The Mekong - Applications of Value at Risk (VaR) and Conditional Value at Risk (CVaR) simulation to the benefits, costs and consequences of water resources development in a large river basin.

[1]Webby, R.B., [1]P.T. Adamson, [2]J. Boland, [2]P.G. Howlett, [1]A.V. Metcalfe and [2]J. Piantadosi

[1]School of Mathematical Sciences, The University of Adelaide, Australia.
E-Mail: roger.webby@adelaide.edu.au
[2]Centre for Industrial and Applied Mathematics, University of South Australia, Australia.

ABSTRACT

The world's tenth largest river in terms of annual flow volume, the Mekong is comparatively undeveloped in terms of its exploitable water resources, so that its hydrological regime remains largely natural and unregulated. However, this is about to change given the scale of water resources schemes proposed over the next two decades. These include over 23 cubic kilometres of reservoir storage behind a cascade of hydropower dams already under construction on the mainstream in Yunnan, China, and the additional hydropower regulation on major tributary systems that is planned in Laos. There are further proposals to divert substantial volumes of flow from the main stream into the water stressed regions of northeast Thailand, while the potential for irrigation expansion, particularly in Cambodia, is considerable.

The wider socio-economic benefits of such development are potentially offset by costs and penalties. Examples of the benefits are accelerated economic growth, an increase in irrigable land and local electricity supplies. As for the costs and penalties, changes to the volume and timing of the flood regime could have significant and negative impacts upon the seasonal refilling of the Great Lake (Tonle Sap) in Cambodia, the fishery of which is reported to support almost half of the economically active population of the country. These changes could be man-induced or the result of climate change. The risks to the sustainability of the fishery are an emerging cause for international concern.

Conditional Value at Risk (CVaR) was developed as a coherent measure of expected loss given that actual loss exceeds some Value at Risk (VaR) threshold. To date the concept has been primarily used to support quantitative risk assessment for investment decisions and portfolio management, using stochastic financial models to minimise the risk of unacceptable monetary loss. Intriguingly, the models and concepts are potentially adaptable to water resources planning and operational problems. This paper explores the application of CVaR within the context of identifying the risk of macro-economic damage to the fishery resources of Tonle Sap given reduced volumes of flow on the mainstream Mekong during the flood season. Emphasis is placed on simulating the linkages between the seasonally available flows in the Mekong mainstream, Tonle Sap water levels, annual fish catch and its economic value.

We present scenarios using real hydrological and fish catch data along with exploratory concepts of contingency fund costs in terms of national and international aid requirements. The objective is to estimate the potential economic loss at a prescribed level of probability and to illustrate how VaR and CVaR may be calculated in this context. We demonstrate the properties of these risk measures through their behaviour under continuous and discontinuous loss distributions. We show that CVaR has advantages over VaR even under a relatively simple modelling approach. In the case where a loss distribution has discontinuities, VaR is potentially a poor measure of risk as it can vary unacceptably with a small increase in probability level. CVaR is stable in these situations. Here we find that when the loss distribution is continuous the CVaR is only marginally higher than the VaR. However, for the more realistic model where the loss distribution is discontinuous, the CVaR is substantially greater.

We demonstrate the potential use of these two risk measures on a simple set of models of the Tonle Sap fishery in Cambodia. The sustainability of this fishery is crucial to the country in order to avoid even further dependence on international donor aid. Estimating the financial risk to which the national government and potential aid donors might be exposed given any damage to the fishery is the essence of this exploratory study of VaR and CVaR.

