Calibration of a Daily Rainfall-Runoff Model to Estimate High Daily Flows

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EXTENDED ABSTRACT

Rainfall-runoff models are becoming an increasingly indispensable tool in flood studies and operational flood forecasting for integrated catchment planning and flood emergency management. In flood studies, the objective of a rainfall-runoff model is to estimate flood hydrographs using precipitation (rainfall and snowfall), which is the major flood-producing forcing, as the key input.

Flood estimation can be performed on the basis of event-based simulation or continuous simulation. A model with continuous water balance accounting has the attractive feature that dynamic factors affecting runoff production may be represented explicitly (Lamb 1999). In particular, a conceptual rainfall-runoff model can simulate the changing antecedent moisture conditions that determine the catchment runoff response for a given rainfall event, and the problem of baseflow estimation in event-based approach can be circumvented. In addition, the design flood estimate need not be associated with specific design storm, instead it is derived directly from continuous rainfall records, or from Monte-Carlo simulation using stochastic rainfall data. With the advent of computing capability, flood frequency estimation by continuous modelling approach has been widely applied (e.g. Lamb 1999, Cameron et al. 2000).

Although there is a plethora of rainfall-runoff models ranging from very simple black box schemes to complex, differential, distributed models, they differ little in their ability to simulate daily flows. The choice of the model depends on the objective of the study, the aspects that are considered to be of primary importance in the physical rainfall-runoff transformation process, and the availability of data and resources. Usually, a simpler conceptual modelling approach that conceptualise a catchment as a number of interconnected storages with mathematical

functions describing the movement of water into, between and out of them is suitable for most applications.

This paper describes the improvement and calibration of a daily conceptual rainfall-runoff model SIMHYD (<u>http://www.toolkit.net.au/rrl</u>) to estimate high daily flows for flood risk assessment. SIMHYD is chosen because it is simple (seven parameters), easy to calibrate, and has been used extensively to simulate flows across Australian catchments (Chiew and McMahon 1994, Chiew et al. 2002). Several modifications are made to improve the model performance in flood simulation. An internal Muskingum flow routing routine is included to improve the timing of daily flows, and a degree-day snow component is added to model the snow processes.

More significantly, a modified objective function with emphasis placed on comparing high flows during model calibration is introduced to ensure that flood events are properly captured. Accurate modelling of high flows is central to the determination of the annual exceedence probability (AEP) of extreme flood levels in any flood study.

The improved SIMHYD model is calibrated on six major river catchments flowing into the Gippsland Lakes with 25 years of observed daily rainfall and flow data. The results show that the modified SIMHYD model that is calibrated using the new objective function that emphasises on high daily flows, is capable of simulating continuous daily flows that capture the principal characteristics of floods including the 1-day and 3-day flows, while preserving the overall hydrologic balance of the system and the realism of the model parameters.

The improved and calibrated SIMHYD model, capable of simulating high flows more accurately, can then be used to simulate continuous daily flows driven by stochastically generated daily rainfall data for flood risk assessment.

1. INTRODUCTION

Rainfall-runoff models are becoming an increasingly indispensable tool for estimating flood hydrographs. Models with continuous water balance accounting has the advantage over models with event-based simulation in that dynamic factors affecting runoff production may be represented explicitly. In addition, the design flood estimate need not be associated with specific design storm, instead it is derived directly from continuous rainfall records, or from Monte-Carlo simulation using stochastic rainfall data.

Daily rainfall-runoff models can be categorised into three groups: black box, process and conceptual models (Chiew et al. 1993). In the black box approach, empirical functions (e.g. simple mathematical expressions, time series equations or artificial neural network) are used to relate runoff (output) to rainfall (input), and only the input and output have physical meanings. On the other hand, process models such as Système Hydrologique Européen (SHE) (Abbott et al. 1986) and Institute of Hydrology Distributed Model (IHDM) (Beven et al. 1987) attempt to simulate, in a spatially distributed manner, the catchment hydrologic processes using partial differential equations governing various physical processes and equations of continuity. While process models are intuitively more precise and/or accurate than the black box models, many parameters need to be calibrated (hence practically prohibitive due to data and resources limitation), and they suffer from scaling problem (in model structure) relating theoretical equations that describe hydrologic processes on small laboratory scales to temporally and spatially heterogeneous catchment (Beven 1989). A simpler conceptual modelling approach (e.g. Sacramento (Burnash et al 1973) and SIMHYD (Chiew et al. 2002), that mimics a catchment as a number of interconnected storages with mathematical functions describing the movement of water into, between and out of them may be suitable for most applications. Catchment average rainfall and potential evapotranspiration (PET) are used as inputs to calibrate the model to reproduce the observed flows. The calibrated model is then run in simulation mode. Large catchments may be divided into smaller sub-catchments (as separate lumped models) and the estimated flows are routed through a stream network to the outlet.

