, Ng Tan Tiat and Ng Yi for the computations and programming of the graphs; to Mr. Tan Jenn Yee and Mr. Kon Chee Keat of UOB Asset Management Ltd. for the daily market data on the Hang Seng and S&P 500 Index Options; and to the staff at Research Lab 3 of the School of Electrical and Electronics Engineering at NTU for their helpful advice on visualization of multidimensional data sets.

4.2 References

Bharadia, M., N. Christofides and G. Salkin, Computing the Black-Scholes Implied Volatility: Generalization of a Simple Formula, *Adv. in Futures and Options Res.*, 8, 15-29, 1995.

Bharadia, M. A. J., N. Christofides, and G. R. Salkin. A Quadratic Method for the Calculation of Implied Volatility Using the Garman-Kohlhagen Model, *Fin. Anal. J.*, 52(2), 61 - 64, 1996.

Black, F. and M. Scholes, The Pricing of Options and Corporate Liabilities, *J. of Pol. Econ.*, 81(3), 369-381, 1973.

Bloomberg, *Help for Call Options Analysis Table (COAT)*, 12 pp., 1997.

Brenner, M. and M. Subrahmanyam, A Simple Solution to Compute the Implied Standard Deviation, *Fin. Anal. J.*, 44(5), 80-83, 1988.

Chance, D. M., Leap into the Unknown, *Risk*, 6(5), 61-66, 1992.

Corrado, C. and T. Miller, Volatility Without Tears, *Risk*, 9(7), 49-52, 1996.

Derman, E., I. Kani and J. Z. Zou, The Local Volatility Surface: Unlocking the Information in Index Option Prices, *Fin. Anal. J.*, 52(4), 25-36, 1996.

Dupire, B., Pricing with a Smile, *Risk*, 7(1), 18-20, 1994.

Feinstein, S., A Source of Unbiased Implied Volatility Forecasts, *Fed. Res. B. of Atlanta, WP*, 88-89, 1988.

Hall, S. S., *Mapping the Next Millennium*. Vintage Books/Random House, 477 pp., 1992.

Hull, J., *Options, Futures and Other Derivative Securities*, Prentice Hall , (3rd Ed.), 572 pp., 1995.

Macbeth, J. and L. J. Merville, An Empirical Examination of the Black-Scholes Call Option Pricing Model, *J. of Fin.*, 34(5), 1173-1186, 1979.

Macbeth, J. and L. J. Merville, Tests of the Black-Scholes and Cox Call Option Valuation Models. *J. of Fin.*, 35(2), 285-303, 1980.

Price, J. F., Optional Mathematics is not Optional, *Not. Am. Math. Soc.*, 43(9), 964-971, 1996.

Tomkins, R., *Options Explained: Advanced Issues in Volatility Estimation*, Macmillan Press, 597 pp, London, 1995.

Wolff, R. S. and L. Yaeger, *Visualization of Natural Phenomena*, Springer- Verlag, 373 pp., New York, 1993.

Table 1: Sample Set of Empirical Data of Hang Seng Index November 1996 Call Option

Hang Seng Index November Call Options								
Date	23-Sep	25-Sep	26-Sep	27-Sep	30-Sep	1-Oct	2-Oct	3-Oct
Hang Seng Index	11622.13	11603.53	11636.13	11759.39	11902.43	11921.22	11951.88	12014.56
US T-Bill Rate (%)	5.01	4.98	4.92	4.94	4.95	4.90	4.82	4.82
Hang Seng Index November Call Options Prices								
11000	752	666	751	862	981	994	1020	1076
11200	568	482	593	695	805	817	842	894
11400	459	357	459	549	647	657	679	663
11600	317	221	317	394	448	458	456	575
11800	229	138	227	260	330	340	357	384
12000	153	77	152	155	227	236	248	272
12200	101	93	100	100	85	161	166	189
12400	0	0	0	0	0	0	105	115
12600	0	0	0	0	0	0	0	67
Historical Volatility	9.92%	10.98%	10.99%	11.02%	9.12%	9.01%	8.74%	8.82%
Black-Scholes Implied Volatility								
11000	13.78%	13.78%	13.52%	13.52%	13.52%	13.52%	13.52%	13.52%
11200	11.90%	11.90%	13.13%	13.13%	13.13%	13.13%	13.13%	13.13%
11400	13.15%	13.15%	13.15%	13.15%	13.15%	13.15%	13.15%	7.11%
11600	11.95%	11.95%	11.95%	11.95%	10.25%	10.25%	10.25%	13.17%
11800	12.16%	12.16%	12.16%	10.89%	10.80%	10.91%	10.91%	10.39%
12000	12.08%	12.08%	12.08%	10.06%	10.84%	10.99%	10.96%	10.68%
12200	12.08%	12.09%	12.14%	10.38%	7.82%	11.27%	11.08%	11.08%
12400	0	0	0	0	0	0	11.12%	10.82%
12600	0	0	0	0	0	0	0	10.74%

