Bayesian Structural Equation Models: A Health Application

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EXTENDED ABSTRACT

Concepts of health are often multivariate or multidimensional. Structural equation modelling (SEM) is a multivariate method that incorporates ideas from regression, path-analysis and factor analysis. A Bayesian approach to SEM may enable models that reflect hypotheses based on complex theory. The development and application of Bayesian approaches to SEM has, however, been relatively slow but with modern technology and the Gibbs sampler, is now possible. This paper contributes to the knowledge of Bayesian methods in the SEM framework by illustrating how different sources of uncertainty in data can be incorporated into the modelling process. The particular aim is to develop a preliminary Bayesian approach to SEM for the longitudinal relationship between life events and health, which is extended to account for the suspected effect of telescoping. Telescoping is a tendency to recall events from the past as having happened more recently than when they actually occurred, and is suspected to have occurred due to the time recall component of some questions. It is expected that this basic model be extended to include other issues as required.

A preliminary Bayesian approach to SEM is initially considered that is posed independent of data characteristics. A model with uninformative priors, in the form of very large variances on prior distributions, is adopted. Given the inadequate convergence of this model, an approach that accounts for data characteristics is proposed. This latter approach takes three forms. Data is first centred as it is believed this may improve convergence. The same uninformative priors as previously are adopted with initial values randomly sampled from priors. Convergence though remained poor with variables demonstrating poor mixing and long-term trend.

An informed model is then proposed whereby the observed variables are centred around zero and scaled to a standard deviation of one. Informative priors on the parameters of interest are adopted, with means taken from an exploratory data analysis with tighter but still strongly overdispersed variances. Priors that were moderately informed with strongly overdispersed variances produced more stable results than alternative less informative priors. The importance of carefully choosing priors that allow the Markov Chain Monte Carlo (MCMC) algorithm to start in a region of high posterior probability is illustrated.

There is still concern that the algorithm fails to explore the entire space and may identify only a local mode and starting the algorithm from positions far away from this region of high probability did not lead to strong confirmation of other joint posterior modes. Further evaluation of this concern was undertaken by simulating data with a much stronger signal than in the dataset considered here. Much better mixing of all variables was observed and convergence was obtained in a much shorter number of iterations. The strong priors imposed in the model indeed enabled the algorithm to start in a region of high probability but did not constrain it to staying in that exact region.

The Bayesian framework additionally has the ability to model particular features of the data. Given the a priori anticipation that subjects would experience approximately the same number of life events in the two time periods considered, an investigation of this possible measurement error bias was undertaken by constructing an additive model. This application illustrates the ease with which the Bayesian hierarchical model can accommodate features such as measurement error.

Considerable issues in applications of repeated sampling remain relatively unexplored. As demonstrated here, the Bayesian framework provides a flexible coherent way of doing this. Further research remains into the best interpretation of coefficients and their posterior distributions under the Bayesian framework. These are general Bayesian problems of current international interest.

1. INTRODUCTION

It is common for notions of health and behaviour to be multidimensional. However univariate analysis of correlated outcomes remains the dominant form of analysis in the health sciences, which may result in distorted estimates of effect and variation. Structural equation modelling (SEM) incorporates ideas from regression, path-analysis and factor analysis. SEM allows the original predictors and outcomes to be summarised by their underlying latents while also accounting for the anticipated causal relationships between the latents. SEM also allows exploration of relationships using longitudinal data, which can take the form of repeated measures data.

It is postulated that a Bayesian approach to SEM may enable models that are easy to interpret and supported by data, while also portraying research hypotheses based on complex theories. Although Bayesian modelling has been adopted vigorously in the health research community, and despite some long-standing Bayesian discussion of certain aspects of factor analysis and SEM in the classical framework [eg Lee 1981; Press and Shigemasu, 1989], the development and application of Bayesian approaches to SEM has been relatively slow.

Congdon (Chapter 8) identifies some advantages of the Bayesian approach, including linking structural equation concepts to multi-level hierarchical models [Ansari et al. 2001] and estimation of conventional unidentifiable models [Scheines et al. 1999]. Recent developments of computer intensive sampling methods of estimation have revolutionalised the application of Bayesian methods in many fields including biostatistics [Congdon 2001].

