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Fuzzy Parameters for Evaluating Capital Investments

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Abstract: In the process of doing capital budgeting, one of the required steps is to evaluate the investment value of the candidates, while the net present value (NPV) is always a trustworthy indicator. This work presents a series of pragmatic algorithms for calculating the return and risk parameters of NPV in fuzzy numbers, which are anticipated to do a better job in an uncertain environment than traditional crisp-number equations due to their effectiveness in capturing the abstract qualitative information as well as the quantitative information. These algorithms cope with the randomness of outcomes and the vagueness of data estimation and can efficiently integrate the information. The return parameters (EFNPVs) include the expected fuzzy NPV, the equivalent annuity, and the equivalent annuity to infinity. The risk parameter goes to the fuzzy lower partial moment (FLPM), which measures the absolute value of the expected below-target loss in terms of a capitalist's risk tolerance and is more realistic than the commonly used standard deviation. A performance ratio is further defined as the expected excess-target return to LPM. The candidate with a high EFNPV, a low FLPM, and/or a high performance ratio is a priority.

Keyword: *capital budgeting, fuzzy net present value, fuzzy equivalent annuity, fuzzy equivalent annuity to infinity, fuzzy lower partial moment*

1. INTRODUCTION

The financial decisions of an organization begin with the choice of a business strategy which is designed to create the wealth of shareholders. The strategy is then implemented by making capital investments. A capital investment is defined as an outlay that is expected to result in future benefits. Because capital investments are so important to the success of an organization, most companies have formal policies guiding the decision process. A typical process includes several steps: (1) Establishing a firm goal. The abstract goal of wealth creation is then translated into some concrete goals against which performance can be measured, such as ROE, share price, and sales growth. (2) Developing strategies. The strategy sets the general direction of an organization and provides the framework within which capital investment opportunities are sought. (3) Searching for investment opportunities. (4) Appraising and deciding investment opportunities. (5) Implementing and monitoring. (6) Post-auditing. This includes a comparison of the actual performance with the forecast and the modification of goals, strategies, and operations. [18]

At the stage of making an appraisal and decision, there are several popular methods, such as net present value (NPV), profitability index, internal rate of return, payback period, and accounting rate of return. Among these methods, NPV is always trustworthy, because it measures a project's net wealth contribution to a company. [18] However, an investment with a satisfactory ex-ante NPV may still incur a loss or get back an amount that is less than expectation. This is due to uncertainty towards the future. It is therefore an important issue to analyze and gauge the uncertainty.

One common method for the analysis is through forecasting the future outcomes of an investment. The traditional field of finance deals with the problem that future outcomes have randomness and can describe the randomness by the parameters of random variables. It dates back to 1952 when Markowitz [13] identified the return of an investment with the expected value, and the risk with the variance of outcomes around the mean value. However, the common perception of risk focuses on the likelihood of losses, or the "downside," and not on the upside or variability. Markowitz [14] in 1959 recognized this idea and agreed that only downside risk or safety first is relevant to an investor and that security distribution may not be normally distributed. He provided a below-mean semivariance and a below-target semivariance. In 1975 Bawa [1] defined the lower partial moment (LPM, to be detailed in Section 3.2), which describes the below-target risk in terms of individual risk tolerance. By comparison, LPM measures the possibility of losses in a whole set of utility functions, while the variance and semivariance only provide one utility function.

Traditional finance does not deal with the problem that the data estimation of outcomes comes with imprecision, or vagueness. For example, the profits and losses of a business are available only when the operation ends and all assets and liabilities have been liquidated and paid, yet in practice the Generally Accepted Accounting Principles require a business to cut its duration into many accounting periods in order to provide users with financial statements incorporating timely information. The figures on financial statements are actually an approximation based on some artificial rules. For another example, when we estimate the amount of future cash flow from an investment, it is only an approximation and may not actually be that amount. It may be more effective to incorporate a flexible range into estimation.