The choice of model therefore depends on the objective of the study, the aspects that are considered to be of primary importance in the physical rainfall-runoff transformation process, and the availability of data and resources. Detailed discussion on model types and selection

are given in Grayson and Chiew (1994), McMahon and Chiew (1998), and Chiew and McMahon (1999).

This paper describes the improvement and calibration of a daily conceptual rainfall-runoff model SIMHYD (http://www.toolkit.net.au/rrl) to estimate high daily flows for flood risk assessment. SIMHYD is chosen because it is simple (seven parameters), easy to calibrate, and has been used and tested extensively to simulate flows across Australian catchments (Chiew and McMahon 1994, Chiew et al. 2002). Several modifications are made to improve the model performance in high flow simulation. Internal Muskingum flow routing (two parameters) is included to improve the timing of daily flows, and a degree-day snow component (one parameter) is added to model the snow processes. Significantly, a modified objective function that emphasises high flows during calibration is introduced. The improved SIMHYD model is calibrated on six major river catchments flowing into the Gippsland Lakes with 25 years of observed daily rainfall and flow data.

2. SIMHYD MODEL AND DATA

2.1. Basic Structure

The basic structure of the SIMHYD model (seven parameters) and the equations representing the various hydrologic processes are shown in Figure 1 (Chiew et al. 2002). Input data for SIMHYD are daily catchment average rainfall and PET.

In SIMHYD, daily rainfall is first intercepted by an interception store (maximum daily interception is the lesser of the interception store capacity and PET). Incident rainfall, which occurs if rainfall exceeds the maximum daily interception, is then subjected to an infiltration function. The incident rainfall that exceeds the infiltration capacity becomes infiltration excess runoff. A soil moisture function diverts the infiltrated water to the river (as saturation excess runoff/interflow), groundwater store (as recharge) and soil moisture store. The saturation excess runoff/interflow is estimated as a linear function of the soil wetness (soil moisture level divided by soil moisture Groundwater recharge is capacity). then estimated, also as a linear function of soil wetness. The remaining moisture flows into the soil moisture store. Evapotranspiration from soil moisture store is modelled as a linear function of soil wetness, but cannot exceed the potential rate (PET minus intercepted water). The soil moisture store has a finite capacity and overflows into the groundwater store. Baseflow is modelled as a linear recession from the groundwater store. There are therefore three runoff components: infiltration excess runoff, saturation excess runoff/interflow (both occur on the day rainfall occurs), and baseflow (occurrence depends solely on the groundwater store).



Figure 1. Structure of the SIMHYD model

2.2. Addition of Routing and Snow Modules

The existing SIMHYD model does not have a routing algorithm. An internal flow routing scheme is added to SIMHYD to account for the effects of routing, particularly for daily flow simulation in large catchments. Total flow (surface runoff, interflow and baseflow) generated by SIMHYD on each day is first discretised into 24 equal hourly flows, and the hourly flows are routed based on the Muskingum method (USACE 1994). These hourly routed flows are aggregated into the daily routed flows at the catchment outlet. The internal routing scheme thus acts as a filter to distribute part of the generated total flows on the present day to the catchment outlet on subsequent days, with two parameters K_{MUSK} and X_{MUSK} .

