Asymmetry and Leverage in Stochastic Volatility Models: An Exposition

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EXTENDED ABSTRACT

The accurate specification and modelling of risk are integral to optimal portfolio and risk management. In this context, a wide variety of conditional and stochastic volatility models has been used to estimate latent volatility (or risk). In both the conditional and stochastic volatility literature, there has been some confusion between the definitions of asymmetry and leverage. This paper examines alternative univariate SV models that have recently been developed and estimated in to order to understand the differences and similarities between the definitions of asymmetry and leverage. Five univariate SV models, namely the basic SV model, SV model with leverage, and three different types of asymmetric SV models, are analysed in order to clarify the distinction between asymmetry and leverage. Alternative specifications of SV models are defined according to the use of standardized or unstandardized returns, with or without leverage, in order to evaluate the differential impacts of positive and negative returns on future volatility, namely symmetry, asymmetry, type I asymmetry (or leverage), type II asymmetry, and type III asymmetry.

1. INTRODUCTION

The accurate specification and modelling of risk are integral to optimal portfolio and risk management, and for calculating Value-at-Risk (VaR) forecasts and optimal capital charges under the Basel Accord. In this context, a wide variety of conditional and stochastic volatility models has been used to estimate latent volatility (or risk). McAleer (2005) provides a comprehensive discussion of both univariate and multivariate Stochastic Volatility (SV) models in the literature.

In both the conditional and stochastic volatility literature, there has been some confusion between the definitions of asymmetry and leverage. This paper examines alternative univariate SV models that have recently been developed and estimated in to order to understand the differences and similarities between the definitions of asymmetry and leverage.

The plan of the paper is as follows. Section 2 presents five univariate SV models, namely the basic SV model, SV model with leverage, and three different types of asymmetric SV models, in order to clarify the distinction between asymmetry and leverage. Alternative specifications of SV models are defined in Section 3 according to the use of standardized or unstandardized returns, with or without leverage, in order to evaluate the differential impacts of positive and negative returns on future volatility, namely symmetry, asymmetry, type I asymmetry (or leverage), type II asymmetry, and type III asymmetry. Some concluding remarks are given in Section 4.

2. MODEL SPECIFICATION

Let the returns on a financial asset, y_t, be given by

$$y_t = \mu_t + e_t, \qquad e_t = h_t^{1/2}\varepsilon_t, \quad \varepsilon_t : N(0,1), \tag{1}$$

where $\mu_t = E\left(y_t \mid \Im_{t-1}\right)$ denotes expected returns on the financial asset, and $\Im_t$ is the past information available at time t.

In order to understand the differences and similarities among alternative univariate SV models that have been developed recently, consider the following SV models for h_t and $\alpha_t = \log h_t$:

Model 1: Basic SV Model Without Leverage

$$\begin{aligned} &\alpha_{t+1} = \mu + \phi\alpha_t + \eta_t, \\ &\eta_t : N\left(0,\sigma_\eta^2\right), \\ &E\left(\varepsilon_t\eta_t\right) = 0 \end{aligned} \tag{2}$$

The basic SV model is symmetric as positive and negative returns have identical effects on future volatility.

Model 2: SV Model With Leverage

$$\begin{aligned} &\alpha_{t+1} = \mu + \phi\alpha_t + \eta_t, \\ &\eta_t : N\left(0,\sigma_\eta^2\right), \\ &E\left(\varepsilon_t\eta_t\right) = \rho\sigma_\eta \end{aligned} \quad . \tag{3}$$

This asymmetric model was suggested by Harvey and Shephard (1996) as a discrete time SV model. Given current returns and volatility in equation (3), the leverage SV model is given as

$$\begin{aligned} &\alpha_{t+1} = \mu + \phi\alpha_t + \rho\sigma_\eta e_t \exp\left(-0.5\alpha_t\right) + \eta_t^*, \\ &\eta_t^* : N\left(0,\sigma_\eta^2\left(1-\rho^2\right)\right), \\ &E\left(\varepsilon_t\eta_t^*\right) = 0. \end{aligned} \tag{4}$$

where e_t is defined in equation (1).

Model 3: Asymmetric SV Model - Unstandardized Returns Without Leverage

$$\alpha_{t+1} = \mu + \phi\alpha_t + \gamma_1 e_t + \gamma_2 |e_t| + \eta_t,$$

$$\eta_t : N\left(0, \sigma_\eta^2\right), \qquad (5)$$

$$E\left(\varepsilon_t \eta_t\right) = 0.$$

This asymmetric SV model was proposed by Danielsson (1994), and was estimated in Asai and McAleer (2005). Equation (5) uses the unstandardized returns in forecasting future volatility.

Model 4: Asymmetric SV Model - Unstandardized Returns With Leverage

$$\alpha_{t+1} = \mu + \phi\alpha_t + \gamma|e_t| + \eta_t,$$

$$\eta_t : N\left(0, \sigma_\eta^2\right), \qquad (6)$$

$$E\left(\varepsilon_t \eta_t\right) = \rho\sigma_\eta.$$

This asymmetric SV model was suggested by Asai and McAleer (2005) to capture both leverage and asymmetric effects. Similar algebraic manipulation as for Model 2 yields

$$\alpha_{t+1} = \mu + \phi\alpha_t + \rho\sigma_\eta e_t \exp\left(-0.5\alpha_t\right) + \gamma|e_t| + \eta_t^*,$$

$$\eta_t^* : N\left(0, \sigma_\eta^2\left(1-\rho^2\right)\right), \qquad (7)$$

$$E\left(\varepsilon_t \eta_t^*\right) = 0.$$

Model 4 is an alternative to Model 3 for capturing the asymmetric effects of positive and negative returns. Equation (6) also uses the unstandardized returns in forecasting future volatility.

