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# Can Momentum Returns be Optimised?

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**Abstract:** The paper reports on an investigation of various techniques to optimise momentum returns from share trading. Eight different processes are applied to share returns from five countries, using United States dollars as the common currency. The aim is to determine whether one method is clearly superior to other algorithms in maximising the momentum returns for the synthesised portfolios over a period of time.

This is the first study of its type where optimising programmes are applied to momentum returns and portfolio selection. The analysis includes varying lengths of time periods with the longest data set pertaining to the United States, covering the period 1973-2007 and the shortest is India ranging from 1993 to 2007. The five countries under investigation are Canada, India, Japan, United Kingdom, and the United States.

Various optimisation models have been widely used in capital market research for a number of years and discussed extensively in the literature. Each has received attention relating to its conceptual and practical relevance for portfolio construction. Opposing views have been extolled by multiple researchers and in part their respective findings may have been influenced by the particular application they were addressing. This new research area provides further insight into the efficiency of each model in a totally new context.

Momentum studies have previously concentrated on finding the cause of this anomaly or to document whether abnormal returns are present only in a particular dataset, e.g. effected by size, volume etc. Very little attention is paid to the portfolio weighting and the extant literature primarily uses an equal-weighted or value-weighted approach to allocate money to each share of the portfolio. This study proposes an alternative way of allocating money to check if momentum returns can outperform equal-weighted or value-weighted momentum returns.

The practical importance of the research relates to the potential to increase profits from trading using a momentum strategy through superior information processing which in turn will generate greater returns for specified risk levels.

## 1. INTRODUCTION

The possibility of making significant returns in stock trading based on past price movement of securities has had appeal for a long time. Chartism and other forms of technical analysis continue to attract exponents. The efficient market hypothesis debunked these approaches demonstrating that returns follow a more random walk and that information about future returns is not present in the historical series of returns. This wisdom gained a broad acceptance, at least among academics, although criticisms of the EMH were made. A more serious assault arose from DeBondt and Thaler (1985) who point to a failure of the EMH when they document the possibility of making abnormal profit in the stock market by studying past stock prices.

Since the seminal work of DeBondt and Thaler (1985) considerable research has been undertaken using their "model" and many arguments proposed for and against the contrarian strategy. Jegadeesh and Titman (1993) consider medium-term periods, of 3 to 12 months for stock returns and state that significant abnormal profit can be made by buying (short selling) best (worst) performing stocks, which is popularly known as momentum strategy.

In both the DeBondt & Thaler (1985) and the Jegadeesh & Titman (1993) studies stocks are assigned equal weight in each portfolio, i.e. investors are expected to allocate an equal amount of investment to each stock. Another approach to allocating money to stocks is a value-weighted system. In this approach the weighting for each stock should be proportional to its market capitalisation. Small firm bias, suggested by Fama (1998), can be partially eliminated through this weighting system as a higher percentage of money will be allocated to the higher market capitalisation stocks.

Surprisingly, there has been no obvious advancement in the approach to allocating the weight for stocks when contrarian or momentum portfolio returns are calculated. This is despite the impact of alternative weighting being subject to intensive investigation during the past 20 years. The literature has so far ignored the potential benefits of using portfolio optimisation to determine how money should be allocated in a portfolio to maximise return or minimise risk. The question arises as to whether sophisticated methods of allocation can be used in contrarian or momentum strategies to further increase portfolio returns or decrease portfolio risk.

Construction of optimised portfolios and comparison with the frequently used methods will provide evidence of any real advantage. If results confirm superiority of optimisation techniques in generating extra returns or decreasing risk compared to traditional equal- or value weighted method, then it is an important finding with implications for portfolio management, viz. momentum portfolio holders can adopt portfolio optimisation technique to further improve their portfolio performance. Michaud (1998) notes that effective asset management is not simply a matter of finding attractive investments; it also requires optimally structuring the portfolio of the assets. This is because the investment behaviour of a portfolio is typically different from that of the constituent assets.