Amongst the six river catchments, the Mitchell is the only catchment having substantial area located at high elevation (Victorian Alps) where snow processes can significantly affect the rainfallrunoff response in winter. Two SIMHYD submodels are used, one modified to include snowfall, snow storage, and snowmelt for the proportion of catchment above 800 m (Mitchell Up), and the standard version for the proportion of catchment below 800 m (Mitchell Low). Total flows generated from the two sub-models are summed and routed internally to the catchment outlet. The snow module (one parameter: melt factor, *SRATE*) added to SIMHYD is based on the concept of degree-day (Beven 2000, Singh and Frevert 2002). When the temperature is below 0°C, precipitation falls as snow. The snow modelling component has two variables, snow store, and snowpack temperature calculated as the accumulation of daily air temperature since the snow store started to fill. Evapotranspiration from the soil moisture store does not occur when there is snow in the snow store. Snowmelt occurs if the snowpack temperature is greater than 0°C. Snowmelt is calculated as a melt factor times the air temperature above 0°C. Published melt factors of 3.0-4.2 mm/°C/day depending on the month (Schreider et al. 1997) are tested and melt factors of 3.0, 3.6 and 4.2 are found to be satisfactory for pre-September, September and post-September respectively for the Mitchell Up sub-catchment. The snowmelt is added to the soil moisture store.

2.3. Study Area and Data

Figure 2 shows the six major Gippsland Lakes catchments comprising the Tambo, Nicholson and Mitchell (eastern catchments) flowing into Lake King, and the Latrobe, Thomson/Macalister and Avon (western catchments) into Lake Wellington.



Figure 2. Six major Gippsland Lakes catchments

Continuous daily catchment average rainfall, PET (and daily air temperature data for Mitchell Up) are required as inputs into the SIMHYD model. Approximately 25 years (1976-2000) of historical data are used in this paper. Catchment average rainfall is derived from the SILO 0.05° x 0.05° daily girded rainfall data (QDNRM 2000). Mean monthly areal potential evapotranspiration (APET) is obtained from the Evapotranspiration Maps produced jointly by the CRC for Catchment Hydrology and Australian Bureau of Meteorology (BOM 2002). Mean monthly APET is used since

APET has little influence on flood behaviour at daily time scale and the inter-annual variability of APET is relatively small (Chiew et al. 2002). Daily air temperature is obtained from the Bureau of Meteorology, and daily river flows are obtained from the Victorian Water Resources Data Warehouse (http://www.vicwaterdata.net).

3. MODEL CALIBRATION

3.1. Standard Calibration

In the 'standard' SIMHYD model calibration, eight model parameters – seven runoff parameters (see Figure 1) and one routing parameter (K_{MUSK} is optimised, while X_{MUSK} is set to zero to maximise attenuation of the disaggregated daily flows) are optimised for each catchment except the Mitchell. The Mitchell model has 15 parameters – seven runoff parameters for each of the two subcatchments, Mitchell Up (includes snow module but *SRATE* is fixed) and Mitchell Low, plus one combined routing parameter. The model is run for one extra year prior to the calibration period to remove the effect of initial conditions in the model storage levels. Model parameters are optimised to minimise an objective function,

$$OBJ = \sum_{i=1}^{n} \left(MDL_i - OBS_i \right)^2 \tag{1}$$

where MDL is the modelled daily flow, OBS the observed daily flow and n the number of days with observed flow data. Penalty is applied to the

objective function $(OBJ = OBJ \ge 5)$ if the total modelled and observed flow volumes (*VOL*) differ by more than 10%, or if the modelled and observed quickflow ratios (*QFR*) differ by more than 20%. The penalty attempts to ensure that the model is realistic with regard to water balance, and that it is mimicking the actual processes by correctly partitioning the total flow into surface and subsurface components.

An automatic pattern search optimisation method is used to calibrate the model, with ten parameter starting points used to increase the likelihood of finding the optimum global values.

Figures 3a and 3b show the modelled vs. observed 1-day and 3-day flows in the standard calibration based on daily flows (Figures 3c and 3d are for high flow calibration, see Section 4 for discussion). The results indicate that the standard calibration leads to satisfactory simulation of medium flows, but high flows that are important in flood studies are grossly underestimated. The models are also calibrated against the observed 3day flow volumes (results not shown here). This is carried out because there may be small time offsets between the observed rainfall and flows at the daily time scale, which can lead to mismatch in the modelled and observed flows. In addition, 3-day flow volumes are also found to be dominant in the analysis of estuarine flood level variations in the Gippsland Lakes (Tan 2004). However, no noticeable improvement resulted, and the parameters are optimised based on daily flows.



Figure 3. Scatter plots of 1-day and 3-day modelled vs. observed flows in standard and high flow calibration.