With modern computers and the Gibbs sampler, a Bayesian approach to SEM is now possible and hence posterior distributions over parameters of SEM can be approximated with arbitrary precision, even for small samples. Some of the possible advantages of Bayesian analyses in SEM applications are suggested by Scheines et al. [1999] and Lee [1992] in terms of changing formal constraints to permit stochastic uncertainty. Recent developments introducing SEM concepts into maximum likelihood analyses are discussed by Ansari et al. [2001], who include an application of Monte Carlo estimation. Following early work by Lee [1981, 1992] and Press and Shigamasu [1989], Bayesian formulations of structural equation models have been presented by Scheines et al. [1999], Ansari et al. [2001] and Song and Lee [2001]. This literature has been recently reviewed by Congdon [2003, Chapter 8] in the wider context of latent variable models.

The present study aims to develop a basic Bayesian structural equation model which may be extended to similar problems by researchers. It is expected that this basic model be extended to include other issues as required.

People in middle age commonly experience multiple transitions including changes in employment, children leaving home and illness or death of parents. The impact of common life events on health is not well known, in particular among middle aged Australian women. A previous study that assesses the temporal relationship between life events and health will form the basis for the present study. It is based on retrospective data collected at two points in time, 1996 and 1998, for the mid age cohort of the Australian Longitudinal Study on Women's Health (ALSWH) [Brown et al. 1998]. A description of the data used for the study that assesses the relationship between life events and health is described elsewhere in more detail [Stojanovski, 2005]. In summary, theory postulates that life events affect both physical and mental health [Wilcox et al. 2003].and that mental health affects physical health [Ader et al. 1995]. The model depicting these relationships is displayed in Figure 1. This path diagram describes the causations among the latent variables in the model. The items used to represent each latent are not presented in the diagram but described elsewhere [Stojanovski, 2005] in detail and briefly here. A list of age-group specific life events relevant to women in Australia in the 1990s were developed for the ALSWH project. In both the baseline and follow-up survey participants were asked if they had experienced each event in the last twelve months. Events were summed to create the total number of events experienced at each time point. Mental health in was measured by dimensions of the Short-Form Health Status survey (SF-36) [Ware et al. 1994], a widely used measure of health related quality of life and physical health by the physical health domains of SF-36 along with doctor visits, medical conditions, prescribed medications and physical symptoms. A composite score was created for the set of items comprising physical and mental health.

The previous study by Stojanovski demonstrates that baseline health measures mediate the relationship between life events at baseline and the corresponding health measures at follow-up. The fully mediated model, whereby life events at baseline affect the health measures at follow-up only via their effect on health at baseline, is extended here using a Bayesian approach to illustrate the ease with which different prior

distributions can be incorporated. This enables a more informative portrayal of the relationships of interest. It is acknowledged that results from other studies can only be used as a guide in developing prior types as their characteristics do not match exactly those of the present study. Priors considered for this study will be centred on the evidence from these external studies and hence will be overdispersed. This is still much more informative than the typical uninformative or vague priors that are used to represent no prior information at all.

Due to the time recall component of the life event items asked in the questionnaires, respondents are expected to make errors because of incorrect recollection of information. Some differences in the number of experienced life events were expected due to variations in exposure to some events in the twelve months prior to each survey but was not expected for most events. A bias in reporting the number of life events is evident with an increased incidence of reporting life events in 1996 compared to the respective rates in 1998 (Figure 2). A concern is that this bias is due to

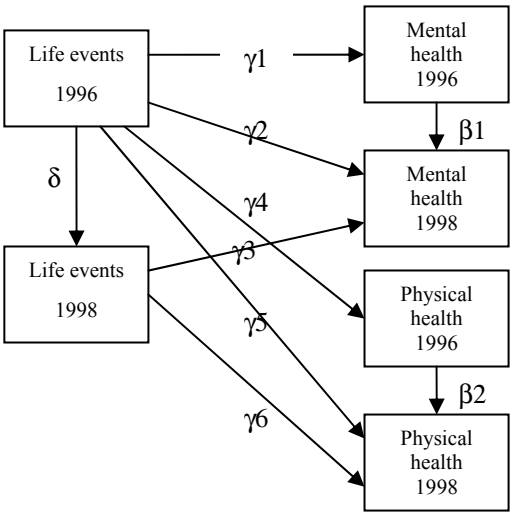

Figure 1. Structural Equation Model relating life events to health over time.

the limited response categories for items from questionnaires related to life events at baseline which can have the effect of bringing events from the past to a more recent time in memory, a concept known as telescoping [Fowler 1988].