Several considerations are pertinent:

i. Portfolio optimisation techniques are developed in a manner by which the portfolio return is maximised or risk is minimised. This can be achieved by wisely allocating money among the stocks given certain inputs. Therefore, instead of using traditional equal- or value-weighted approach as a portfolio optimisation technique could be applied to allocate weights for the stocks.

ii. Some optimisation techniques provide an option for including risk preferences when calculating the optimum allocation, i.e. an allocation can be made according to investors risk preference. Risk averse investors prefer less risky portfolios than risk-seeking investors and a portfolio optimisation technique can calculate the optimal portfolio where risk is minimised. Similarly, risk seekers can use an optimisation technique to allocate money in a portfolio where return is maximised.

iii. Use of the efficient frontier of return and risk in an optimisation technique provides the investors with a range of options for investing in the portfolio. Therefore, the efficient frontier provides a platform where investors with varying risk tolerance can pick an efficient portfolio to meet their requirements.

Portfolio optimisation potentially offers certain advantages. Nevertheless its application in allocating weights in portfolio construction has so far been ignored in the contrarian and momentum strategy literature. Researchers are reluctant to accept portfolio optimisation techniques due to researchers:

i. having ignored the importance of allocating of weights among stocks and the benefit arising from high return and low risk in calculating momentum or contrarian strategies.

ii. believing that using any portfolio optimisation technique will fail to make difference in generating extra return or decreasing risk.

iii. thinking that including portfolio optimisation technique in the study will make calculations more complex and difficult to compute.

## 2. DATA & METHODOLOGY

Monthly stock price and market capitalisation are downloaded for all stocks listed in China, India, Japan, UK, and US stock market from Datastream database. Various issues relating to the problems noticed in the Datastream database highlighted by Ince and Porter (2006) are addressed before the analysis is conducted. Adoption of an arbitrary minimum number of 1000 stocks for all the sample countries in any given month is set as a precondition for inclusion in the sample. The restriction of 1000 stocks in any month ensures an adequate number of stocks in each portfolio for proper analysis as well as increased statistical reliability. For example, the Loser and Winner portfolio will contain around 100 stocks each if the whole sample is divided into 10 portfolios to calculate momentum returns. Similarly, the Loser and Winner portfolio will have at least 33 stocks each if the whole sample is divided into three sub-samples based on the market-capitalisation of each stock. The number of stocks within the Winner and Loser portfolios are expected to decrease as a stock is only included in the Winner or Loser portfolio if 60-months of prior historical data are available.

Certain inputs are needed to optimise a portfolio, viz. expected mean, standard deviation, correlation etc. In reality, true expected return, standard deviation and correlation data are not available and historical estimates are used as a proxy to expected returns, standard deviation and correlation/covariance. This study uses the prior 60 months of historical data of each stock to compute expected return, standard deviation and covariance/correlation. Jobson and Korkie (1981) and Chopra et al. (1993) consider that the prior 60 months of data is a fairly reasonable period over which to calculate different inputs needed for an optimisation process. Therefore, availability of data for the previous 60 months becomes another condition determining whether a stock will be included in the Loser or Winner portfolio in any given month.

The various optimisation techniques considered in this study are:

### (a) Markowitz method

Markowitz analysis requires three inputs to calculate the optimal combination, viz.:

(i) expected return for stock *j*
(ii) standard deviation or variance of stock *j*
(iii) covariance or correlation between stock *j* and *k*

Jobson and Korkie (1981) note that the Sharpe reward-to-variability ratio is a common performance measure. They further add that the portfolio weight calculated using Sharpe's ratio substantially dominates the portfolio formed from the traditional Markowitz technique, i.e. to optimise a portfolio by specifying a certain return target or a maximum risk. The objective is to the maximise Sharpe ratio by changing the weight of each stock of the portfolio. Several constraints are imposed in while maximising Sharpe's reward-to-variability ratio, including:

(i) the sum of all individual stock weights shall be equal to one.
(ii) all the stock weights shall be positive, i.e. no short-selling is allowed within the portfolio.
(iii) the maximum percentage of allocation to each stock of a portfolio cannot exceed 5%. This is to avoid the corner solution problem often observed in Markowitz optimisation. Jorion (1985) notes that the Markowitz approach often allocates a high percentage of weight to stocks with high expected returns when the objective of the optimisation is set to maximise returns. Cohen and Pogue (1967) propose a maximum allocation of 2.5% to a stock when there are around 150 stocks in the portfolio and 5% when the number of stocks in the portfolio drops to 75.
(iv) Jorion (1985) documents corner solution problem in the Markowitz method when some stocks in the portfolio are assigned zero weight. A trial run in this study also exhibits the same pattern where 20 out of 100 stocks are allocated a weight close to 5% each and the remaining 80 stocks are allocated a weight close or equal to 0%. To minimise this problem and to reduce idiosyncratic risk i.e. risk arising from a particular company, another constraint is imposed in the optimisation process where the weight of each stock of the portfolio should be greater than 0.1%. The figure 0.1% is arbitrary but primarily chosen to reduce idiosyncratic risk.

These four constraints are imposed in all optimising techniques considered in this study.

### (b) Markowitz method excluding extreme returns

This approach proposes to exclude the extreme 5% of stocks within the Winner and Loser portfolio. The primary reason for excluding the extreme 5% returns is the high sensitivity of the Markowitz approach to inputs used in the optimisation. Michaud (1989) finds that a small change in input estimation can make a big difference in the distribution of the optimum portfolio weights. The problem can be controlled, to some extent, by excluding stocks with extreme high/low returns and placing constraints on the optimisation process as explained above.

**(c) Single Index Model**

One of the criticisms noted of the Markowitz approach is that a large number of inputs are required to complete the optimisation process. This problem can be reduced by decreasing the number of inputs required to compute the optimised weight of each stock of the portfolio. This reduction of inputs also effectively reduces the "error maximization" problem noted in the Markowitz approach. For example, 1325 estimates are needed to calculate the optimal portfolio allocation involving 50 stocks. However, only 152 inputs (3n+2) are needed to optimise a portfolio of 50 stocks under the Single Index model. The Single Index model assumes that the stock returns generally vary consistently with an overall market aggregate and therefore estimates of correlation/covariance among individual stocks are not required.

**(d) Single Index Model with adjusted beta**

One of the most important inputs of the Single Index model is the beta of each stock. The reliability of the beta estimate can greatly influence optimisation results. One of the concerns raised in the literature is whether the past stock beta is an appropriate estimate to calculate expected portfolio mean and variance. Blume (1975) documents how the future beta is closely related to the past data and historical beta can be effective in predicting the next period beta. Regressing beta of one period over the next period, Blume (1975) documents the following result:

$$\beta_{future} = 1/3 + 2/3\beta_{historical}$$

The adjusted-beta has another advantage of converging all betas toward to 1. In reality, true beta coefficients are not available and therefore sampling errors always occur when the beta coefficient is estimated from historical data. This problem can be mitigated by using adjusted-beta as the true beta coefficient is expected to converge toward 1 over a period of time.

**(e) Shrinkage method**

The previous literature notes that the Markowitz optimisation results can change significantly with a small change in input estimates. One of the approaches proposed to address this problem is to shrink the historical mean return of each stock to a grand mean return. This will control the dispersion of stock returns within the portfolio and hence the optimisation results will be less sensitive to inputs. This popular technique commonly referred to as the James-Stein shrinkage method after Stein (1955) who demonstrates that the estimation errors will decrease as the individual mean is converged to the grand mean.