3.2. Calibration to Estimate High Daily Flows

To improve the modelling of daily high flows, SIMHYD is calibrated against a modified objective function that places more importance in comparing high modelled and observed daily flows. This 'high flow' objective function is defined as

$$OBJ = \sum_{i=1}^{n} \left(MDL_{i} - OBS_{i} \right)^{2} \times \max \left(MDL_{i}, OBS_{i} \right)^{\lambda} (2)$$

where λ is an arbitrary power. The same penalty to the objective function ($OBJ = OBJ \ge 5$), as used in the standard calibration, is also applied here. In addition, the flow is subjected to the objective function only if the modelled or observed flow is greater than a threshold based on the percentile (e.g. top 1%) of the observed daily flows.

4. RESULTS AND DISCUSSION

To improve daily high flow modelling, SIMHYD model for the six catchments is calibrated using the 'high flow' objective function. Thresholds based on 2%, 1%, 0.5% and 0.2% of the observed daily flows are investigated (results not shown here). In general, basing the flow threshold on lower percentiles (i.e. higher flows) improves extreme flow modelling, but leads to overestimation of more frequent events (i.e. medium/low flows). The 1% observed flow threshold is adopted for all the six catchments because the medium to high flows are reasonably modelled.

Several λ values (i.e. weights for high daily flow events) are investigated. Increasing the value of λ leads to better modelling of extreme events, but with increasing overestimation of medium to high flows. The final choice of an appropriate value of λ for each catchment is made by subjective assessment of the scatter plots and residual plots of modelled vs. observed 1-day and 3-day flows.

Two options relating to the infiltration capacity parameters (*COEFF* and *SQ*) in the SIMHYD model are also tested. In the first option, default values are used for *COEFF* and *SQ* (200 and 1.5 respectively) to alleviate potential problems of parameter cross-correlation resulting from the use of too many model parameters (see Chiew and McMahon 1994). In the second option, these parameters are optimised to provide an opportunity for additional fine-tuning to the quickflow component, resulting in some improvements in most catchments. In both the standard and high flow calibration, SIMHYD model for the six catchments are calibrated with flow volume (VOL) and quickflow ratio (QFR) constraints of 10% and 20% respectively except the Mitchell. These constraints are not used for the Mitchell catchment because the high flows cannot be modelled satisfactorily with the constraints.

Figures 4a and 4b present the scatter plots and residual plots of the modelled vs. observed 1-day and 3-day flows for the Tambo catchment based on the high flow calibration by testing the two infiltration parameter options: default (Def.) and optimised (Opt.). For each option, different λ values (ranging from 1.0 to 4.0) are investigated. The best model is determined subjectively based on the scatter plots. In this case, the model selected for the Tambo catchment is $\lambda = 3.0$ using the default infiltration parameters.

The final results and parameters of the SIMHYD model calibrated based on the high flow objective function (Eqn.2) for all the six catchments are presented in Table 1b. The optimised parameter values and results for the standard calibration (Eqn.1) are also given in Table 1a for comparison.

The scatter plots comparing the modelled vs. observed 1-day and 3-day flows for the standard calibration (Figures 3a and 3b) and high flow calibration (Figures 3c and 3d) clearly indicate that the high flow calibration leads to significantly better simulation of the high flows. Table 1 also indicates that this is achieved with satisfactory simulation of the total flow volume (VOL) and quickflow ratio (QFR), and that the optimised parameter values are realistic.

The high flow calibration generally leads to higher total flow volume and quickflow ratios. This is not surprising, since the modified objective function artificially forced the model to fit the high flows. However, both the standard and high flow calibration approaches are equally valid, the choice of which will depend upon the main purpose of the model (i.e. for water resources or flood studies).

Note that the coefficient of efficiency (*COE*) (Nash and Sutcliffe 1970) comparing the modelled and observed daily flows in the high flow calibration (which is based on only daily flow data above the 1% flow threshold) is always equal to or lower than the *COE* in the standard calibration (which is based on all daily flow data), hence they are not directly comparable.



Figure 4. Scatter and residual plots of 1-day and 3-day modelled vs. observed flows for Tambo using λ from 1.0 to 4.0 in high flow calibration with infiltration capacity parameters options: (a) default, and (b) optimised.

Table 1. Optimised SIMHYD parameters.

