Hydrodynamic modelling of coastal inundation

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EXTENDED ABSTRACT

Modelling the effects on the built environment of natural hazards such as riverine flooding, storm surges and tsunami is critical for understanding their economic and social impact on our urban communities. Geoscience Australia and the Australian National University are developing a hydrodynamic inundation modelling tool called AnuGA to help simulate the impact of these hazards.

The core of AnuGA is the fluid dynamics module, called pyvolution, which is based on a finite-volume method for solving the shallow water wave equation. The study area is represented by a mesh of triangular cells. By solving the governing equation within each cell, water depth and horizontal momentum are tracked over time.

A major capability of pyvolution is that it can model the process of wetting and drying as water enters and leaves an area. This means that it is suitable for simulating water flow onto a beach or dry land and around structures such as buildings. Pyvolution is also capable of modelling hydraulic jumps due to the ability of the finite-volume method to accommodate discontinuities in the solution.

To set up a particular scenario the user specifies the geometry (bathymetry and topography), the initial water level, boundary conditions such as tide, and any forcing terms that may drive the system such as wind stress or atmospheric pressure gradients. Frictional resistance from the different terrains in the model is represented by predefined forcing terms.

A mesh generator, called pmesh, allows the user to set up the geometry of the problem interactively and to identify boundary segments and regions using symbolic tags. These tags may then be used to set the actual boundary conditions and attributes for different regions (e.g. the Manning friction coefficient) for each simulation.

Most AnuGA components are written in the object-oriented programming language Python. Software written in Python can be produced quickly and can be readily adapted to changing requirements throughout its lifetime. Computationally intensive components are written for efficiency in C routines working directly with the Numerical Python structures. The animation tool developed for AnuGA is based on OpenSceneGraph, an Open Source Software (OSS) component allowing high level interaction with sophisticated graphics primitives.

The inundation model will be released under an OSS license in 2006. This strategy will enable free access to the software and allow the risk research community to use, validate and contribute to the development of AnuGA.

1. INTRODUCTION

Sudden-onset natural hazards regularly disrupt communities in Australia, causing fatalities, serious injuries, substantial financial losses and social dislocation. Post-disaster analyses suggest that recovery of the social fabric of communities affected by severe events takes months and sometimes years (Emergency Management Australia, 2002), while recovery of affected local, state and national economies may require in excess of twenty years (Wittwer, 2004). The magnitude of this disruption has generated significant interest in understanding natural hazards so that the nature and extent of their impact may be better anticipated and their social and economic effects mitigated.

Floods are the single greatest cause of death due to natural hazards in Australia, causing almost 40% of the fatalities recorded between 1788 and 2003 (Blong, 2005). Analysis of buildings damaged between 1900 and 2003 suggests that 93.6% of damage is the result of meteorological hazards, of which almost 25% is directly attributable to flooding (Blong, 2005).

Flooding of coastal communities may result from surges of near-shore waters caused by severe storms. The extent of inundation is critically linked to tidal conditions, bathymetry and topography; as recently exemplified in the United States by Hurricane Katrina. While the scale of the impact from such events is not common, the preferential development of Australian coastal corridors means that storm-surge inundation of even a few hundred metres beyond the shoreline has increased potential to cause significant disruption and loss.

Coastal communities also face the small but real risk of tsunami. Fortunately, catastrophic tsunami of the scale of the 26 December 2004 event are exceedingly rare. However, smaller-scale tsunami are more common and regularly threaten coastal communities around the world. Earthquakes which occur in the Java Trench near Indonesia (e.g. Tsuji et al., 1995) and along the Puysegur Ridge to the south of New Zealand (e.g. Lebrun et al., 1998) have potential to generate tsunami that may threaten Australia's northwestern and southeastern coastlines.

Research at Geoscience Australia is aimed at increasing our understanding of the temporal and spatial distribution of sudden-onset natural hazards in Australia and the region. A component of this research is hazard modelling, which simulates the physical characteristics and behaviour of the hazard so that the nature and extent of its impact may be determined. Hydrodynamic modelling allows flooding, storm-surge and tsunami hazards to be better understood, their impacts to be anticipated and, with appropriate planning, their effects to be mitigated. Geoscience Australia in collaboration with the Mathematical Sciences Institute, Australian National University, is developing a software application called AnuGA to model the hydrodynamics of floods, storm surges and tsunami.

2. MODEL

AnuGA uses a finite-volume method for solving the shallow water wave equations (Zoppou and Roberts, 1999). The study area is represented by a mesh of triangular cells in which water depth *h*, and horizontal momentum (*uh*, *vh*), are determined. The size of the triangles may be varied within the mesh to allow greater resolution in regions of particular interest.

