Comparison of Two Stochastic Spatial Daily Rainfall Generation Approaches

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EXTENDED ABSTRACT

Daily rainfall is a key input into models that simulate water resources, agricultural and ecological systems. Stochastic rainfall data provide alternative realisations that are equally likely to have occurred, and are often used to drive hydrological and other models to quantify uncertainty in environmental systems associated with climatic variability.

This paper describes the comparison of two stochastic spatial daily rainfall generation approaches: a multi-site two-part model (M2P) and a transition probability matrix-random cascade model (CAS), using 101 years of rainfall data across the eastern Gippsland region in south-east Victoria, Australia.

The M2P model consists of a two-state, first-order Markov chain for rainfall occurrences and a two-parameter Gamma distribution for rainfall amounts. It generates rainfall simultaneously at multiple locations by driving a collection of individual models with serially independent but spatially correlated random numbers. In the CAS approach, daily regional rainfall is first generated using a first-order transition probability matrix with a two-parameter Gamma distribution for rainfall amounts in the largest state. The spatial rain field is then simulated using a non-homogeneous random cascade model that utilises scaling invariance features in the historical rain field.

The M2P and CAS models are used to generate 20 replicates of 101-year daily concurrent catchment average rainfall time series for five catchments (Tambo, Nicholson, Mitchell Low, Mitchell Up and Avon) across the eastern Gippsland region. These generated rainfall time series are then used as inputs into a calibrated daily conceptual rainfall-runoff model for each of the four major catchments (Tambo, Nicholson, Mitchell and Avon) to generate 20 replicates of 101-year daily concurrent catchment flow time series.

The stochastic flow simulations using rainfall inputs from M2P and CAS are assessed by comparing key statistics (spatial and temporal) in the stochastic replicates with those of the historical data. The statistics assessed are: correlations of 1-day, 3-day and annual flows between catchments; mean annual flow, standard deviation of annual flow and 5-year low flow total in the four catchments; and 1-day and 3-day annual exceedance probabilities (AEPs) in the four catchments. Runoff (or flow), rather than rainfall, are assessed because it is the variable directly affecting catchment yield and flood studies. In any case, the general results for stochastic rainfall and flow simulations from the M2P and CAS models are similar, but with the errors accentuated in the flow.

The results indicate that both models slightly overestimate mean annual flow, simulates the inter-annual variability well, and the 5-year low flow total reasonably. M2P underestimates the spatial 1-day and 3-day correlations slightly while CAS overestimates the correlations, which will lead to slight underestimations and overestimations respectively in regional flood estimates. M2P also underestimates the spatial annual correlations, which will lead to underestimation of droughts in system simulations. The CAS model simulates 1-day and 3-day flow AEP characteristics much better than the M2P model, and is therefore a better model for regional flood studies.

Many of the limitations in the M2P model can be overcome with model improvements, and the paper provides some suggestions. The main limitation of the CAS model is the absence of space-time correlation of rain fields on consecutive days, and the limitation in simulating the clustering (i.e. spatial correlation) of daily rain field during extreme storm events, both of which are difficult to overcome and require further research.

1. INTRODUCTION

Daily rainfall is a key input into hydrological models that estimate flow, sediment, pollutant loads and other hydrological fluxes and state variables. Rainfall is highly variable over space (e.g. point, catchment, regional) and time (e.g. daily, seasonal, inter-annual) scales. The use of historical rainfall time series as input into hydrologic models provides results that are based on only one realisation of the past climate. Stochastic rainfall data provide alternative realisations that are equally likely to have occurred, and are often used as inputs into hydrologic models to quantify uncertainty in environmental systems associated with climatic variability, allowing informed risk-based design, system operation and environmental management decisions to be made (McMahon et al. 1996).

Stochastic rainfall data are random numbers that are generated so that they have the same statistical characteristics (e.g. mean, variance, long-term persistency, auto-correlations) as the historical data from which they are based. Different characteristics are important for different applications (e.g. extreme rainfall for floods, dry spell and long-term persistency for droughts). Each stochastic replicate (sequence) is different and has different characteristics compared to the historical data, but the average (and distribution) of each characteristic from all the stochastic replicates should be the same as the historical data. Generation of daily rainfall data at a single site is a well-researched area. However, for accessing environmental systems at the regional scale, the spatial dependence of rainfall must be accounted for, in addition to the preservation of statistical properties of rainfall series at each site.