		(a) SIMHYD Parameters in Standard Calibration							(b) SIMHYD Parameters in High Flow Calibration						
Parameter	Description	Tambo	Nicholson	Mitchell Up (snow)	Mitchell Low (rain)	Avon	Thomson/ Macalister	Latrobe	Tambo	Nicholson	Mitchell Up (snow)	Mitchell Low (rain)	Avon	Thomson/ Macalister	Latrobe
NSC	Interception store capacity (mm)	2.50	0.50	5.00	5.00	2.38	4.90	5.00	1.55	0.78	5.00	1.50	1.38	0.51	2.66
COEFF	Maximum infiltration loss (mm)	68	85	240	400	118	300	95	104	200	348	300	130	200	74
SQ	Infiltration loss exponent	1.23	1.50	1.50	3.10	2.20	1.00	1.65	0.99	1.50	2.56	1.50	3.20	1.50	1.50
SMSC	Soil moisture store capacity (mm)	175	160	500	153	180	300	305	160	139	60	108	160	265	269
SUB	Proportionality constant (interflow)	0.19	0.15	0.40	0.50	0.33	0.20	0.20	0.25	0.30	0.00	0.85	0.20	0.14	0.23
CRAK	Proportionality constant (groundwater)	0.20	0.15	1.00	0.00	0.30	0.13	0.43	0.20	0.00	0.93	0.07	0.20	0.00	0.51
Κ	Baseflow linear recession parameter	0.15	0.26	0.003	0.10	0.30	0.22	0.11	0.27	0.30	0.02	0.30	0.30	0.30	0.30
K _{MUSK}	Internal routing flood wave travel time (hr)	36.0	27.5	48.0		23.0	48.0	48.0	10.7	10.5	41.7		14.6	29.7	48.0
X _{MUSK}	Internal routing attenuation parameter	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0		0.0	0.0	0.0
SRATE	Snowmelt rate (mm/°C/day) (Aug,Sep,Oct)	-	-	3.0, 3.6, 4.2		-	-	-	-	-	3.0, 3.6, 4.2		-	-	-
-	COEFF & SQ option (Default/Optimised)	Opt.	Opt.	Opt.		Opt.	Opt.	Opt.	Opt.	Def.	Opt.		Opt.	Def.	Opt.
-	Flow threshold in objective function (mm)	-	-	-		-	-	-	3.36	5.59	4.61		6.09	3.54	3.07
λ	Arbitrary power in high flow obj. function	-	-			-	-	-	3.0	2.0	4.0		2.5	4.0	3.0
Ε	Coefficient of efficiency (daily)	0.70	0.71	0.30		0.66	0.52	0.62	0.58	0.64	-0.69		0.53	0.32	0.42
	Coefficient of efficiency (monthly)	0.89	0.88	0.31		0.77	0.81	0.82	0.84	0.84	-1.40		0.73	0.55	0.77
VOL	Volume ratio (Modelled/Observed)	0.97	1.07	1.60*		1.09	1.00	0.91	1.10	1.10	2.20*		1.10	1.10	1.10
QFR	Quick flow ratio (Modelled)	0.56	0.60	0.40*		0.65	0.51	0.36	0.52	0.70	0.29*		0.74	0.51	0.36
-	Quick flow ratio (Observed)	0.53	0.64	0.	44*	0.72	0.56	0.40	0.53	0.64	0	.44*	0.72	0.56	0.40

* Flow volume (VOL) and quickflow ratio (QFR) constraint of 10% and 20% respectively applied in all catchments except Mitchell.

5. CONCLUSIONS

An existing daily conceptual lumped rainfallrunoff model SIMHYD has been modified and calibrated for continuous simulation of daily flows for six major rivers flowing into the Gippsland Lakes. An internal Muskingum routing scheme is included, and a degree-day snow module is added to the SIMHYD model to improve daily flow modelling. Significantly, a modified objective function with emphasis placed on comparing high flows during model calibration is introduced to ensure accurate modelling of daily high flows, which is central to any flood study.

The results show that the SIMHYD model, calibrated for high daily flows, is capable of simulating continuous daily flows that capture the principal flow characteristics of floods including the 1-day and 3-day floods, while preserving the overall hydrologic balance of the system and the realism of the model parameters. The calibrated model can then be used to simulate continuous daily flows driven by stochastically generated daily rainfall data for flood risk assessment.

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