2. METHODS

A preliminary Bayesian approach to SEM is considered in Section 2.1. Given the inadequate MCMC convergence of this model, an approach that accounts for data characteristics is proposed in Section 2.2. A preliminary Bayesian approach to account for the suspected effect of telescoping is presented in Section 2.3.

2.1 Description of Model

A general Bayesian structural equation model is posed independent of data characteristics. The following notation is used throughout this paper.

- Let $\underline{Y}_j$, j=1,..,4 be n-dimensional vectors representing the observed composite scores for the responses in the present study, namely mental health in 1996, mental health in 1998, physical health in 1996 and physical health in 1998, respectively, where n is the number of subjects. Denote the corresponding endogenous latent variables by θ_j, j=1,..,4.
- Similarly, let $\underline{X}_j$, j=1,2 be n-dimensional vectors representing observed composite scores for life events in 1996 and 1998, respectively. Denote the corresponding exogenous latent variables by θ_j , j=1,..,4.
- Factor loadings are denoted by λ_{Yj}, j=1,..,4, and λ_{Xj} , j=1,2. Similarly, error precisions are denoted by $1/\tau_{Yj}$, j=1,..,4, and $1/\tau_{Xj}$, j=1,2.

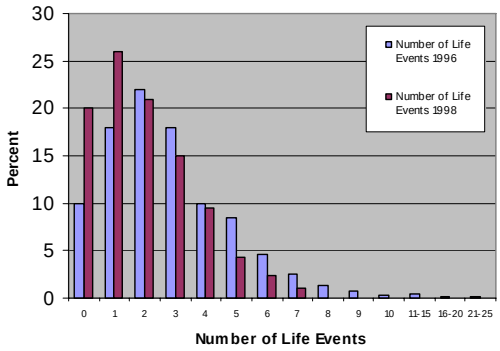

Figure 2. Graph Comparing the Number of life events reported in 1996 and 1998

Based on earlier considerations, λ_Y, λ_X, τ_X and τ_Y are assumed known. In a more general setting, this assumption could be relaxed by imposing prior distributions on the loadings and precisions. For example, a Gamma distribution for λ with parameters chosen to allow centring at the assumed value with appropriate uncertainty could be imposed. Similarly, the precision could be represented by $1/(\sigma^2 W)$, where W is the estimated variance-covariance matrix and

$$1/\sigma^2 \sim \chi_\nu^2 / \nu,$$

where ν is the corresponding degrees of freedom. These alternatives are not pursued here, given the preliminary nature of the present analysis.

The measurement model describing the

relationships between the observed variables Y and the latent health scores θ are given by

$$Y_{ij} \sim \mathrm{N}(\lambda_{Yj}\ \theta_{ji},\ \tau_{Yj})\ ,\ i = 1,..,n;\ j=1,..,4.$$

Similarly, the measurement model describing the relationships between the observed variables X and the latent life events η are given by

$$X_{ij} \sim \mathrm{N}(\lambda_{Xj}\ \eta_{ji},\ \tau_{Xj})\ ,\ i = 1,..,n;\ j=1,..,4.$$

The Gaussian distributions adopted for Y and X reflect the nature of the composite scores used in this analysis. More robust distributions or nonparametric representations could easily be substituted in a more general setting, since the resultant non-standard posteriors can be readily estimated using MCMC [Besag et al. 1995; Congdon 2003]. The correlation induced by the repeated measurements at 1996 and 1998 is represented through the life event latent variables as follows

$$\eta_{2i} = \delta\ \eta_{1i},\ i=1,..,n.$$

A vague hyperprior is imposed on the link δ, although as discussed later, a more informative prior can be constructed based on ancilliary information.

Following Figure 1, the structural models become, for i=1,..,n

$$\theta_{1i} = \gamma_1\ \eta_{1i}$$
$$\theta_{2i} = \beta_1\ \theta_{1i} + \gamma_2\ \eta_{1i} + \gamma_3\ \eta_{2i}$$
$$\theta_{3i} = \gamma_4\ \eta_{1i}$$
$$\theta_{4i} = \gamma_5\ \eta_{1i} + \gamma_6\ \eta_{2i} + \beta_3\ \theta_{3i}\ .$$

Finally, the following vague but proper hyper-priors are introduced

$$\eta_{1i} \sim \mathrm{N}(0,\sigma^2),\ i=1,...,n$$
$$\delta \sim \mathrm{N}(0,\sigma^2)$$
$$\gamma_j \sim \mathrm{N}(0,\sigma^2),\ j=1,..,4$$
$$\beta_j \sim \mathrm{N}(0,\sigma^2),\ j=1,2,.$$

where the variance, σ^2, is a common constant described in Section 2.2.