1 INTRODUCTION

Value at Risk (VaR) is a measure of risk developed by the financial industry and widely used to meet the mandatory regulatory requirements for reporting financial exposure. It is defined as the maximum loss expected to be incurred over a certain time horizon at a given probability. VaR has three main attributes: it allows the potential loss associated with a decision to be quantified, it summarises complex positions in a single figure, and it is intuitive - expressing loss in monetary terms. VaR also has two main drawbacks. First, it does not provide an indication of how much worse than the VaR the loss might be. Second, it is not a coherent risk measure (Artzner *et al.* 1999). For a coherent risk measure, reducing dependence on a single source of income by spreading over several comparable sources does not increase risk. However, this is not relevant here. Conditional Value at Risk (CVaR) has the attributes of VaR, is a coherent risk measure and takes into account the extremely large losses that may occur, albeit at low probabilities, in the tail of a value distribution (see Figure 1). CVaR is defined as the expected loss given that the loss is greater than or equal to the VaR value.
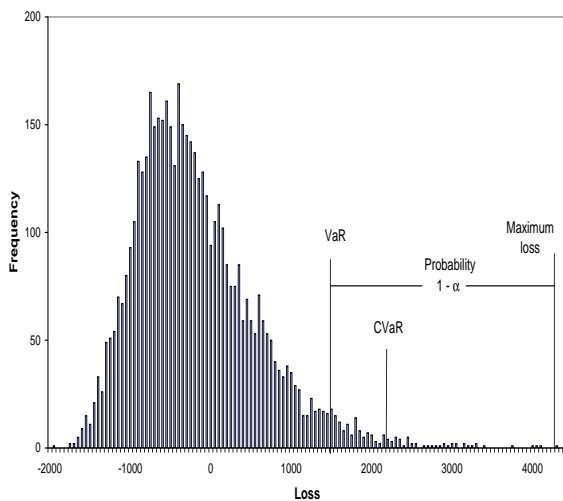

Figure 1. A loss distribution with VaR and CVaR illustrated

Other than in banking and finance, VaR and CVaR have been used as risk measures in agricultural enterprises (Pruzzo *et al.* 2003), electricity generation in a deregulated market (Dahlgren *et al.* 2003) and the product selection and plant dimensioning arena (Sodhi 2005). We extend these ideas and their application to the world's largest single inland fishery.

The Mekong River flows from the Tibetan plateau through the People's Republic of China, Myanmar, the Lao Peoples Democratic Republic, Thailand, Cambodia and Viet Nam to discharge into the South China Sea. The principal feature of the river's hydrological regime is the annual flood hydrograph generated by the SW monsoon which occurs between May and November. The mean annual hydrograph for the period 1924 to 2004 is illustrated in Figure 2 for the mainstream at Kratie, Cambodia where the average annual flow volume is 420 cubic kilometres (km^3).
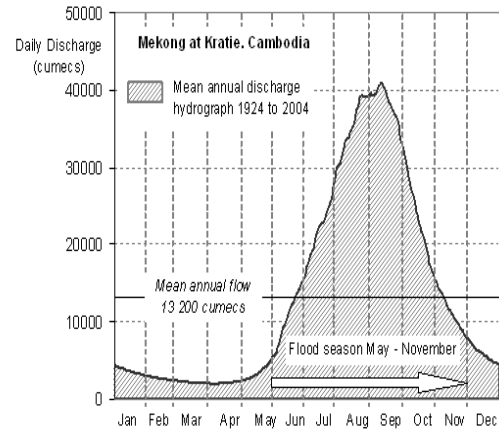

Figure 2. Mekong mainstream mean annual hydrograph at Kratie, Cambodia, 1924-2004

These seasonal floodwaters inundate highly productive floodplains, among them those around the Great Lake (Tonle Sap) system in Cambodia, the single largest area of wetlands in the Lower Mekong Basin (see Figure 3). The Tonle Sap system acts as a huge natural reservoir which expands from a surface area of 2700 km^2 in the dry season to as much as 16000 km^2 during the peak of the flood season, which is associated with a change in depth from an average of 1 meter to 9 metres. These changes are generated not only by seasonal flood inflows from the Great Lake's own catchment but also from flood water entering the system from the Mekong mainstream. This is effected by hydraulic head differences in the very flat regional topography which cause the Tonle Sap River to reverse its flow direction in May. In the flood season, therefore, the flows are to the northwest out of the Mekong and into the lake. This situation reverses once again at the end of the flood season in September when water flows out of the lake into the mainstream.

This flooding, particularly of the riparian forest areas around the lake, provides a huge natural habitat for fish feeding and breeding. Fishery yields are estimated at around 230 thousand tons per year, worth US$150 to 200 million (MAFF 2003). This amounts to 60% of Cambodia's commercial fishery production, contributing up to 10% of the country's gross domestic product, providing 75% of the protein consumption for Cambodia's people, and primary or secondary sources of income and employment for a third of the population (MAFF