Jorion (1985), Jorion (1986) and Golosnoy and Okhrin (2005) find superior Markowitz optimisation results when shrinkage means are used as input. Chopra et al. (1993) compare optimisation performance under Stein-estimator and traditional Markowitz approach and conclude a clear dominance for the Stein method. Although, there are a number of methods available to calculate the percentage of individual mean shrinkage to global mean, this study proposes three weights: 25%, 50%, and 75% to the global mean.

**(f) Markowitz method with zero expected return**

This is the last optimisation method considered in the study and is another version of the shrinkage method where the expected return of each stock is set to zero. All stocks within the portfolio are converged to a common return (zero in this study). This idea, proposed by Chopra and Ziemba (1993), suggests that in the absence of a true expected return for each stock, the best practice is set to set all the stock returns to zero (or a non-zero constant). In their view, the optimisation process, which is the same as Makowitz, will yield better results when using sensible constraints (in this case constraining all expected stock returns to zero) than without constraints.

The Winner and Loser portfolio are optimised, for each of the eight methods discussed above, at the end of every formation period. These optimum weights are used in the beginning of the holding period to allocate money among the Winner and Loser portfolio stocks. Altogether 2,202 months of data are to be optimised under each optimisation technique to calculate momentum returns for five countries. These optimisation months increase to 17,616 data points when all eight optimisation techniques are considered in the study. A breakdown of the number of months to be optimised for each country is:

(i) Canada = 153 Loser months + 153 Winner months
(ii) India = 97 Loser months + 97 Winner months
(iii) Japan = 167 Loser months + 167 Winner months
(iv) UK= 342 Loser months + 342 Winner months
(v) US = 342 Loser months + 342 Winner months

## 3. RESULTS

All results reported below are average-monthly momentum returns using 6 months formation period, 1-month gap, 6 month holding period and computed using 10 portfolios. The stock prices are converted into US dollar and all the momentum strategies place a restriction on availability for 60 months of prior data before the starting of formation period. The equal- and value-weighted Winner-Loser momentum returns are presented in each table for ease of comparison of momentum returns arising from various optimisation techniques. The equal- and value-weighted momentum returns reported are computed only on the large-cap sample as unreported results suggest significant variation in small-size stocks.

### 3.1. Markowitz approach

The momentum returns under the Markowitz approach are reported in Table 1. The momentum returns under the equal- and value-weighted methods using the same restriction of 60 months are reported on the right side of the table. An initial inspection of results indicates the superiority of optimising momentum returns under the Markowitz approach, except for Canada. The momentum returns for all countries outperform the momentum returns computed under the equal- or value-weighted approaches. The most remarkable improvement in return is seen in the Japanese stock market where the average-monthly momentum return tripled from 0.23% (under equal-weighted) to 0.76% (under Markowitz approach). The momentum returns for the Japanese stock market are, however, not statistically significant. An increase of about 0.20% over equal-weighted and 0.60% over value-weighted method are also observed in the UK market. The least benefit of employing optimisation techniques is seen in the Canadian stock market where the average-monthly momentum returns remain the same for the Markowitz and equal-weighted process.

**TABLE 1: Momentum optimisation: Markowitz approach (using only large-cap stocks)**

| | Using Markowitz approach | | | | | | Equal- | Value- |
|---|---|---|---|---|---|---|---|---|
| **Country** | **Loser(L)** | **t-stat** | **Winner(W)** | **t-stat** | **W-L** | **t-stat** | weighted | weighted |
| Canada | -1.33% | -3.45 | 0.89% | 3.96 | 2.22% | 5.42 | 2.22% | 1.99% |
| India | -0.68% | -1.25 | 0.86% | 2.20 | 1.54% | 2.70 | 1.04% | 1.46% |
| Japan | -0.19% | -0.75 | 0.56% | 1.65 | 0.76% | 1.54 | 0.23% | 0.29% |
| UK | -0.83% | -3.54 | 0.78% | 6.00 | 1.60% | 6.62 | 1.42% | 1.06% |
| US | -0.28% | -1.59 | 0.66% | 4.01 | 0.94% | 4.23 | 0.73% | 0.67% |