Uninformative priors are adopted, in the form of very large variances (σ^2=10^6) on prior distributions of δ, γ, β and initial values for MCMC analysis are randomly sampled from the priors. As shown in the results, convergence for this model was very poor, with many variables showing very little movement from their initial values and other variables showing definite non-stationarity over very long run lengths. The Bayesian model proposed here is general in that it does not account for characteristics of the data.

2.2 Original, Centred and Informed Models

Exploratory analysis of the data resulted in an evolution of more informed Bayesian models. A second model using centred data was considered in an attempt to improve convergence [Spiegelhalter et al. 2000]. Observed variables are centred around zero and scaled to a standard deviation of one. Uninformative priors are adopted as above (σ^2=10^6) and initial values for MCMC analysis are randomly sampled from priors. Convergence was marginally improved but still remained unacceptably poor, with variables still demonstrating very poor mixing and long-term trend. These results are not presented. The priors used in the model were thus modified from (unrealistically) vague to moderately informed.

For the informed model, observed variables are centred around zero and scaled to a standard deviation of one. Informative priors for δ, γ and β are adopted, with means taken from an exploratory data analysis that was undertaken, and with tighter but still strongly overdispersed variances. The initial values for MCMC analysis are set at the prior means: γ=(3.84, 0.28, -0.061, 4.10, 0.12, 0.011), δ = 0.52 and β = (0.93, 0.94). As shown in Section 3.2, this model that started the MCMC algorithm in a zone of high probability produced much more stable results in that it allowed much better mixing and increased the ability to identify a joint posterior mode.

2.3 Adjusting for Bias in the number of reported life events

The Bayesian framework additionally has the ability to model particular properties of data. It is anticipated *a priori* that on average subjects would experience the same number of life events in the year preceding 1996 as in the year preceding 1998. As demonstrated by the means in Table 2 (3 versus 2), the observed data do not support this. An investigation of this possible measurement error bias was undertaken by constructing an additive model described below.

Closer inspection of the estimates from the earlier Bayesian SEM analyses resulted in a relationship between life events in 1996 (X_1) and in 1998 (X_2) of the form $X_2 = \delta X_1$, where δ denotes a non-zero constant depicting the quantity by which life events in 1996 are multiplied to derive the same number of life events in 1998. Under the above *a priori* expectations, the posterior value of δ should be one. It is suspected though that for the present data, δ, takes on a value less than one, depicting the quantity by which life events in 1996 are multiplied to derive the same number of life events in 1998. This should be able to be assessed by inspecting the posterior distribution of δ in the models described in Sections 2.1 and 2.2.

An alternative additive representation of this bias was also considered. Under this model δ_i is set equal to one in the model, but the relationship $X_{1\text{true}}=X_{1\text{observed}}+\zeta$ was imposed, where ζ indicates the difference between the true number of life events in 1996 (X_{1true}) and the observed number of life events in 1996 ($X_{1observed}$), that is, the degree of over-reporting. Then X_{1true} is used in the remaining model equations. A prior mean of -2 was imposed on ζ. This model is referred to as the Additive Adjusted Model. This second model indicates the ease with which the Bayesian hierarchical model can accommodate features such as measurement error [Congdon 2001; Wolpert and Mengersen, 2005]. Results of this analysis are discussed in section 3.3.

Each analysis described below was run in Win BUGS [Speigelhalter et al. 2000] for a total of 500,000 iterations, with 500,000 iterations burn-in for a sample size of 7537..

3. RESULTS

3.1. Original Model Analysis

Summary statistics for the observed composite scores used as inputs (X,Y) are given in Table 1. The table also shows the small age range of participants in this study, justifying the omission of age from further SEM analysis.

	MH 96	MH 98	PH 96	PH 98	Age 96	LE 96	LE 98
Min	0.00	2.22	0.987	44.3	45.6	0.00	0.00
Q1	77.1	65.0	59.5	65.4	46.7	1.00	1.00
Q2	96.3	82.1	70.6	80.6	48.4	2.00	2.00
Mean	88.6	75.3	65.5	74.7	48.3	3.00	2.00
Q3.	106	90.4	76.5	86.7	49.2	4.00	3.00
Max	118	101	83.7	93.8	50.1	25.0	25.0

Table 1. Summary statistics for inputs to Bayesian SEM.