The shallow water wave equations are a system of differential conservation equations of the form

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial G}{\partial y} = S$$

where $U = [h \quad uh \quad vh]^T$ is the vector of conserved quantities; water depth *h*, *x* momentum *uh* and *y* momentum *vh*. Other quantities entering the system are bed elevation *z* and stage (absolute water level) *w*, where the relation $w = z + h$ holds true at all times. The fluxes in the *x* and y directions, *E* and *G* are given by

$$E = \begin{bmatrix} uh \\ u^2h + gh^2/2 \\ uvh \end{bmatrix} \text{and}\, G = \begin{bmatrix} vh \\ vuh \\ v^2h + gh^2/2 \end{bmatrix}$$

and the source term (which includes gravity and friction) is given by

$$S = \begin{bmatrix} 0 \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{bmatrix}$$

where S_0 is the bed slope and S_f is the bed friction. The friction term is modelled using Manning's resistance law

$$S_{fx} = \frac{u\eta^2\sqrt{u^2+v^2}}{h^{4/3}} \text{ and } S_{fy} = \frac{v\eta^2\sqrt{u^2+v^2}}{h^{4/3}}$$

in which η is the Manning resistance coefficient.

The equations constituting the finite-volume method are obtained by integrating the differential conservation equations over each cell of the mesh. By applying the divergence theorem we obtain for each cell an equation which describes the rate of change of the average of the conserved quantities within each cell, in terms of the fluxes across the edges of the cells and the effect of the source terms. In particular, rate equations associated with each cell have the form

$$A_i \frac{dU_i}{dt} + \sum_j (F_{ij} n_{ij1} + G_{ij} n_{ij2}) l_{ij} = A_i S_i$$

where the subscript *i* refers to the *i*th cell, A_i is the associated cell area, U_i the vector of averaged conserved quantities, and S_i is the source term associated with each cell. The subscript *ij* refers to the *j*th neighbour of the *i*th cell, i.e. it corresponds to the edges of the *i*th cell. We use $F_{ij} n_{ij1} + G_{ij} n_{ij2}$ to denote the approximation of the outward normal flux of material across the *ij*th edge, and use l_{ij} to denote the length of the *ij*th edge.

From the average values of the conserved quantities in each of the cells, we use a second order reconstruction to produce a representation of the conserved quantities as a piece-wise linear (vector) function. This function is allowed to be discontinuous across the edges of the cells, but the slope of this function is limited to avoid unnecessary oscillations. Across each edge, the reconstructed function is generally discontinuous. The Godunov method (Toro, 1992) usually involves approximating the flux across an edge by exactly solving the corresponding one dimensional Riemann problem normal to the edge. We use the central-upwind scheme of Kurganov et al. (2001) to calculate an approximation of the flux across each edge.

We finally obtain a fully discrete updating method by using a total variation diminishing Runge-Kutta method (TVD-RK) as described by Shu and Osher (1988) to discretise the rate equations.

The model output consists of values for *w*, *uh* and *vh* at every mesh vertex at every time step.

3. SOFTWARE

To implement a scenario the user specifies the geometry, the initial water level, boundary conditions (e.g. tide), and forcing terms (e.g. frictional resistance, wind stress, atmospheric pressure gradients). The geometry could be a digital elevation model of an estuary and a boundary could be a collection of time series derived from tide gauges or a deep water tsunami model.

To help with these tasks, a tool called pmesh has been developed which allows the user to set up the geometry of the problem interactively. Pmesh produces a triangular mesh of the study area in which the user can specify the locations and types of boundary conditions that apply as well as identifying regions for either mesh refinement or for assignment of values at run time. Figure 1 shows an example of a geometry generated by pmesh.

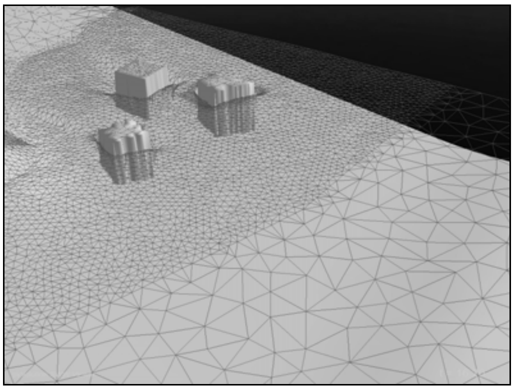

Figure 1. Triangular mesh generated by pmesh for a hypothetical run-up scenario.

When the model is run, the mesh is converted into a domain object which represents the study area, quantities, boundaries and forcing terms with the domain object also containing methods for time stepping, flux calculations, and all other numerical operations pertinent to the model.