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Model 5: Asymmetric SV Model - Standardized Returns Without Leverage

An alternative asymmetric SV model that is different from the two previous asymmetric SV models can be proposed as follows:

$$\alpha_{t+1} = \mu + \phi\alpha_t + \rho_1\sigma_\eta\varepsilon_t + \rho_2\sigma_\eta|\varepsilon_t| + \eta_t^*,$$

$$\varepsilon_t = e_t \exp\left(-0.5\alpha_t\right), \qquad (8)$$

$$\eta_t^* : N\left(0, \sigma_{\eta^*}^2\right), \quad E\left(\varepsilon_t \eta_t^*\right) = 0.$$

This new model is an adaptation of the exponential GARCH (EGARCH) model of Nelson (1991) to the SV literature. In contrast to Model 3, this model uses the standardized returns, ε_t , in forecasting future volatility, and can capture various types of asymmetry and leverage.

3. SV MODEL DEFINITIONS AND COMPARISON

Given the developments presented above, consider the following categories of symmetry and asymmetry SV models, conditional on a negative shock increasing volatility:

Symmetry: Positive and negative returns have identical effects on future volatility;

Asymmetry: Positive and negative returns have different effects on future volatility;

Type I Asymmetry (Leverage): A negative correlation exists between current returns and future volatility;

Type II Asymmetry: Positive and negative shocks increase future volatility, but a negative shock has a larger effect than does a positive shock.

Type III Asymmetry: Positive and negative shocks increase future volatility, but a positive shock has a larger effect than does a negative shock.

Type I Asymmetry is based on the original framework of Black (1976) and Christie (1982), and is also consistent with the definition of leverage in continuous time SV models.

In the conditional volatility literature, the empirical results based on the GJR model of Glosten, Jagannathan and Runkle (1992) and the EGARCH model of Nelson (1991) typically fall into the Type II Asymmetry category. Leverage effects are not possible for the GJR model, whereas leverage is possible, though frequently not observed, for the EGARCH model.

4. CONCLUDING REMARKS

The accurate specification and modelling of risk are integral to optimal portfolio and risk management. Hence, a wide variety of conditional and stochastic volatility models has been used to estimate latent volatility (or risk). In both the conditional and stochastic volatility literature, there has been some confusion between the definitions of asymmetry and leverage.

This paper examined alternative univariate SV models that have recently been developed and estimated in to order to understand the differences and similarities between the definitions of asymmetry and leverage.

Five univariate SV models, namely the basic SV model, SV model with leverage, and three different types of asymmetric SV models, were analysed in order to clarify the distinction between asymmetry and leverage. Alternative specifications of SV models were defined according to the use of standardized or unstandardized returns, with or without leverage, in order to evaluate the differential impacts of positive and negative returns on future volatility, namely symmetry, asymmetry, type I asymmetry (or leverage), type II asymmetry, and type III asymmetry.

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REFERENCES

Asai, M. and M. McAleer (2005), “Dynamic Asymmetric Leverage in Stochastic Volatility Models”, to appear in *Econometric Reviews*, **24**.

Black, F. (1976), “Studies of Stock Market Volatility Changes”, *1976 Proceedings of the American Statistical Association, Business and Economic Statistics Section*, pp. 177-181.

Christie, A.A. (1982), “The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects”, *Journal of Financial Economics*, **10**, 407-432.

Danielsson, J. (1994), “Stochastic Volatility in Asset Prices: Estimation with Simulated Maximum Likelihood”, *Journal of Econometrics*, **64**, 375-400.

Glosten, L., R. Jagannathan and D. Runkle (1992), “On the Relation Between the Expected Value and Volatility of Nominal Excess Returns on Stocks”, *Journal of Finance*, **46**, 1779-1801.

Harvey, A.C. and N. Shephard (1996), “Estimation of an Asymmetric Stochastic Volatility Model for Asset Returns”, *Journal of Business and Economic Statistics*, **14**, 429-434.

McAleer, M. (2005), “Automated Inference and Learning in Modeling Financial Volatility”, *Econometric Theory*, **21**, 232-261.

Nelson, D.B. (1991), “Conditional Heteroskedasticity in Asset Returns: A New Approach”, *Econometrica*, **59**, 347-370.

Table 1: Parametric Restrictions for Various Univariate SV Models

Authors	Basic	Symmetry	Type I Asymmetry (Leverage)	Type II Asymmetry	Type III Asymmetry
Harvey and Shephard (1996)	$\rho = 0$	NA	$\rho < 0$	NA	NA
Danielsson (1994)	$\gamma_1 = 0$ $\gamma_2 = 0$	$\gamma_1 = 0$	$\gamma_1 < 0$ $\lvert\gamma_2\rvert < -\gamma_1$	$\gamma_1 < 0$ $\gamma_1 + \gamma_2 > 0$	$\gamma_2 < \gamma_1 < 0$
Asai and McAleer (2005)	$\rho = 0$ $\gamma = 0$	$\rho = 0$	$\rho < 0$ $\lvert\gamma\rvert \sigma < -\rho\sigma_\eta$	$\rho < 0$ $\rho\sigma_\eta + \gamma\sigma > 0$	$\gamma\sigma < \rho\sigma_\eta < 0$
This paper	$\rho_1 = 0$ $\rho_2 = 0$	$\rho_1 = 0$	$\rho_1 < 0$ $\lvert\rho_2\rvert < -\rho_1$	$\rho_1 < 0$ $\rho_1 + \rho_2 > 0$	$\rho_2 < \rho_1 < 0$

Note: 'σ' is defined as the square root of $E\left[\exp(\alpha_t)\right]$. 'NA' denotes not applicable.