There are two major groups of stochastic spatial rainfall models: multi-site rainfall models and space-time rainfall models. Multi-site rainfall models are extensions to single site stochastic rainfall models that simulate multi-site rainfall concurrently using serially independent but spatially correlated uniform random numbers (e.g. Wilks 1998). More complex conditional multi-site models are hidden state multi-site Markov models where different probability of rainfall occurrence at each site is conditional on a number of hidden states (e.g. regional weather classes, atmospheric circulation patterns). The rainfall intensities at all the sites are then simulated concurrently based on the rainfall correlation structure for each season and for each weather class, or resampled from the historical rainfall record (e.g. Zucchini and Guttorp 1991; Pegram and Seed 1998; Charles et al. 1999). In general, multi-site rainfall models are not parsimonious, difficult to parameterise, and suffer from the deficiency in preserving the cross-correlation as the number of stations increases (Srikanthan and McMahon 2001).

An alternative to multi-site rainfall model is the space-time rainfall model. There are two general approaches to stochastic space-time rainfall modelling: cluster point process; and scaling-based multiplicative random cascade approach. Cluster point process models (Northrop 1998; Cowpertwait et al. 2002) are intermediate stochastic models that combine both physical and stochastic processes into their model structure. They are developed to reproduce the hierarchical spatial-temporal organisation in the observed rain fields (LeCam 1961; Waymire et al. 1984), which is defined such that rain clusters (fields) occur in a temporal Poisson process, rain bands (storms) occur within each field in a spatial Poisson process, and rain cells occur in each storm, clustering in space and time. There are two problems with cluster point process models: the overall number of parameters is quite large and difficult to estimate unambiguously arising from the need to characterise rainfall at each scale separately in the model hierarchy (Sivapalan and Wood 1987); and it cannot describe the statistical structure of rain fields (e.g. intermittency, non-homogeneity) over a large range of scales (Foufoula-Georgiou and Krajewski 1995).

Stochastic multiplicative random cascade models utilise certain scaling invariance features, such as extreme variability and strong intermittence, seen in the observed rain fields to model space-time rainfall (Lovejoy and Schertzer 1990; Gupta and Waymire 1990). Theoretical arguments and empirical evidence suggest that spatial and temporal organisation of rain fields tend to exhibit certain self-similarity in their patterns at different scales, and can be modelled within the multifractal framework (Seed 2003). This self-similarity property enables parsimonious parameterisations of rain fields over a wide range of scales, hence circumventing the problem of separate parameterisation at each scale in the cluster point process approach (Lovejoy and Schertzer 1990). The conceptual basis of multiplicative random cascades originates from the turbulence theory, where a cascade of turbulent eddies is seen as transferring kinetic energy from a large energy scale progressively to smaller dissipation scales (Over and Gupta, 1996). The analogy to rainfall is that total mass of rainfall is disaggregated in a scaling hierarchical manner, such that an area of higher intensity is embedded in larger areas of lower intensity, which are part of even larger structures but of even lower intensity (Jothityangkoon et al. 2000).

This paper assesses the performance of two stochastic spatial daily rainfall generation approaches: a multi-site two-part (M2P) approach; and a transition probability matrix-random cascade (CAS) approach, using 101 years of space-time rainfall data across the eastern Gippsland region in Victoria. Stochastic flows are generated using a calibrated daily conceptual rainfall-runoff model for each catchment, using rainfall inputs from M2P and CAS. The stochastic flow (and also rainfall) series are then assessed by comparing the key statistics (spatial and temporal) in the stochastic replicates with those of the historical data. Runoff (or flow), rather than rainfall, are assessed because it is the variable directly affecting catchment yields and floods.

2. MODEL DESCRIPTION

2.1. Daily Multi-Site Two-Part Model (M2P)

The daily multi-site two-part model (M2P) consists of a two-state, first-order Markov chain for rainfall occurrences and a two-parameter Gamma distribution for rainfall amounts. The model generates rainfall simultaneously at multiple sites by driving a collection of individual models with serially independent but spatially correlated random numbers based on the procedure described in Wilks (1998). Seasonality is considered by model calibration and simulation in different months. Nesting of the daily model in monthly and annual rainfall models resulted in improved preservation of monthly and annual characteristics (Srikanthan 2005). M2P model is one of the stochastic models in SCL (Stochastic Climate Library, http://www.toolkit.net.au/scl), a software product in the Catchment Modelling Toolkit designed to facilitate the generation of stochastic climate data.