The conserved quantities updated by the numerical scheme are stage *w*, through the conserved quantity *h*, and horizontal momentum. Bed elevation and friction are quantities that are not updated. Setting initial values for quantities is done through the method

```
domain.set_quantity(name, X, location, region)
```

where `name` is the name of the quantity (e.g. ‘stage’, ‘xmomentum’, ‘ymomentum’, ‘elevation’ or ‘friction’). The variable `X` represents the source data for populating the quantity and may take a variety of forms including constants, linear combinations of other quantities and arbitrary functions f(x, y), where x and y are assumed to be vectors in which case the quantity in question will take values for f at each location within the mesh. The source data can also take the form of an arbitrary set of points with associated values. The points need not coincide with triangle vertices or centroids and a penalised least squares technique is employed to populate the quantity in a smooth and stable way. Since the least squares technique can be time consuming for large problems, we employ a caching technique which automatically decides whether to perform the computations or retrieve them from a cache. This will typically speed up the build by several orders of magnitude after each computation has been performed once.

The parameter location determines whether the values should be assigned to triangle edge midpoints or vertices and region allows the operation to be restricted to a region specified by a symbolic tag or a set of indices.

Boundary conditions are bound to symbolic tags through the method domain.set_boundary which takes as input a lookup table implemented as a Python dictionary. The boundary objects are all assumed to be callable functions of vectors x and y. Several predefined standard boundary objects describe Dirichlet, Reflective, Transmissive, Temporal, and Spatio-Temporal boundaries. Additional problem-specific custom boundaries may also be readily defined.

Forcing terms can be written according to a fixed protocol and added to the model using the idiom domain.forcing_terms.append(F) where F is assumed to be a user-defined callable object.

When the simulation is running, the length of each time step is determined from the maximal speeds encountered and the sizes of triangles in order not to violate the Courant-Friedrichs-Levy (CFL) condition which specifies that no information should skip any triangles in one time step. With large speeds and small triangles, time steps can become very small. In order to access the state of the simulation at regular time intervals, AnuGA uses the method evolve:

```
For t in domain.evolve(yieldstep, duration):
      <do whatever>
```

The parameter duration specifies the time period over which evolve operates, and control is passed to the body of the for-loop at each fixed yieldstep. The internal time stepping is thus decoupled from the overall time stepping so that outputs may be stored, displayed or interrogated. The evolve method has been implemented using a Python generator.

Figure 2 shows a simulation of water flowing through structures in a hypothetical run-up scenario.

Figure 2. Hypothetical run-up scenario.

Most of the components of AnuGA are written in Python, an object-oriented programming language which has gained increasing popularity in the scientific computing community over the past decade due to its clarity, efficiency and reliability. With Python it is possible to develop complex pieces of software without undue distractions in dealing with idiosyncrasies of the software language syntax. Consequently, software written in Python can be produced quickly and can be readily adapted to changing requirements throughout its lifetime.

4. VALIDATION

The process of validating the AnuGA application is in its early stages, however initial indications are encouraging.

As part of the Third International Workshop on Long-wave Runup Models in 2004 (http://www.cee.cornell.edu/longwave), four benchmark problems were specified to allow the comparison of numerical, analytical and physical models with laboratory and field data. One of these problems describes a wave tank simulation of the 1993 Okushiri Island tsunami off Hokkaido, Japan (Matsuyama and Tanaka, 2001). A significant feature of this tsunami was a maximum run-up of 32 m observed at the head of the Monai Valley. This run-up was not uniform along the coast and is thought to have resulted from a particular topographic effect. Among other features, simulations of the Hokkaido tsunami should capture this run-up phenomenon.

The wave tank simulation of the Hokkaido tsunami was used as the first scenario for validating AnuGA. The initial dataset provided bathymetry and topography along with initial water depth and the wave specifications. The dataset also contained water depth time series from three wave gauges situated offshore from the simulated inundation area.

Figure 3 compares the observed wave tank and modelled AnuGA water depth (stage height) at one of the gauges. The plots show good agreement between the two time series, with AnuGA closely modelling the initial draw down, the wave shoulder and the subsequent reflections. The discrepancy between modelled and simulated data in the first 10 seconds is due to the initial condition in the physical tank not being uniformly zero. Similarly good comparisons are evident with data from the other two gauges. Additionally, AnuGA replicates exceptionally well the 32 m Monai Valley run-up, and demonstrates its occurrence to be due to the interaction of the tsunami wave with two juxtaposed valleys above the coastline. The run-up is depicted in Figure 4.

This successful replication of the tsunami wave tank simulation on a complex 3D beach is a positive first step in validating the AnuGA modelling capability. Subsequent validation will be conducted as additional datasets become available.