This work suggests using fuzzy sets [23] to address the factor of inherent vagueness and combines them with the parameters of random variables to report randomness. The application of fuzzy sets to finance began in the 1980s [2]. Capital budgeting that applies fuzzy sets has been academically popular in recent years [3,4,8,9,11,16,20,21], and many contributions have gone into the expansion of theorems, such as the discussion of evaluation indicators and risk simulation, but most financial practitioners actually seldom apply fuzzy sets. One of the more likely reasons is that the calculation equations are too academic to follow. This work therefore develops step-by-step computational algorithms for the purpose of practical use.

Assume that the chances of outcomes for a random experiment [12,15] are modeled by possibilities, which connect with fuzzy sets and allow a reasoning to be carried out on imprecise or vague knowledge. [25] First, the capitalist assumes the scenarios of economic prospects and assesses the corresponding possibilities. Second, the possibilities are expressed in linguistic terms [24], which better catch the intrinsic human thought than does quantitative appraisal when there is no similar event for referral to. Fuzzy numbers, which allow the flexibility on estimation, are then adopted to represent the linguistic terms and estimate cash flows and costs of capital.

The return parameters (EFNPVs) include the expected fuzzy NPV for a single project, expected fuzzy equivalent annuity for mutually exclusive projects with equal life spans, and expected fuzzy equivalent annuity to infinity for projects with unequal risks. The risk parameter goes to the fuzzy lower partial moment (FLPM) instead of standard deviation. FLPM measures the absolute value of expected below-target return in terms of a capitalist's risk tolerance. It is more realistic than the standard deviation's measuring volatility because the capitalist feels unsafe only on a loss, and not at a profit. A performance ratio is further defined as the expected excess-target return to LPM. The priority of the candidate investment projects is given by considering three criteria: high EFNPV, low FLPM, and/or high performance ratio.

The rest of this work is organized as follows. Section 2 briefly describes fuzzy numbers and fuzzy arithmetic. Section 3 develops the fuzzy algorithms. Conclusions are made in Section 4.

2. FUZZY NUMBERS AND FUZZY ARITHMETIC

2.1. Fuzzy Numbers

Fuzzy sets describe the classes of objects encountered in the real physical world with no precisely defined criteria of membership. It is a "class" with a continuum of grades of membership that provide a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets. [23]

An any-shape fuzzy number $\tilde{A}=[a,b,c,d]$ (in square bracket), $-\infty<a\le b\le c\le d<\infty$, $a,b,c,d\in R$, is described as any fuzzy subset of the real line R with the membership function $f_{\tilde{A}}(x)$. The membership function $f_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]; $f_{\tilde{A}}(x)$ is strictly increasing on $x\in[a,b]$; $f_{\tilde{A}}(x)$=1 for $x\in[b,c]$; $f_{\tilde{A}}(x)$ is strictly decreasing on $x\in[c,d]$; $f_{\tilde{A}}(x)$=0 for all $x\in(-\infty,a]$ and $x\in[d,\infty)$. [6] A fuzzy number $\tilde{B}=(e,f,g,h)$ (in round bracket) is defined as a trapezoidal one if $f_{\tilde{B}}(x)$ is given by: [10]

$$f_{\tilde{B}}(x)=\begin{cases}(x-e)/(f-e), & e\le x\le f,\\ 1 & f\le x\le g\\ (x-h)/(g-h), & g\le x\le h,\\ 0, & otherwise.\end{cases} \tag{1}$$

2.2. Fuzzy Arithmetic

The α-cut, also called the interval of confidence for the level of α, of a fuzzy number $\tilde{A}$ is defined as: [10]

$${}^{\alpha}\tilde{A}=\{x|f_{\tilde{A}}(x)\ge\alpha,\},\ x\in R,\ \alpha\in[0,1], \tag{2}$$

where ${}^{\alpha}\tilde{A}$ is a non-empty bounded closed interval contained in R. Hereafter it is denoted by ${}^{\alpha}\tilde{A}=[{}^{\alpha}\underline{A},{}^{\alpha}\overline{A}]$, where ${}^{\alpha}\underline{A}$ and ${}^{\alpha}\overline{A}$ are respectively the lower and upper bounds of the closed interval for the α level. The α-cut of a trapezoidal fuzzy number $\tilde{B}=(e,f,g,h)$ is expressed as:

$${}^{\alpha}\tilde{B}=[{}^{\alpha}\underline{B},{}^{\alpha}\overline{B}]=[(f-e)\alpha+e,(g-h)\alpha+h],\ \alpha\in[0,1] \tag{3}$$