2003). This productivity is enhanced by the sediment from the Mekong mainstream being deposited on the floodplains and the subsequent uptake of its nutrient load into the Tonle Sap system food chain.
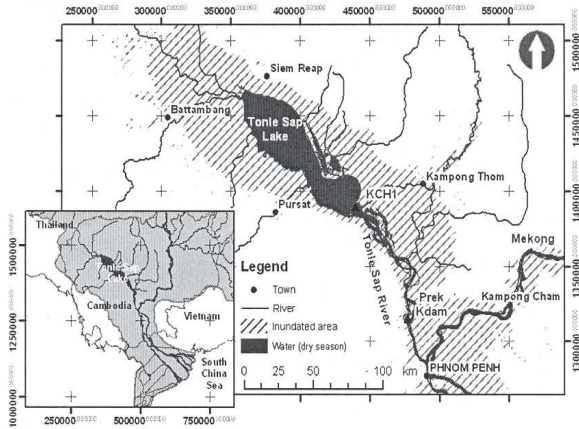

Figure 3. Tonle Sap (Kummu *et al.* 2004)

The generally accepted indicator of fish catch in the system is the dai fishery which is licensed and operates during the months of receding flood levels from September onwards. This dai catch is strongly influenced by the maximum level of floodplain inundation and therefore the area of fish feeding habitat available in a particular year. Any upstream developments or climatic influences that would systematically reduce the seasonal flood hydrograph or change its timing could have the potential to seriously damage the sustainability of the fishery (Kummu *et al.* 2004).

Any reduction in fish yield implies a monetary loss, the magnitude of which affects the risk. Collectively, the financial hazard is faced by the national government and, in the case of Cambodia, by the international donor agencies given the potential scale of relief required. Here we attempt to estimate the level and magnitude of this risk, conditional upon unfavourable flow conditions during the flood season on the mainstream. Essentially, this means reduced seasonal flood volumes and therefore depths and durations of floodplain inundation.

2 METHODOLOGY

2.1 Definitions of VaR and CVaR

Let $x \in X \subset \Re^n$ be a decision vector, and $y \in Y \subset \Re^m$ be a vector representing the values of a contingent variable influencing the loss. Let $z = f(x,y)$ be a function that describes the loss generated by x and y. VaR and CVaR are associated with a particular confidence level, $\alpha \in (0,1)$, commonly set at 0.95 or

0.99. The VaR_α of the loss associated with a decision x is defined as,

$$\text{VaR}_\alpha(x) = \min\{z \mid G(x,z) \geq \alpha\}, \qquad (1)$$

where $G(x,z)$ is the cumulative density function (cdf) for loss associated with decision x. The CVaR_α of the loss associated with a decision x is defined (Rockafellar and Uryasev 2002) as,

$$\text{CVaR}_\alpha(x) = \text{E}\{z \mid G(x,z) \geq \alpha\}, \qquad (2)$$

where E denotes the expectation operator. Each scenario presented in this paper represents a single decision so that the x variable can be considered as a constant.

These definitions are straightforward when the cdf of loss is continuous and strictly increasing. But, in some applications these conditions do not apply. For the situation where there is a discontinuity in the loss distribution with an associated atom of probability at VaR we define the upper α quantile,

$$\alpha^+ = \lim_{z \to \text{VaR}_\alpha^+} G(z).$$

In this situation, CVaR is composed of the probability between α^+ and α multiplied by VaR plus the expected value of the loss greater than VaR, all scaled to conform as a probability distribution. That is, the area under the probability density function of loss, $g(z)$, above VaR_α plus $(\alpha^+ - \alpha)$ is to equal one. So,

$$\text{CVaR}_\alpha = \frac{1}{1-\alpha} \left\{ \text{VaR}_\alpha[\alpha^+ - \alpha] + \int_{(\text{VaR}_\alpha, \infty)} z \, d_z G(z) \right\}. \qquad (3)$$

Rockafellar and Uryasev (2002) give more details on the development of this equation.

2.2 Fish Catch and Flood Volume

Mattson (2005) reports a linear relationship between the total wet season flood volume (fl_vol) in the Mekong River and the annual catch in the Tonle Sap River dai fishery (see Figure 2). The seasonal flood volume not only indicates the magnitude of the river's peak flow rate but also the duration of the inundation period, and is considered to be the single most useful summary statistic of the year to year flood regime (Adamson 2005).