2.2. Daily TPM and Cascade Model (CAS)

The stochastic daily space-time rainfall model (CAS) used here comprises a transition probability matrix-based temporal areal rainfall model, and a scaling-based spatial rainfall disaggregation model (Jothityangkoon et al. 2000; Tan 2004). Daily temporal regional rainfall is first generated using a first-order TPM (transition probability matrix) with a two-parameter Gamma distribution for rainfall amounts in the largest state. The generated daily regional rainfall time series is then disaggregated into daily spatial rain fields using a modified non-homogeneous random cascade model. Concurrent daily catchment average rainfall time series for each of the five catchments are then derived from the simulated daily rain fields.

In the TPM modelling, the daily regional rainfall can occur in one of up to ten states: state 1 is dry (rainfall less than 0.1 mm), and states 2 to 9 are intermediate rainfall states with lower and upper bounds, and state 10 is the largest rainfall state with no upper bound (rainfall greater than 25 mm). A shifted Gamma distribution is used to model rainfall amounts in the unbounded largest state, while a linear distribution is used for states 2 to 9. The parameters in the model, which are estimated from the historical data, are therefore the transition probabilities of being in a particular state given the state on the previous day, and the two parameters of the Gamma distribution for the largest state. The seasonality in occurrence and magnitude of daily rainfall are taken into account by considering each month separately (three-running month is used here). Boughton's adjustment (Boughton 1999) is used to reproduce the rainfall inter-annual variability.

A non-homogeneous random cascade model is used here to disaggregate the generated daily regional rainfall into rain field. It is a modified version (Tan 2004) of the model described by Jothityangkoon et al. (2000) that improved the realism of simulated rain fields, notably during extreme events. The model also has a deterministic component to explicitly model the non-homogeneities due to systematic spatial gradient in the historical daily rain fields, and strong seasonal dependence of both the spatial gradients and rainfall intensity.

3. STUDY AREA, DATA AND RAINFALL-RUNOFF MODELLING

Figure 1 shows a meso-scale square region of 128 km x 128 km (i.e. 32 x 32 cells of 4 km x 4 km each) covering the eastern region of the Gippsland Lakes catchment, in south-east Victoria, Australia. The square region is devised so as to adequately and tightly cover the five catchments comprising the Tambo, Nicholson, Mitchell Low, Mitchell Up and Avon to suit the random cascade modelling approach.

Daily catchment average rainfall used here is 101 years (1900-2000) of historical data derived from the SILO 0.05º x 0.05º daily girded rainfall (QDNRM 2000). Mean monthly potential evapotranspiration (PET) values (required for the rainfall-runoff modelling) are obtained from the PET maps produced jointly by the CRC for Catchment Hydrology and Australian Bureau of Meteorology (Wang et al. 2000). Twenty five years of river flow data are obtained from the Victorian Water Resources Data Warehouse (http://www.vicwaterdata.net).

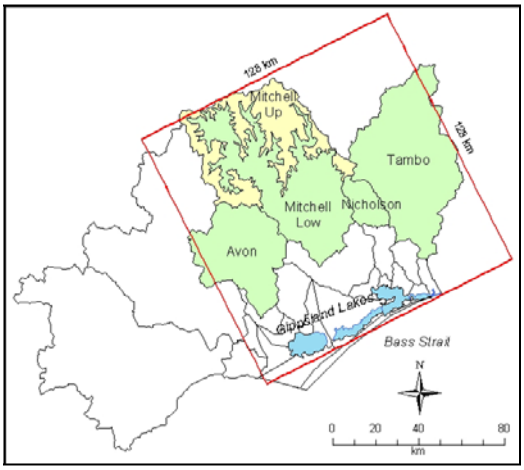

Figure 1. Eastern Gippsland Lakes catchments enclosed in a meso-scale modelling square box.

The M2P and CAS approaches are used to generate 20 replicates of 101-year daily concurrent catchment average rainfall time series for the five major catchments across the eastern Gippsland region. These generated rainfall time series are then used as key inputs into a calibrated daily conceptual rainfall-runoff model SIMHYD for each of the four major catchments (Tambo, Nicholson, Mitchell combined and Avon) to generate 20 replicates of 101-year daily concurrent catchment flow series. SIMHYD (http://www.toolkit.net.au/rrl) is chosen because it is simple (seven parameters), easy to calibrate, and has been used extensively to simulate flows across Australia catchments (Chiew and McMahon 1994, Chiew et al. 2002). The SIMHYD models for catchments flowing into the Gippsland Lakes are well-calibrated against the 25 years of historical flow data (here models with standard calibration are used, see Tan et al. 2005).