Equations (4)~(7) are the standard fuzzy-arithmetic operational rules for $\tilde{A},\tilde{B}\in R$: [10]

$${}^{\alpha}(\tilde{A}+\tilde{B})={}^{\alpha}\tilde{A}+{}^{\alpha}\tilde{B}=\left[{}^{\alpha}\underline{A}+{}^{\alpha}\underline{B},\ {}^{\alpha}\overline{A}+{}^{\alpha}\overline{B}\right], \tag{4}$$

$${}^{\alpha}(\tilde{A}-\tilde{B})={}^{\alpha}\tilde{A}-{}^{\alpha}\tilde{B}=\left[{}^{\alpha}\underline{A}-{}^{\alpha}\overline{B},\ {}^{\alpha}\overline{A}-{}^{\alpha}\underline{B}\right], \tag{5}$$

$${}^{\alpha}(\tilde{A}\times\tilde{B})=\left[\min({}^{\alpha}\underline{A}\cdot{}^{\alpha}\underline{B},\ {}^{\alpha}\underline{A}\cdot{}^{\alpha}\overline{B},{}^{\alpha}\overline{A}\cdot{}^{\alpha}\underline{B},{}^{\alpha}\overline{A}\cdot{}^{\alpha}\overline{B}),\max({}^{\alpha}\underline{A}\cdot{}^{\alpha}\underline{B},{}^{\alpha}\underline{A}\cdot{}^{\alpha}\overline{B},{}^{\alpha}\overline{A}\cdot{}^{\alpha}\underline{B},{}^{\alpha}\overline{A}\cdot{}^{\alpha}\overline{B})\right], \tag{6}$$

$${}^{\alpha}(\tilde{A}\div\tilde{B})=\left[\min({}^{\alpha}\underline{A}/{}^{\alpha}\underline{B},\ {}^{\alpha}\underline{A}/{}^{\alpha}\overline{B},{}^{\alpha}\overline{A}/{}^{\alpha}\underline{B},{}^{\alpha}\overline{A}/{}^{\alpha}\overline{B}),\max({}^{\alpha}\underline{A}/{}^{\alpha}\underline{B},{}^{\alpha}\underline{A}/{}^{\alpha}\overline{B},{}^{\alpha}\overline{A}/{}^{\alpha}\underline{B},{}^{\alpha}\overline{A}/{}^{\alpha}\overline{B})\right], \tag{7}$$

2.3. Ranking Fuzzy Numbers by the Method of Average Relative Regions

A ranking procedure is suggested to defuzzify the fuzzy parameters to crisp values and put them in order. Among many ranking methods developed in the past few years [5], one that can consistently rank positive and negative fuzzy numbers – the method of average relative regions [22] – is applied in this work:

$$S(\tilde{A}_i)=\frac{1}{2}\left(S_L(\tilde{A}_i)+S_R(\tilde{A}_i)\right)=\frac{1}{2}\left(\left(\left(b_i-\min_i(a_i)\right)\times1-\int_{a_i}^{b_i}f_{\tilde{A}_i}^{\mathrm{L}}(x)\,\mathrm{d}x\right)+\left(\left(c_i-\min_i(a_i)\right)\times1+\int_{c_i}^{d_i}f_{\tilde{A}_i}^{\mathrm{R}}(x)\,\mathrm{d}x\right)\right). \tag{8}$$

The terms $S_L(\tilde{A}_i)$ and $S_R(\tilde{A}_i)$ respectively denote the value of the left and right relative regions of an any-shape fuzzy number $\tilde{A}_i=[a_i,b_i,c_i,d_i]$; they are respectively defined as the area stretching from the left and right membership functions of $\tilde{A}_i$ to the axis at the minimal value of the lower bounds of $\tilde{A}_i$. The term $S(\tilde{A}_i)$ denotes the average of the relative left and right regions, where there are values $S_L(\tilde{A}_i), S_R(\tilde{A}_i), S(\tilde{A}_i)\geq 0$. A larger fuzzy value of $\tilde{A}_i$ is anticipated to have a larger crisp value of $S(\tilde{A}_i)$.