The regression equation is,

$$\text{dai\_catch} = 2.1555 + 0.02648 \times \text{fl\_vol}, \qquad (4)$$

where dai_catch has units of thousands of tonnes and fl_vol has units of cubic kilometres. The equation was estimated using catch and flood volume data collected between 1995 and 2003. The highest and lowest flood volumes experienced in this period were 500 and 260 km^3 respectively. Although short, this period is representative of the flood hydrology in that it includes the highest (in 2000), the fifth highest (in 2002) and the third lowest (in 1998) annual flood volumes observed on the mainstream at Kratie in the 81 year sample. The estimated standard deviation of the errors about the line is 2.16. Although there is a suggestion that the catch levels off above a flood volume of about 380 km^3, the equation gives a working approximation, over a range of realistic flood volumes, for the present purposes.
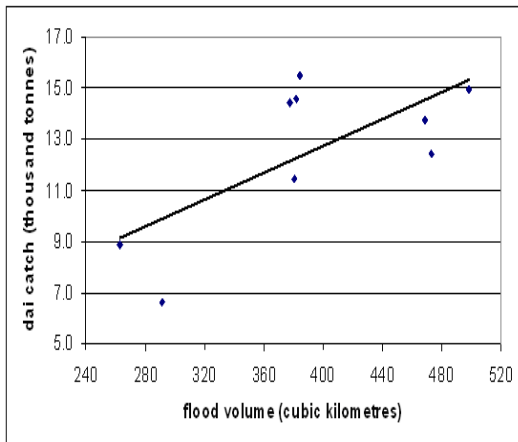

Figure 4. Dai catch on flood volume

2.3   Model Assumptions

In finance, the time horizon over which VaR or CVaR would be typically calculated is much briefer (often, one day ahead) than that for which natural resource-based investments would be considered. An assumption of VaR and CVaR is that the variables influencing the loss distribution are constant over the time horizon considered. Here, we calculate VaR and CVaR for the discrete time period of one wet season with flood volume found from an assumed statistical distribution, and we assume that loss is directly proportional to flood volume.

In order to assign a monetary value to the dai catch, we assume the return to fishers per kilogram of fish is constant across years. Initially, we can set an arbitrary value of, say, 2,000 riel per kilogram. (The currency of Cambodia, the riel, is named after one of the major species, the trey riel, caught in the dai fishery and indicates the importance of fishing to the national economy). Then return per thousand tonnes of fish is 2 billion riel or 2 B riel. (Four thousand riel is approximately one U.S. dollar). From (4), catches in

the dai fishery fall in the range of 9.0 thousand tonnes (for a flood volume of 260 km^3) to 15.4 thousand tonnes (for a flood volume of 500 km^3). The catch value is in the range [18.0, 30.8] B riel.

To generate a loss distribution, we assume that the fishery breaks even in years of mean flood volume. Then loss is positive for flood volumes in the range [260, 380] km^3. For the scenarios in this paper we initially assume that flood volumes have a uniform distribution, with cumulative density function :-

$$P(\text{fl\_vol}) = \begin{cases} 0 & \text{for fl\_vol} < 260, \\ \frac{\text{fl\_vol} - 260}{240} & \text{for } 260 \leq \text{fl\_vol} \leq 500, \\ 1 & \text{for fl\_vol} > 500, \end{cases}$$

We now propose four simple scenarios with different cost structures associated with unfavourable flood season flows to apply the methodologies used in calculating VaR and CVaR for the fishery.

3   EXAMPLES

3.1   Scenario a: Continuous Loss Distribution

Here we consider loss to the fishing community if the seasonal flood is below average. We assume that the loss is determined by (4) and, initially, do not include random variation about the line. It follows therefore that in common with the annual flood volumes, annual losses also have a uniform distribution.

The value of the catch has a range of 30.8 - 18 = 12.8 B riel. With mean flood volume generating a loss of zero, loss falls in the range ±12.8/2 or [-6.4, 6.4] B riel.

Any flood volume in the range [260, 500] has equal probability of occurrence. The cumulative density function for loss, (z), (in B riel) is thus,

$$G(z) = \begin{cases} 0 & \text{for z} < \text{-6.4}, \\ \frac{z - (-6.4)}{30.8 - 18} & \text{for -6.4} \leq \text{z} \leq \text{6.4}, \\ 1 & \text{for z} > \text{6.4}. \end{cases}$$

A graph of the cumulative density function for loss is given in Figure 5.