One of the important attributes of using the Markowitz approach, as well as other approaches outlined later in this study, is the surety that the Winner and Loser portfolios are well diversified. This is due to the constraint of placing maximum 5% allocation in any one stock. This is in contrast with the value-weighted approach where there is a high possibility that stocks with high market capitalisation will be allocated a major weight and the rest of the stocks will be allocated a very small weight. The equal-weighted approach follows naïve diversification as 1/n weight is allocated to each stock of the portfolio.

### 3.2. Markowitz approach after excluding extreme 5%

In this approach, the momentum returns are calculated after excluding the extreme 5% of the stocks within the Winner and Loser portfolio. The exclusion of extreme stock returns decreases the variability of returns within each portfolio. This action is taken into consideration in the literature where it is noted that the Markowitz method has a high sensitiveness to various inputs and how a small change in input can lead to a big change in the allocating of weight to a stock.

The results presented in Table 2 indicate a decrease in momentum returns compared to the Markowitz approach without excluding any stocks. This decrease in momentum returns is primarily due to lower Loser portfolio returns compared to the Loser portfolio returns of the Markowitz method without excluding any stocks. The Winner portfolio returns are, however, not different when compared to the Winner portfolio returns under the Markowitz approach without excluding any stock. The momentum returns for all countries except Canada and India show improvement of returns over the equal- and value-weighted approach.

**TABLE 2: Momentum optimisation: Markowitz approach excluding extreme 5% (using only large-cap stocks)**

| | Using Markowitz approach excluding extreme 5% | | | | | | Equal- | Value- |
|---|---|---|---|---|---|---|---|---|
| **Country** | **Loser(L)** | **t-stat** | **Winner(W)** | **t-stat** | **W-L** | **t-stat** | weighted | weighted |
| Canada | -1.23% | -3.01 | 0.88% | 3.78 | 2.11% | 4.89 | 2.22% | 1.99% |
| India | -0.55% | -1.00 | 0.85% | 2.18 | 1.40% | 2.45 | 1.04% | 1.46% |
| Japan | 0.01% | 0.05 | 0.56% | 1.69 | 0.55% | 1.13 | 0.23% | 0.29% |
| UK | -0.84% | -3.85 | 0.72% | 5.99 | 1.56% | 6.91 | 1.42% | 1.06% |
| US | -0.31% | -1.86 | 0.59% | 3.81 | 0.91% | 4.30 | 0.73% | 0.67% |

**3.3. Single Index Model**

The previous two tests show the Markowitz optimising method without excluding extreme stocks generates impressive returns when compared to the equal- and value-weighted approaches. However, the computational burden is high for the Markowitz model and a large number of inputs are required. Further, as the previous literature postulates some estimates, e.g. covariance among stocks, may not be a very important factor in optimising results. A Single Index model is tested next to verify if this model with fewer estimates can outperform the Markowitz method.

The momentum returns presented in Table 3 offer mixed result, with Canada, UK, and US generating higher returns under the Single Index model and a decline in the momentum returns for India and Japan when compared to the Markowitz method. The momentum returns for all countries except India, however, outperform equal- and value-weighted momentum approach returns. The momentum returns for the Japanese stock market show the most promising improvement but remain not statistically significant under all approaches. The average-monthly momentum returns in the US market also stood at 1.05%, an impressive increase of about 0.32% over the equal-weighted approach and 0.38% over the value-weighted approach.