3. FUZZY NET PRESENT VALUE AND FUZZY LOWER PARTIAL MOMENT

3.1. Fuzzy Net Present Value

3.1.1. For a Single Project

The scenarios of economic prospects and the corresponding possibilities in linguistic terms are assumed in linguistic terms. For example, let Ω={recession, standoff, low growth, medium growth, high growth} be the set of economic scenarios, and let Λ ={no, low, fair, high, absolute} be the set of possibilities. Let trapezoidal fuzzy number $\tilde{P}_i^{'}=(o_i^{'},p_i^{'},q_i^{'},r_i^{'})$ represent the linguistic possibility for scenario *i*, where $0\leq o_i^{'}\leq p_i^{'}\leq q_i^{'}\leq r_i^{'}\leq 1$. Standardize $\tilde{P}_i^{'}$ to $\tilde{P}_i$:

$$\tilde{P}_i=\left(o_i^{'}\Big/\sum_{i=1}^{m}r_i^{'},\, p_i^{'}\Big/\sum_{i=1}^{m}r_i^{'},\, q_i^{'}\Big/\sum_{i=1}^{m}r_i^{'},\, r_i^{'}\Big/\sum_{i=1}^{m}r_i^{'}\right)=(o_i,p_i,q_i,r_i)\,, \tag{14}$$

so as to ensure $\sum_{i=1}^{n_k}r_i=1$. The evaluator further estimates the cash flow $\tilde{F}_{kij}=(a_{kij},b_{kij},c_{kij},d_{kij})$ and the cost of capital $\tilde{R}_{ki}^{'}=(e_{ki}^{'},f_{ki}^{'},g_{ki}^{'},h_{ki}^{'})$ in trapezoidal fuzzy numbers, where $k=1,2,...,l$, $i=1,2,...,m$, $j=1,2,...,n_k$, $-\infty<a_{kij}\leq b_{kij}\leq c_{kij}\leq d_{kij}<\infty$, and $0<e_{ki}^{'}\leq f_{ki}^{'}\leq g_{ki}^{'}\leq h_{ki}^{'}$. Let $(1+\tilde{R}_{ki}^{'})=\tilde{R}_{ki}$ $=(1+e_{ki}^{'},1+f_{ki}^{'},1+g_{ki}^{'},1+h_{ki}^{'})$ $=(e_{ki},f_{ki},g_{ki},h_{ki})$. According to Eq. (3), the α-cuts of $\tilde{F}_{kij}$, $\tilde{R}_{ki}^{'}$, $\tilde{R}_{ki}^{\;j}$, and $\tilde{P}_i$ are: ${}^{\alpha}\tilde{F}_{kij}=\left[(b_{kij}-a_{kij})\alpha+a_{kij},(c_{kij}-d_{kij})\alpha+d_{kij}\right]=\left[{}^{\alpha}\underline{F_{kij}},{}^{\alpha}\overline{F_{kij}}\right]$, ${}^{\alpha}\tilde{R}_{ki}^{'}=\left[(f_{ki}^{'}-e_{ki}^{'})\alpha+e_{ki}^{'},(g_{ki}^{'}-h_{ki}^{'})\alpha+h_{ki}^{'}\right]=\left[{}^{\alpha}\underline{R_{ki}^{'}},{}^{\alpha}\overline{R_{ki}^{'}}\right]$, ${}^{\alpha}\tilde{R}_{ki}^{\;j}=\left[((f_{ki}-e_{ki})\alpha+e_{ki})^{j},((g_{ki}-h_{ki})\alpha+h_{ki})^{j}\right]=\left[{}^{\alpha}\underline{R_{ki}}^{j},{}^{\alpha}\overline{R_{ki}}^{j}\right]$, and ${}^{\alpha}\tilde{P}_i=\left[(p_i-o_i)\alpha+o_i,(q_i-r_i)\alpha+r_i\right]=\left[{}^{\alpha}\underline{P_i},{}^{\alpha}\overline{P_i}\right]$.