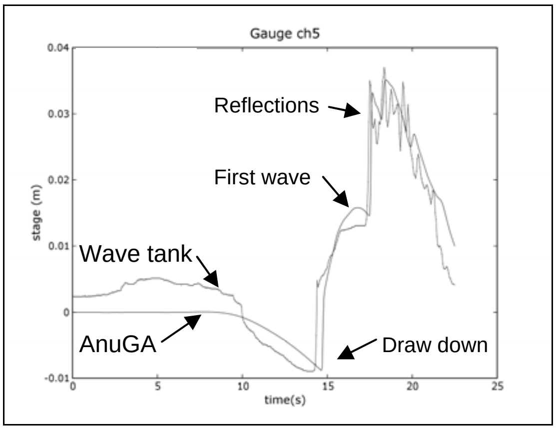

Figure 3. Comparison of wave tank and AnuGA water stages at gauge 5.

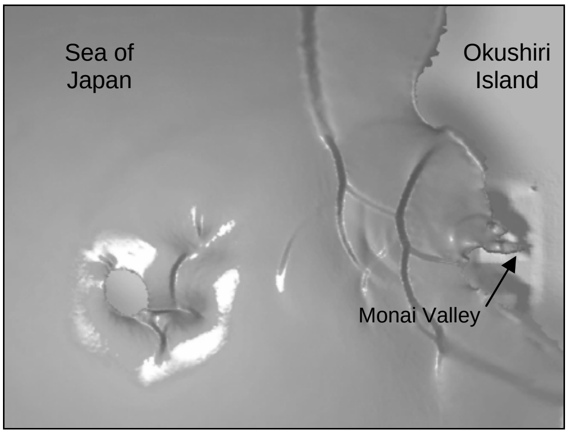

Figure 4. Complex reflection patterns and run-up into Monai Valley simulated by AnuGA.

5. APPLICATION

As part of an assessment of emergency service response to a catastrophic event, the impact of a tsunami at Wollongong on the NSW coast has been modelled. The initial scenario simulated a 6 m tsunami impacting a 7 km stretch of coastline from Bellambi Point south to Flagstaff Point along the south east coast of Australia (see Figure 5). This simulation demonstrated the extent of inundation and identified the suburbs and infrastructure most at risk.

Subsequent studies with greater spatial resolution were conducted for other sections of the coastline. Using the results from this modelling exercise the impact on buildings and other structures was estimated and the expected fatalities, injuries and economic losses quantified. Such information is invaluable to emergency management services for training purposes to enhance response capabilities and for long-term planning and mitigation strategies.

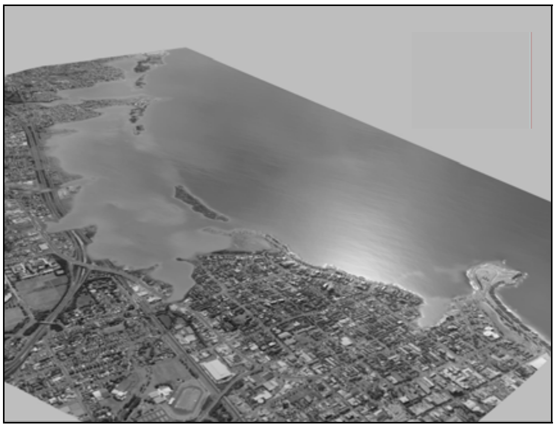

Figure 5. Simulated tsunami impact on Wollongong coast of NSW, Australia.

The AnuGA model works well for detailed inundation modelling of small sections like those mentioned above but currently less well for synoptic scenarios. To capture e.g. the source modelling and propagation across large regions we use deep water tsunami models such as MOST (Titov 1997) to provide boundary conditions for AnuGA. In fact, Geoscience Australia has embarked on a program using this approach to predict which coastlines are most at risk taking into account the return period of submarine earthquakes, tsunami wave propagation and the non-linear effects of local bathymetries.

6. CONCLUSIONS

AnuGA is a flexible and robust modelling system that simulates hydrodynamics by solving the shallow water wave equation in a triangular mesh. It can model the process of wetting and drying as water enters and leaves an area and is capable of capturing hydraulic shocks due to the ability of the finite-volume method to accommodate discontinuities in the solution.

AnuGA can take as input bathymetric and topographic datasets and simulate the behaviour of riverine flooding, storm surge and tsunami. Initial validation using wave tank data supports AnuGA's ability to model complex scenarios. Further validation will be pursued as additional datasets become available.

AnuGA is already being used to model the behaviour of hydrodynamic natural hazards. This modelling capability is part of Geoscience Australia's ongoing research effort to model and understand the potential impact from natural hazards in order to reduce their impact on Australian communities.

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