**TABLE 3: Momentum optimisation: Single Index Model (using only large-cap stocks)**

| | Using Single Index Model | | | | | | Equal- | Value- |
|---|---|---|---|---|---|---|---|---|
| **Country** | **Loser(L)** | **t-stat** | **Winner(W)** | **t-stat** | **W-L** | **t-stat** | weighted | weighted |
| Canada | -1.32% | -3.36 | 1.00% | 4.24 | 2.32% | 5.48 | 2.22% | 1.99% |
| India | -0.55% | -0.93 | 0.87% | 2.03 | 1.42% | 2.33 | 1.04% | 1.46% |
| Japan | -0.11% | -0.42 | 0.60% | 1.65 | 0.71% | 1.37 | 0.23% | 0.29% |
| UK | -0.85% | -3.32 | 0.79% | 5.94 | 1.64% | 6.37 | 1.42% | 1.06% |
| US | -0.31% | -1.75 | 0.74% | 4.03 | 1.05% | 4.41 | 0.73% | 0.67% |

**3.4. Single Index Model with adjusted beta**

The momentum returns under the Single Index model with adjusted beta show the best results when compared to other optimisation methods and the traditional equal- and value-weighted approaches. The results presented in Table 4 show that the momentum returns can be increased over the traditional value-weighted approach from an average-monthly 0.08% (India: over value-weighted) to as much as average-monthly 0.59% (UK: over value-weighted). Similarly, when the momentum returns are compared to the equal-weighted approach, the minimal increase is 0.09% (Canada: over equal-weighted) and the highest increase is 0.56% (Japan: over equal-weighted).

The highest average-monthly momentum returns is seen in the Canadian stock market at 2.31% which is statistically significant and the lowest is in the Japanese stock market with an average-monthly return of 0.79% which is not statistically significant. The momentum returns in the US stock market increase under this method but the increase in returns is very small (an average-monthly 0.01%) when compared to the single index model without adjusted beta.

**TABLE 4: Momentum optimisation: Single Index Model with adjusted beta (using only large-cap stocks)**

| | Using Single Index Model with adjusted beta | | | | | | Equal- | Value- |
|---|---|---|---|---|---|---|---|---|
| **Country** | **Loser(L)** | **t-stat** | **Winner(W)** | **t-stat** | **W-L** | **t-stat** | weighted | weighted |
| Canada | -1.33% | -3.33 | 0.99% | 4.23 | 2.31% | 5.44 | 2.22% | 1.99% |
| India | -0.64% | -1.07 | 0.91% | 2.12 | 1.54% | 2.55 | 1.04% | 1.46% |
| Japan | -0.17% | -0.63 | 0.62% | 1.78 | 0.79% | 1.59 | 0.23% | 0.29% |
| UK | -0.86% | -3.33 | 0.79% | 5.69 | 1.65% | 6.37 | 1.42% | 1.06% |
| US | -0.30% | -1.69 | 0.76% | 3.97 | 1.06% | 4.51 | 0.73% | 0.67% |

These increases in return also outperform the previous best results of the Markowitz approach without excluding extreme stocks, as well as the shrinkage method results discussed next. The analysis of results does confirm that the optimisation techniques can generate higher returns when an appropriate model is selected and constraints are imposed correctly.

**3.5. Shrinkage method**

Four shrinkage methods are presented together in Table 5. The 25% to global mean suggests that the expected return of each stock consists of 25% of the stock mean + 75% of the global mean (in this case market return). The same logic applies to 50% and 75% to global mean. The results of zero-expected returns are also included in the same table as all expected returns are set to zero and therefore, in a sense, all individual stock means are converged to a grand mean (zero in this case).