By Eq. (9), the α-cut of fuzzy NPV (FNPV) for project *k* under scenario *i* is defined as:

$$\begin{aligned}{}^{\alpha}\tilde{V}_{ki}&={}^{\alpha}\left(\sum_{j=0}^{n_k}\frac{\tilde{F}_{kij}}{\tilde{R}_{ki}^{\;j}}\right)=\left[\sum_{j=0}^{n_k}\min\left(\frac{{}^{\alpha}\underline{F_{kij}}}{{}^{\alpha}\underline{R_{ki}}^{j}},\frac{{}^{\alpha}\underline{F_{kij}}}{{}^{\alpha}\overline{R_{ki}}^{j}},\frac{{}^{\alpha}\overline{F_{kij}}}{{}^{\alpha}\underline{R_{ki}}^{j}},\frac{{}^{\alpha}\overline{F_{kij}}}{{}^{\alpha}\overline{R_{ki}}^{j}}\right),\sum_{j=0}^{n_k}\max\left(\frac{{}^{\alpha}\underline{F_{kij}}}{{}^{\alpha}\underline{R_{ki}}^{j}},\frac{{}^{\alpha}\underline{F_{kij}}}{{}^{\alpha}\overline{R_{ki}}^{j}},\frac{{}^{\alpha}\overline{F_{kij}}}{{}^{\alpha}\underline{R_{ki}}^{j}},\frac{{}^{\alpha}\overline{F_{kij}}}{{}^{\alpha}\overline{R_{ki}}^{j}}\right)\right]\\&=\left[\sum_{j=0}^{n_k}{}^{\alpha}\underline{V_{kij}},\sum_{j=0}^{n_k}{}^{\alpha}\overline{V_{kij}}\right]=\left[{}^{\alpha}\underline{V_{ki}},{}^{\alpha}\overline{V_{ki}}\right]\end{aligned} \tag{15}$$

By Eqs. (10) and (15), the α-cut of the expected value of FNPV for project *k* is:

$${}^{\alpha}\tilde{\mu}_{\tilde{V}_k}={}^{\alpha}\left(\sum_{i=1}^{m}\tilde{V}_{ki}\tilde{P}_i\right)=\left[\sum_{i=1}^{m}\min\left({}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\underline{P_i},{}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\overline{P_i},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\underline{P_i},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\overline{P_i}\right),\sum_{i=1}^{m}\max\left({}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\underline{P_i},{}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\overline{P_i},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\underline{P_i},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\overline{P_i}\right)\right]=\left[{}^{\alpha}\underline{\mu_{\tilde{V}_k}},{}^{\alpha}\overline{\mu_{\tilde{V}_k}}\right]. \tag{16}$$

3.1.2. For Mutually Exclusive Projects

Let $\tilde{A}_{n_k,\tilde{R}_{ki}^{'}}$ denote the PVIFA in fuzzy costs of capital. Define the discount factor $\tilde{G}_{ki}$ as the reciprocal of $\tilde{A}_{n_k,\tilde{R}_{ki}^{'}}$:

$${}^{\alpha}\tilde{G}_{ki}={}^{\alpha}\left(1/\tilde{A}_{n_k,\tilde{R}_{ki}^{'}}\right)={}^{\alpha}\left(\tilde{R}_{ki}^{'}/\left(1-(1+\tilde{R}_{ki}^{'})^{-n_k}\right)\right)={}^{\alpha}\left(\tilde{R}_{ki}^{'}/\left(1-\tilde{R}_{ki}^{\;-n_k}\right)\right) \tag{17}$$