From (1), the Value at Risk is,

$$\text{VaR}_{0.95} = \min\{z \mid G(z) \geq 0.95\}$$

which is the 95th percentile of this continuous distribution and thus, VaR = 5.76 B riel. In comparison, from (3), the Conditional Value at Risk
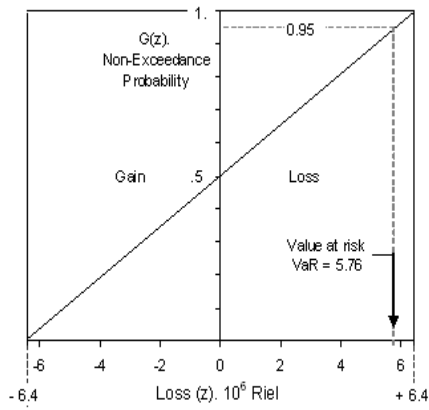
Figure 5. Loss distribution for scenario a

is significantly higher :-

$$
\text{CVaR}_{0.95} \quad = \quad \frac{1}{1 - 0.95} \left\{ 5.76(0.95 - 0.95) \right.
$$

$$
\left. + \int_{(5.76, 6.4)} \frac{1}{12.8} z\, dz \right\} = 6.08\text{B riel.}
$$

If the loss distribution is continuous both VaR and CVaR are strictly increasing with α, and are in one-to-one correspondence. Nevertheless, CVaR is far more informative as it gives the expected loss given that a loss occurs. This cannot be inferred from VaR alone unless the distribution of loss is also specified.

We now include random variation about the line in the regression model for catch. A $N(0, 2.16^2)$ is used to model the distribution of errors. Since loss is directly proportional to catch, its marginal distribution changes from uniform to a mixed normal. Pseudo-random sampling from a normal distribution is used to generate an empirical distribution for loss and the results are ranked in order to find VaR and CVaR. Running the sampling algorithm 1000 times each with 10000 replicates gave a mean value for $\text{VaR}_{0.95}$ of 9.41 B riel with standard error less than 0.003. Similarly, we found a mean for $\text{CVaR}_{0.95}$ of 11.59 B riel with standard error less than 0.003.

In reality, loss distributions are not continuous in this way but are characterised by jumps and discontinuities. These reflect shifts in the magnitude of the loss as the process crosses critical thresholds. For example, fisheries dynamics are such that the late arrival of the flood hydrograph combined with reduced flood volumes and areas of inundation can cause sudden collapses in breeding activity and therefore a discontinuous relationship between catch and flood hydrology. This is illustrated in the following scenarios.

3.2 Scenario b: Horizontal Discontinuities

Assume that international aid has up to 2 billion riel available for disbursement under various crisis scenarios within the fishery and adopted the schedule of payments indicated in Table 1. For below average flood volumes lying between the 1 in 10 (284 km^3) and the 1 in 20 (272 km^3) annual recurrence interval, up to 0.5 B riel is available on a sliding scale. For more extreme drought conditions, 1 to 2 B riel is available, also on a sliding scale. What cost in aid is being risked under this policy at the 95% level? Figure

Table 1. Schedule of costs under scenario b

fl_vol (km^3)	500 - 284	284 - 272	272 - 260
cost (B riel)	0	0 - 0.5	1 - 2

6 is a graph showing the probability of contingency fund cost (w). At $\alpha = 0.95$,
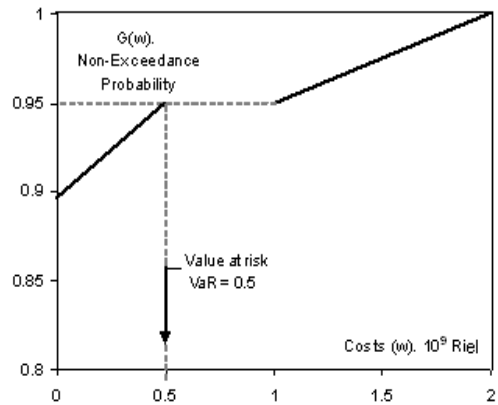

Figure 6. Contingency fund cost under scenario b

$$
\text{VaR}_{0.95} = \min\{w \mid G(w) \geq 0.95\} = 0.5\text{B riel.}
$$

$$
\text{CVaR}_{0.95} \quad = \quad \frac{1}{1 - 0.95} \left\{ 0.5(0.95 - 0.95) \right.
$$

$$
\left. + \int_{(1,2)} \frac{(1 - 0.95)}{(2 - 1)} w\, dw \right\}
$$

$$
= \quad 1.5\text{B riel.}
$$

Note that if α is slightly higher than 0.95 in this scenario, VaR doubles in value (to 1 B riel) while CVaR increases in a continuous manner. In general, whenever the probability distribution is discontinuous in loss with respect to the cost structure, (1) is unstable. That is, a small change in the value of the parameter (α) produces a significant change in the value of VaR.