4. RESULTS

The plots in Figure 2 summarise the spatial correlations of 1-day flow, 3-day flow and annual flow between the four catchments simulated by SIMHYD using 20 replicates of 101-year daily rainfall time series generated by M2P and CAS (box and whisker plots), and compare them against those of the historical data (OBS) (solid red squares). The blue dash line is the median of the statistic from the 20 generated replicates, the upper and lower box indicate the 25th and 75th percentiles, solid circles give the 5th and 95th percentiles, and crosses represent the 2.5th and 97.5th percentiles, while the whiskers indicate the minimum and maximum values.

The plots in Figure 3 show the mean and standard deviation of annual flow, and 5-year low flow total. The left y-axis corresponds to the box and whisker plots that indicate the distribution of the ratio between the statistic in the stochastic replicates and that of the historical data. The right y-axis corresponds to the red solid squares that indicate the absolute values of the historical data.

The plots in Figure 4 show the annual exceedence probability (AEP) curves for 1-day (circles) and 3-day (triangles) annual maximum flows. The solid blue symbols represent the historical data, while the generated flows are plotted in hollow red (M2P) and green (CAS) symbols.

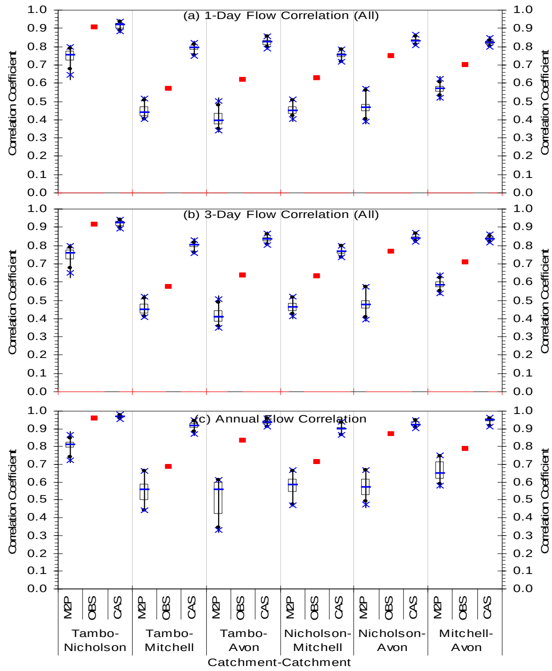

Figure 2. Spatial correlation of flows.

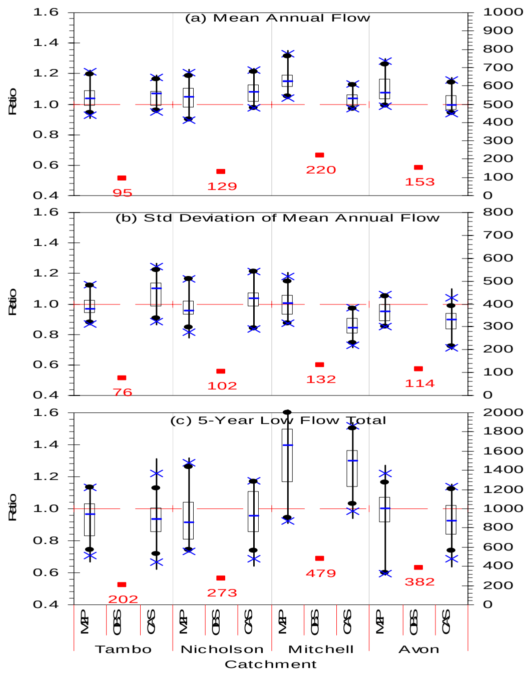

Figure 3. Annual flow statistics.