3.2. Informed/Adjusted Model Analysis

The original Bayesian model proposed in Section 2.2 was applied to the data. Plots of the posterior densities are depicted in Figure 3 for parameters linking health measures at baseline with the corresponding measures at follow-up. It can be seen that these parameters are unstable. The overall precision has some very large values consistent with its vague Gamma distribution. The instability of all these estimates is confirmed with the trace plots which are not presented. The centred model was then applied which only marginally improved the convergence and stability of the MCMC runs. Under the informed model, convergence diagnostics in WinBUGS were passed.

Compared to the original model, the standard deviations of the simulated gamma estimates are much smaller relative to the corresponding means and Monte Carlo methods. From these results the most influential variables in this analysis are γ_1, corresponding to Life Events 1996 and Mental Health 1996; γ_4, corresponding to Physical Health 1996 and Physical Health 1996; β_1 and β_2 corresponding to Physical Health 1996 and Physical Health 1998; and δ, corresponding to Life Events 1996 and Life Events 1998.

Estimates of the posterior mean, standard deviation and 95% credible intervals along with other summary statistics generated from the informed model are presented in Table 2.

3.3. Adjusting for Bias in Baseline Life Events

An investigation of the possible measurement

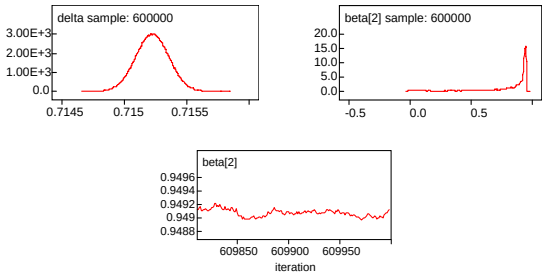

Figure 3. Posterior densities for the original Bayeisan model

Node	Mean	SD	MC Error	2..5	Median	97.5%
γ_1	5.8740	3.82E-4	1.659^{E}-5	5.8730	5.8740	5.8750
γ_2	0.2696	0.001027	5.424^{E}-5	0.2675	0.2696	0.2716
γ_3	0.5182	8.429E-4	3.053^{E}-5	0.5166	0.5182	0.5199
γ_4	6.2400	4.048E-4	1.762^{E}-5	6.2390	6.2400	6.2410
γ_5	0.1119	9.905E-4	5.141^{E}-5	0.1099	0.112	0.1137
γ_6	0.6455	8.416E-4	2.967^{E}-5	0.6439	0.6455	0.6472
B_1	0.8685	1.974E-4	1.074^{E}-5	0.8682	0.8685	0.8689
B_2	0.8895	1.744E-4	9.076^{E}-6	0.8891	0.8894	0.8898
τ	-0.0510	1.551E-5	4.175E-7	-0.05103	-0.0510	-0.05107

Table 2. Summary Statistics of Posterior Distribution generated from the Informed model.

error bias in reporting of life events between periods reported in 1996 compared to those reported in 1998 was undertaken through the activities of more closely interpreting the original model and constructing an additive model. Summary statistics are presented in Table 3. It is recognised that this measurement error is likely due to the fewer response options to questions related to life events at baseline. Women appear to over-report on life events in the present time frame if no previous time frames are provided as

response options, this effect known as telescoping as described previously.

The posterior mean for τ is 0.72 with very small standard deviation (1.33E-4). This indicates a 28% inflation in the average number of life events reported in 1996, compared with 1998 and reflects the suspicion, described in Section 2.3, of biased reporting of life events.