For projects with unequal lives, the α-cut of fuzzy equivalent annuity (FEA) for project *k* under scenario *i* is:

$${}^{\alpha}_{EA}\tilde{V}_{ki}={}^{\alpha}\tilde{V}_{ki}\times{}^{\alpha}\tilde{G}_{ki}=\left[\min\left({}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\underline{G_{ki}},{}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\overline{G_{ki}},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\underline{G_{ki}},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\overline{G_{ki}}\right),\max\left({}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\underline{G_{ki}},{}^{\alpha}\underline{V_{ki}}\,{}^{\alpha}\overline{G_{ki}},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\underline{G_{ki}},{}^{\alpha}\overline{V_{ki}}\,{}^{\alpha}\overline{G_{ki}}\right)\right]=\left[{}^{\alpha}\underline{{}_{EA}V_{ki}},{}^{\alpha}\overline{{}_{EA}V_{ki}}\right]. \tag{18}$$

For projects with unequal risks, the α-cut of fuzzy equivalent annuity to infinity (FEAI) for project *k* under scenario *i* is:

$$^{\alpha}{}_{EAI}\tilde{V}_{ki} = \frac{^{\alpha}{}_{EA}\tilde{V}_{ki}}{^{\alpha}\tilde{R}^{'}_{ki}} = \left[\min\left(\frac{^{\alpha}\underline{_{EAI}V_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\underline{_{EAI}V_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EAI}V_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EAI}V_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}\right), \max\left(\frac{^{\alpha}\underline{_{EAI}V_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\underline{_{EAI}V_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EAI}V_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EAI}V_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}\right)\right] = \left[^{\alpha}\underline{_{EAI}V_{ki}}, ^{\alpha}\overline{_{EAI}V_{ki}}\right] \quad (19)$$

The α-cuts of the expected FEA ($^{\alpha}\tilde{\mu}_{_{EA}\tilde{V}_k}$) and expected FEAI ($^{\alpha}\tilde{\mu}_{_{EAI}\tilde{V}_k}$) are defined by substituting $_{EA}\tilde{V}_{ki}$ and $_{EAI}\tilde{V}_{ki}$ for $\tilde{V}_{ki}$ in Eq. (16), respectively:

$$^{\alpha}\tilde{\mu}_{_{EA}\tilde{V}_k} = {}^{\alpha}\left(\sum_{i=1}^{m} {}_{EA}\tilde{V}_{ki}\tilde{P}_i\right) = \left[^{\alpha}\underline{\mu_{_{EA}\tilde{V}_k}}, ^{\alpha}\overline{\mu_{_{EA}\tilde{V}_k}}\right]; \quad ^{\alpha}\tilde{\mu}_{_{EAI}\tilde{V}_k} = {}^{\alpha}\left(\sum_{i=1}^{m} {}_{EAI}\tilde{V}_{ki}\tilde{P}_i\right) = \left[^{\alpha}\underline{\mu_{_{EAI}\tilde{V}_k}}, ^{\alpha}\overline{\mu_{_{EAI}\tilde{V}_k}}\right].$$

3.2. Fuzzy Lower Partial Moment

3.2.1. For a Single Project

When calculating the FLPM, it is necessary to set a target return, $\tilde{T}_{ki}$, which in this work is defined as the original cash outflow times the compound risk-free rate:

$$\tilde{T}_{ki} = -\tilde{F}_{ki0}\cdot\left((1+{}_f\tilde{R}^{'}_i)^{n_k}-1\right) = -\tilde{F}_{ki0}\cdot{}_f\tilde{R}_{ki} = (s_{ki}, t_{ki}, u_{ki}, v_{ki}), \quad (20)$$

where $\tilde{F}_{ki0}$ and $_f\tilde{R}_{ki}$ respectively denote the original cash outflow and the fuzzy risk-free rate of return of project k under scenario i, $_f\tilde{R}^{'}_i$ denotes the risk-free rate of return under scenario i, and n_k is the duration of project k. The α-cut of fuzzy target return is: $^{\alpha}\tilde{T}_{ki} = [(t_i - s_i)\alpha + s_i, (u_i - v_i)\alpha + v_i] = \left[^{\alpha}\underline{T_{ki}}, ^{\alpha}\overline{T_{ki}}\right]$.