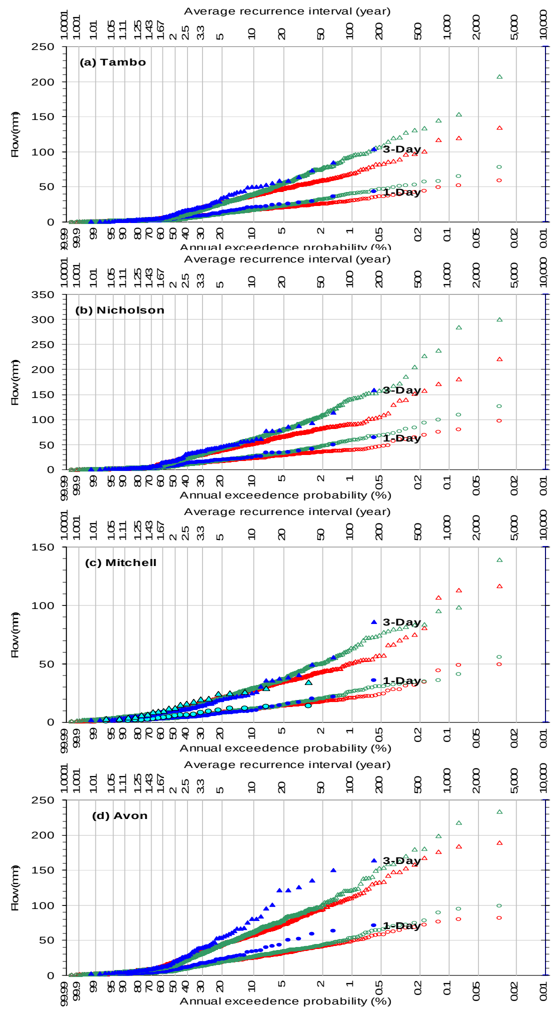

Figure 4. Flow AEP curves.

Runoff (or flow), rather than rainfall, are assessed because it is the variable directly affecting catchment yields and floods. In any case, the general results for stochastic rainfall and flow simulations from the M2P and CAS models are similar, but with errors accentuated in the flow.

5. DISCUSSION

5.1. Spatial Correlation

The correlations at short time scales (e.g. daily, 3-day total) are important for regional flood studies, while correlation at longer time scale (e.g. annual) are important for assessing regional water resources and drought. The results indicate that M2P model underestimates the spatial correlation at both daily, 3-day and annual time scales (Figures 2a-c). This is because the model attempts to preserve the spatial dependence structure in the historical data by generating correlated pseudo-random numbers (standard normal variates), and underestimation occurs due to downward transformation bias (from normal variates into uniform variates). This bias becomes larger in accumulations over longer periods (e.g. annual), and is further accentuated through the rainfall-runoff process. This could possibly be improved by forcing the correlation upward in calibration to match the historical rainfall correlation.

The CAS approach, on the other hand, overestimates the spatial correlation at all time scales. This is expected, as the multiplicative random cascade approach in CAS is known to produce simulated rain fields that tend to decorrelate too quickly and hence appear to be less clustered (e.g. Seed et al. 1999; Tan 2004). This also leads to more simulated rainy days with light drizzle than the historical rainy days at the catchment scale. For these reasons, CAS will always overestimate spatial correlation with poorer stochasticity.

The implications of the results are that: slight underestimation of the spatial correlations in 1-day and 3-day flows in M2P could lead to slight underestimation of regional floods, while underestimation in annual flows is a shortcoming for regional water yield and drought assessment.

Overestimation of spatial correlations in CAS may not be a serious drawback because flows in close-by catchments tend to be highly correlated (particular in annual time scale). However, this can be a problem for regions with weakly correlated flows in close-by catchments. The lack of stochasticity in the generated spatial correlations in CAS may also pose a problem for risk-based assessment.

Although both models have problems in reproducing the exact cross-correlations in the historical data, they are able to reproduce the pattern in the spatial dependence structure. Note that the four catchments span across an area more than 100 km in the east-west direction, but there is no discernible relationship between the strength of spatial dependence with distance.

5.2. Annual Characteristics

Annual characteristics, such as the mean and standard deviation of annual flow are key characteristics for assessing system yields and hydro-climatic variability, while the 5-year low rainfall total reflects persistent low rainfall conditions over several years, and is an important characteristic in drought studies.

The results show that both M2P and CAS slightly overestimate the mean annual flow by 5-10% (median) for all the four catchments (Figure 3a). M2P also produces slightly larger spread in the mean annual rainfall than CAS in the 20 stochastic replicates. Overall, CAS reproduces mean annual flow (similar to rainfall, not shown here) slightly better because unlike M2P which generates all rainfall amounts using a two parameter Gamma function, the TPM model in CAS has many intermediate states and a largest state (fitted with two parameter Gamma function), hence simulates the rainfall amount better.

The standard deviations of generated mean annual flow for all the catchments are also preserved with the medians largely confined within 10% of the historical values (Figure 3b). Slight underestimation is observed in the Mitchell and Avon (notably in CAS), both catchments have the highest standard deviation of historical mean annual flow amongst the four catchments.