The alternative additive adjusted model was also fitted to these data. Noticeably greater mixing of the MCMC chains for all variables was observed under this model. Model parameters were inspected for stability and conformity to the anticipated collection. The corresponding density plots are shown in Figure 4 and support convergence. As expected, the posterior estimates are different under the two models, but the pattern of estimates is the same and the same variables impose most influence. Interestingly, despite a prior mean of -2 being imposed on the τ under the additive adjusted model, the posterior mean is very tightly around -0.051. Note that it is difficult to relate this difference, obtained using the centred data, back to the original scale under this additive model. However, it does strongly support the suspicion of a bias in the number of life events

Table 3. Comparison of the original and the additive models

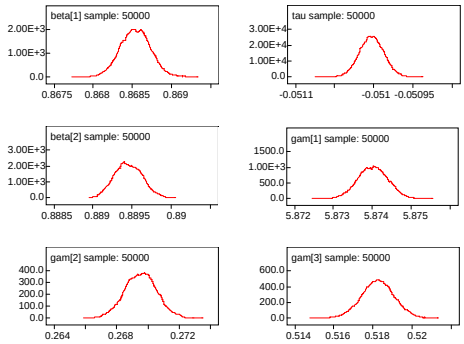

Figure 4. Posterior densities of parameter estimates generated from the Adjusted Additive Model.

4. DISCUSSION AND CONCLUSIONS

The Bayesian structural equation model formulated above follows the LISREL representation as described by Congdon [2003]. Model generalisations can be easily introduced as considered appropriate. The model developed in Section 2 takes advantage of both the distributional properties of the composite scores and the corresponding error structure. It is conceptually easy to augment the model to include a hierarchy that estimates the composite scores using the individual item data. Alternatively, the hierarchy describing the composite scores could be omitted entirely from the model, with estimation of the relationships between the latent variables coming directly from the individual item scores.

Because there are only two time periods under consideration, the bivariate nature of the composite scores over time can be reflected through the single variable, δ. An alternative is to consider a fully multivariate framework for (Y,X) and (θ, η) which would accommodate more general correlation structures between the observed composite scores and between the latent variables. This would also extend more naturally to multiple time periods. This formulation of the model was indeed constructed here, but was difficult to implement in the version of WinBUGS used for analysis. It is anticipated that later versions of this software will better accommodate these higher-dimensional analyses.

Further analysis of the data was performed after a careful examination of the data. As evidenced in the conducted exploratory data analysis reported elsewhere [Stojanovski, 2005], the dataset considered here creates a challenging context for SEM analysis. Many of the variables demonstrate pairwise relationships which, when combined with the uncertainty induced by vague priors, results in a poorly identifiable model and regions of relatively flat posterior probability. As demonstrated by the analyses under the three models above, in this situation it is important to carefully choose priors that allow the MCMC algorithm to start in a region of high posterior probability.

There is still a concern that the algorithm fails to explore the entire space and may identify only a local mode. This concern was partially ameliorated here by starting the algorithm within initial values ±10% of the univariate means. Very similar posterior estimates were obtained from all starting points. Given the tighter variances on the priors in the informed models, it did not make sense to initiate the algorithm in regions further away from this, nor did it make sense to change the prior means. Moreover, as indicated by the poor results of the original and centred models, starting the algorithm from positions far away from this region of high probability did not lead to strong confirmation of other joint posterior modes.

Further evaluation of this concern was undertaken by simulating data with a much stronger signal than in the dataset considered here. The algorithm

	Original Model		Additive Model	
Node	Posterior Mean	Posterior S.D.	Posterior Mean	Posterior S.D.
γ_1	7.04	7.7^{E}-4	5.87	3.8^{E}-4
γ_2	1.25	1.44	0.27	1.0^{E}-3
γ_3	0.71	0.83	0.52	8.4^{E}-4
γ_4	7.48	8.2^{E}-4	6.24	4.0^{E}-4
γ_5	1.04	1.46	0.11	9.9^{E}-4
γ_6	0.76	0.84	0.65	8.4^{E}-4
β_1	0.72	0.28	0.87	2.0^{E}-4
β_2	0.76	0.27	0.89	1.7^{E}-4
τ	0.72	1.33^{E}-4	-0.051	1.5^{E}-5

returned estimates that were equal (within Monte Carlo error) to the simulated values. Moreover, much better mixing of all variables was observed and convergence was obtained in a much shorter number of iterations (200,000). Reassuringly, the strong priors imposed in the model enabled the algorithm to start in a region of high probability but did not constrain it to staying in that exact region. Using these models we are able to explore the probability distributions of middle aged women exposed to life events under certain scenarios which may be informative and provide some insight about the model and behaviour of the parameters.

Considerable issues in applications of repeated sampling remain relatively unexplored. As demonstrated here, the Bayesian framework provides a flexible coherent way of doing this. Further research remains into the best interpretation of coefficients and their posterior distributions under the Bayesian framework. These are general Bayesian problems of current international interest [Congdon 2003].

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