**TABLE 5: Momentum optimisation: Shrinkage method (using only large-cap stocks)**

| Weighting approach | | Canada | India | Japan | UK | US |
|---|---|---|---|---|---|---|
| Equal-weighted | Winner-Loser | 2.22% | 1.04% | 0.23% | 1.42% | 0.73% |
| | t-stat | 5.11 | 1.95 | 0.60 | 5.75 | 3.52 |
| Value-weighted | Winner-Loser | 1.99% | 1.46% | 0.29% | 1.06% | 0.67% |
| | t-stat | 4.42 | 2.32 | 0.76 | 4.19 | 2.92 |
| 25% to global mean | Winner-Loser | 2.24% | 1.49% | 0.76% | 1.58% | 0.90% |
| | t-stat | 5.48 | 2.67 | 1.56 | 6.54 | 4.13 |
| 50% to global mean | Winner-Loser | 2.23% | 1.46% | 0.61% | 1.55% | 0.87% |
| | t-stat | 5.51 | 2.63 | 1.37 | 6.50 | 4.01 |
| 75% to global mean | Winner-Loser | 2.19% | 1.33% | 0.40% | 1.52% | 0.82% |
| | t-stat | 5.35 | 2.49 | 1.00 | 6.20 | 4.16 |
| Zero-expected return | Winner-Loser | 2.27% | 0.75% | 0.47% | 1.44% | 0.65% |
| | t-stat | 5.03 | 1.28 | 1.20 | 5.06 | 2.42 |

These findings point to momentum returns continuuing to decline as the shrinkage to global mean increase. Of the four shrinkage methods the 25% to global mean generates the best momentum returns. However, the optimised momentum returns under various shrinkage methods fail to outperform momentum returns calculated using single index model with adjusted beta as shown in Table 4. This implies that converging individual means to a grand mean may not be a good idea when the technique is applied in the context of momentum returns.

## 4. Conclusion

Momentum strategy, first documented by Jegadeesh and Titman (1993), still remains an anomaly even after considerable research over the past 15 years. The majority of prior research is directed towards finding the cause of this anomaly rather than how to generate extra returns using the same stock. The literature so far is focussed on narrowing down the best and worst performing stocks to be included in the Winner and Loser portfolios rather than how to use these stocks efficiently for increasing momentum returns. This study provides an important extension to the momentum return strategy evaluating various optimisation techniques can be used successfully in generating extra returns compared to the traditional momentum return approach which has been used extensively in the literature.

The results from this study suggest that the optimisation techniques can be important tools to generate extra momentum returns. The Single Index model with adjusted beta is seen as the best optimising tool in terms of generating superior momentum returns compared to the equal- or value-weighted momentum approaches. The momentum returns in the Japanese stock market jumped from an average-monthly 0.23% under equal-weighted approach to 0.79% under single index model with adjusted beta although not statistically significant. Similarly, the momentum returns in the UK stock market increased substantially from an average-monthly 1.06% under value-weighted to 1.65% under Single Index model with adjusted beta. This increase is achieved by using the same momentum strategy with the same set of Winner and Loser portfolio stocks but with different weights allocated to each stock of the portfolio.

The conventional Markowitz method and Single Index model without adjusted beta also show promising results with both optimising methods outperforming equal- and value-weighted approach in almost all countries under investigation. The momentum return is also seen to decrease under the Markowitz method after excluding extreme return stocks from the Winner and Loser portfolios and this decrease is potentially due to fewer returns from the Loser portfolio. This suggests that excluding extreme return stocks, i.e. excluding stocks that have high sensitiveness to Markowitz input estimates may not lead to superior results when compared to the Markowitz method without excluding any stocks. The likely reason can be traced to the constraint of 5% maximum allocation to each stock placed while maximising Sharpe ratio as discussed in the Data & Methodology section.

The shrinkage methods do not generate impressive momentum returns when compared to other optimisation techniques tested. Further, it is noticed that the momentum returns fall when the percentage of shrinking individual mean to grand mean increases. For example, the average-monthly momentum returns for the Indian stock market fall from 1.49% (25% individual mean shrinkage to grand mean) to 1.33% (75% individual mean shrinkage to global mean). The same drop in returns can be observed in other countries. These results suggest that the shrinkage method may not work best when applied in the context of momentum returns. Setting all expected returns to zero as suggested by Chopra and Ziemba (1993) did not yield any better results either.

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