3.3 Scenario c: Vertical Discontinuities

An alternative schedule of relief costs of up to 2 B riel is shown in Table 2. No costs are incurred for flood volumes above 285 km^3 which is exceeded 90% of the time. Up to 1 B riel is available (on a sliding scale) when flood volumes fall below 285 km^3, and additional funds only become available if the 1 in 20 year threshold is crossed. This corresponds to a critically low flood hydrograph of only 265 km^3. As before we estimate the unconditional and conditional Value at Risk.

Table 2. Schedule of costs under scenario c with flood volume ranges corresponding to: i = 500 - 285, ii = 285 - 275, iii = 275 - 265 and iv = less than 265 km^3

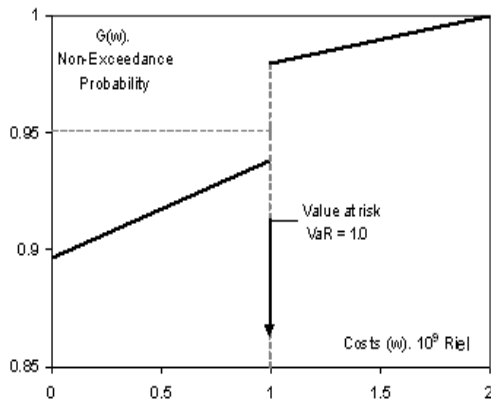| fl_vol (km^3)	i	ii	iii	iv
cost (B riel)	0	0 - 1	1	1 - 2

Figure 7. Contingency fund cost under scenario c

The vertical discontinuity in the probability distribution of cost means that the minimum value of w for which $G(w) \geq 0.95$ here occurs at $G(w) = 15/16$ and thus, from (1),

$$\text{VaR}_{0.95} = \min\{w \mid G(w) \geq 0.95\} = 1\text{B riel.}$$

In this scenario $\alpha^+ = 0.98$ and $\alpha = 0.95$. Then,

$$\text{CVaR}_{0.95} = \frac{1}{1-0.95}\left\{1(0.98-0.95)\right.$$
$$\left.+ \int_1^2 \frac{1-0.98}{2-1}wdw\right\}$$
$$= 1.21\text{B riel.}$$

3.4 Scenario d: Vertical and Horizontal Discontinuities

Now we consider a system of costs whereby a fixed amount of money is available for relief when flood volumes fall into a set of discrete ranges. For example, in a year when flood volume is in the range [285, 275] km^3 0.5 B riel is made available. This scenario implies that a season with flood volume in a relatively narrow range would deliver a relatively fixed or capped loss to the fishery. These discrete costs are defined in Table 3 and mapped in Figure 8.

Table 3. Schedule of costs under scenario d with flood volume ranges corresponding to: i = 500 - 285, ii = 285 - 275, iii = 275 - 265 and iv = less than 265 km^3

fl_vol (km^3)	i	ii	iii	iv
cost (B riel)	0	0.5	1	2

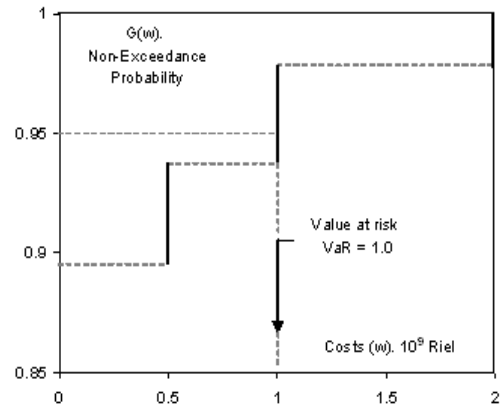**Figure 8.** Contingency fund cost under scenario d

$$\text{VaR}_{0.95} = \min\{w \mid G(w) \geq 0.95\} = 1\text{B riel.}$$

$$\text{CVaR}_{0.95} = \frac{1}{1-0.95} \times$$
$$\left\{1(0.98-0.95) + \int_1^2 2wdw\right\}$$
$$= 20\{1(0.03) + 2(1-0.98)\}$$
$$= 1.42\text{B riel.}$$

Once again, the magnitude of the increase of the cost when CVaR is taken into account (over 40%) indicates that when the underlying process, in this case the floods, has a heavy tailed distribution, VaR as a risk measure is inappropriate.