Both M2P and CAS also preserve the 5-year low flow total, with the medians in the 20 replicates within 10% of the historical values in all catchments except the Mitchell (Figure 3c). The uncertainty in the 5-year low flow in the 20 replicates for all four catchments is generally high (40-60% higher than the historical values). This result is not surprising, given that the 5-year low total is a statistic that reflects longer-term variability (assuming stationary in the climate) which can only be reliably modelled with longer historical record. The corresponding uncertainty in the rainfall replicates is 20-30%) Similar pattern of uncertainty in the generated flow is also observed in the generated vs. historical rainfall (results not shown here), indicating that the errors are accentuated from the generated rainfall into flow thru the non-linear rainfall-runoff process.

5.3. Annual Exceedence Probabilities

For meaningful flood risk assessment, a key feature that must be preserved in a stochastic rainfall generation model is the characteristic of extreme high rainfall and flow. 1-day and 3-day AEP curves of the annual maxima in the 20 generated flow replicates of M2P and CAS are plotted against those of the historical data at each of the four catchments (Figure 4a to 4d). The plots show that CAS reproduces 1-day and 3-day flood characteristic remarkably well for all catchments except the Avon, while M2P slightly underestimates the 1-day flow AEP, but grossly underestimates the 3-day flow AEP (notably for events rarer than 5% AEP (i.e. 20-year average recurrence interval, ARI). In the Avon, both M2P and CAS underestimate the historical 1-day and 3-day AEP. It is interesting to note that slightly different patterns are observed in the comparison of the rainfall AEP curves (results not shown here). For 1-day rainfall AEP, both M2P and CAS produce results that match the historical curves, with M2P performing slightly better in some catchments. However, for the 3-day rainfall AEP, the performance of CAS is better except in the Avon, while M2P performs poorly in all catchments.

M2P is unable to simulate 3-day rainfall because the approach does not account for serial correlation in rainfall amount generation (only in rainfall occurrence), leading to deterioration in not just the 3-day flow, but also the daily flow, since the memory length of floods in the hydrologic system may last for a few days (due to antecedent soil moisture condition). This shortcoming in M2P may be improved by incorporating a multi-site Markov models for rainfall amount generation. In CAS, serial correlation is captured during the generation of daily rainfall at the regional scale using a first-order TPM. The correlation is propagated into daily rainfall at the catchment scale as the random cascade model merely disaggregates the generated daily regional rainfall into daily spatial rain field. However, the lack of daily space-time correlations (no memory between spatial rain fields over consecutive days) in the random cascade model is a challenge for further research into space-time coupling in daily rain field simulation. In CAS, the inability of the random cascade model in simulating the clustering (i.e. spatial correlation) of daily rain field during storms (within the same day) could be the reason why the extreme rainfall (and flow) in the Avon (which has seen a number of relatively more concentrated storms in the historical record) is underestimated.

6. CONCLUSIONS

This paper assesses the performance of two stochastic spatial daily rainfall generation approaches: a multi-site two-part model (M2P) and a transition probability matrix-random cascade model (CAS), using 101 years of rainfall data across the eastern Gippsland region in south-east Victoria, Australia.

The M2P and CAS models are used to generate 20 replicates of 101-year daily concurrent catchment average rainfall time series for five catchments (Tambo, Nicholson, Mitchell Low, Mitchell Up and Avon) across the eastern Gippsland region. These generated rainfall time series are then used as inputs into a calibrated daily conceptual rainfall-runoff model for each of the four major catchments (Tambo, Nicholson, Mitchell and Avon) to generate 20 replicates of 101-year daily concurrent catchment flow series.

The stochastic flow simulations using rainfall inputs from M2P and CAS are assessed by comparing key statistics (spatial and temporal) in the stochastic replicates with those of the historical data. The statistics assessed are: correlations of 1-day, 3-day and annual flows between catchments; mean annual flow, standard deviation of annual flow and 5-year low flow total in the four catchments; and 1-day and 3-day annual exceedance probabilities (AEPs) in the four catchments. Runoff (or flow), rather than rainfall, are assessed because it is the variable directly affecting catchment yield and flood studies. In any case, the general results for stochastic rainfall and flow simulations from the M2P and CAS models are similar, but with the errors accentuated in the flow.

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Many of the limitations in the M2P model can be overcome with model improvements, and the paper provides some suggestions. The main limitations of the CAS model is the absence of space-time correlation of rain fields on consecutive days, and in simulating the clustering (i.e. spatial correlation) of daily rain field during extreme storm events, both of which are difficult to overcome and require further research.

7. ACKNOWLEDGMENTS

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