Table 2: Sample Set of Empirical Data of S&P 500 Index Futures December 1996 Call Option

S&P 500 Index Futures December Call Option										
DATE	25-Oct-96	28-Oct-96	29-Oct-96	30-Oct-96	31-Oct-96	1-Nov-96	4-Nov-96	5-Nov-96	6-Nov-96	7-Nov-96
S&P 500 Index	700.92	697.26	701.50	700.90	705.27	703.77	706.73	714.14	724.59	727.65
US T-Bill Rate (%)	4.79	4.83	4.86	4.85	4.89	4.89	4.91	4.90	4.93	4.91
S&P 500 Index Futures December Call Option Prices										
720	9.4	7.85	9.85	8.65	11.25	9.45	10.65	13.05	20.05	21.45
725	7.45	6.15	7.85	6.8	9.05	7.45	8.45	10.55	16.8	18.1
730	5.75	4.7	6.1	5.25	7.15	5.8	6.55	8.35	13.85	15.1
735	4.4	3.55	4.65	3.95	5.55	4.4	4.95	6.5	11.2	12.35
740	3.25	2.65	3.4	3	4.25	3.25	3.65	4.95	8.85	9.8
745	2.4	1.95	2.5	2.15	3.15	2.35	2.55	3.7	6.8	7.65
750	1.7	1.4	1.8	1.6	2.35	1.6	1.8	2.7	5.2	5.85
Black-Scholes Implied Volatility										
720	13.23%	13.26%	13.95%	13.21%	14.26%	13.31%	13.84%	13.41%	15.08%	14.98%
725	12.81%	12.89%	13.51%	12.81%	13.79%	12.89%	13.37%	12.98%	14.45%	14.38%
730	12.39%	12.52%	13.06%	12.47%	13.37%	12.56%	12.92%	12.58%	13.89%	13.88%
735	12.07%	12.23%	12.67%	12.14%	12.98%	12.23%	12.51%	12.23%	13.36%	13.40%
740	11.73%	11.99%	12.23%	11.96%	12.68%	11.92%	12.14%	11.93%	12.86%	13.02%
745	11.49%	11.80%	11.96%	11.65%	12.34%	11.63%	11.70%	11.67%	12.38%	12.42%
750	11.21%	11.68%	11.71%	11.57%	12.14%	11.27%	11.43%	11.43%	12.05%	12.04%
Bharadia et al. Implied Volatility										
720	15.59%	13.84%	17.60%	15.78%	20.81%	17.88%	21.41%	23.22%	41.13%	44.86%
725	12.35%	10.84%	14.03%	12.40%	16.74%	14.10%	16.98%	21.47%	34.46%	37.85%
730	9.53%	8.29%	10.90%	9.58%	13.22%	10.97%	13.16%	16.99%	27.57%	31.57%
735	7.30%	6.26%	8.31%	7.20%	10.27%	8.33%	9.95%	13.22%	22.99%	26.90%
740	5.39%	4.67%	6.07%	5.47%	7.86%	6.15%	7.34%	10.07%	18.16%	20.50%
745	3.98%	3.44%	4.47%	3.92%	5.83%	4.45%	5.13%	7.53%	13.95%	16.00%
750	2.82%	2.47%	3.22%	2.92%	4.35%	3.03%	3.62%	5.49%	10.67%	12.24%