4 CONCLUSION

In this paper we have considered the linkage between flood hydrology, fishery and a range of policies that might be implemented in terms of financial relief when unfavourable hydrological conditions cause unacceptable reductions in fish catch and therefore serious socio-economic problems. The value at risk is the cost of the aid required at a prescribed level of probability, given the amount of financial relief potentially available and the manner in which it is paid out. Here, we have considered both continuous and discontinuous schedules of payment. In the latter case, the amount of money available increases in steps as a function of the deficit in the flood hydrology and therefore the potential damage to the fishery.

In the introductory case (Scenario a) it was demonstrated that CVaR is far more informative than VaR, as it gives the expected loss given that VaR is exceeded. We then considered more realistic scenarios which have different types of discontinuity in the loss function. The VaRs and CVaRs for each of these are shown in Table 4. Note that :-

Table 4. Values of VaR and CVaR for scenarios b - d

Scenario	$VaR_{0.95}$ (B riel)	$CVaR_{0.95}$ (B riel)
b	0.5	1.5
c	1	1.21
d	1	1.42

- Scenario b produced the lowest value of VaR but the highest value for CVaR, indicating that minimisation of VaR may not lead to the lowest financial risk to the national government and the international aid agencies

- CVaR indicates that the preferred option is described by Scenario c, which indicates a significantly reduced financial risk

Although much simplified, we have demonstrated the potential of the methodology, in particular CVaR, to identify the financial risks that could arise when a natural resource is threatened from human-made intervention or climate change. The inland fisheries of Cambodia make a substantial contribution to the country's food security, and a method of estimating risk would enable aid agencies such as the World Food Program to disburse their funds in the most equitable way. More generally, information and statistics on the fisheries are important for identifying strategies for their sustainable development and management. Considerable work has been done recently on improving the quality of such information

for Tonle Sap fisheries. CVaR provides a measure of potential large loss. It is one item of information that could be used in developing a policy for management of the resource.

Extensions of this study are fairly obvious, though as the loss models become more detailed analytic solutions are unlikely to be available. Rockafellar and Uryasev (2002) have shown that under fairly general conditions, decision variables and loss distributions can be incorporated into a finite and convex function, presenting a relatively straightforward minimisation problem. Then CVaR can be optimised against a range of possible policies. We intend to extend the applications into a general water resources framework covering flood risk and flood protection costs and the agricultural and economic damage caused by saline intrusion into the Mekong Delta in Vietnam, one of the world's largest rice growing regions.

5 ACKNOWLEDGEMENTS

6 REFERENCES

Adamson, P.T. (2005), An Overview of the Hydrology of the Mekong, Mekong River Commission, Vientiane, Lao PDR, 92 pp.

Artzner, P., F. Delbaen, J-M. Eber and D. Heath (1999), Coherent Measures of Risk, *Mathematical Finance*, **9**, 203–228.

Dahlgren, R., C-C. Liu, and J. Lawarrée, (2003) Risk Assessment in Energy Trading, *IEEE Transactions On Power Systems*, **18**, 503–511.

Kummu, M., J. Koponen and J. Sarkkula (2004), Assessing Impacts of the Mekong Development in the Tonle Sap Lake, Mekong River Commission, Vientiane, Lao PDR, 12 pp.

MAFF (2003), National Sector Review, Fisheries Management, Prepared by Ministry of Agriculture, Forestry and Fisheries and Cambodia National Mekong Committee, Government of Cambodia.

Mattson, N. (2005), Personal Communication, Fisheries Program, Mekong River Commission.

Pruzzo, L., R.J.C. Cantet and C.C. Fioretti (2003), Risk-adjusted Expected Return for Selection Decisions, *Journal of Animal Science*, **81**, 2984–2988.

Rockafellar, R.T. and S. Uryasev (2002), Conditional Value-at-Risk for General Loss Distributions, *Journal of Banking and Finance*, **26**, 1443–1471.

Sodhi, M., (2005), Managing Demand Risk in Tactical Supply Chain Planning for a Global Consumer Electronics Company, *Production and Operations Management*, **14**, 69–79.