Let $\tilde{\Delta}_{\tilde{V}_{ki}} = \tilde{T}_{ki} - \tilde{V}_{ki}$. By Eq. (5), $^{\alpha}\tilde{\Delta}_{\tilde{V}_{ki}} = \left[^{\alpha}\underline{T_{ki}} - {}^{\alpha}\overline{V_{ki}}, {}^{\alpha}\overline{T_{ki}} - {}^{\alpha}\underline{V_{ki}}\right] = \left[^{\alpha}\underline{\Delta_{\tilde{V}_{ki}}}, ^{\alpha}\overline{\Delta_{\tilde{V}_{ki}}}\right]$. By Eq. (13), the α-cut of FLPM of NPV, $\tilde{L}^{\delta}_{\tilde{V}_k}$, is defined as:

$$^{\alpha}\tilde{L}^{\delta}_{\tilde{V}_k} = {}^{\alpha}\left(\sum_{i=1}^{m}\left(\left(\max(0, \tilde{T}_{ki} - \tilde{V}_{ki})\right)^{\delta} \times \tilde{P}_i\right)\right) = \left[\sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\underline{\Delta_{\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\underline{P_i}\right), \sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\overline{\Delta_{\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\overline{P_i}\right)\right] = \left[^{\alpha}\underline{L^{\delta}_{\tilde{V}_k}}, ^{\alpha}\overline{L^{\delta}_{\tilde{V}_k}}\right]. \quad (21)$$

3.2.2. For Mutually Exclusive Projects

By Eq. (11), the EA of fuzzy target return is:

$$^{\alpha}{}_{EA}\tilde{T}_{ki} = {}^{\alpha}\tilde{T}_{ki} \times {}^{\alpha}\tilde{G}_{ki} = \left[\min\left(^{\alpha}\underline{T_{ki}}\,^{\alpha}\underline{G_{ki}}, ^{\alpha}\overline{T_{ki}}\,^{\alpha}\underline{G_{ki}}, ^{\alpha}\underline{T_{ki}}\,^{\alpha}\overline{G_{ki}}, ^{\alpha}\overline{T_{ki}}\,^{\alpha}\overline{G_{ki}}\right), \max\left(^{\alpha}\underline{T_{ki}}\,^{\alpha}\underline{G_{ki}}, ^{\alpha}\overline{T_{ki}}\,^{\alpha}\underline{G_{ki}}, ^{\alpha}\underline{T_{ki}}\,^{\alpha}\overline{G_{ki}}, ^{\alpha}\overline{T_{ki}}\,^{\alpha}\overline{G_{ki}}\right)\right] = \left[^{\alpha}\underline{_{EA}T_{ki}}, ^{\alpha}\overline{_{EA}T_{ki}}\right] \quad (22)$$

Let $\tilde{\Delta}_{_{EA}\tilde{V}_{ki}} = {}_{EA}\tilde{T}_{ki} - \tilde{V}_{_{EA}\tilde{V}_{ki}}$. By Eq. (5), $^{\alpha}\tilde{\Delta}_{_{EA}\tilde{V}_{ki}} = \left[^{\alpha}\underline{_{EA}T_{ki}} - {}^{\alpha}\overline{_{EA}V_{ki}}, {}^{\alpha}\overline{_{EA}T_{ki}} - {}^{\alpha}\underline{_{EA}V_{ki}}\right] = \left[^{\alpha}\underline{\Delta_{_{EA}\tilde{V}_{ki}}}, ^{\alpha}\overline{\Delta_{_{EA}\tilde{V}_{ki}}}\right]$. By Eqs. (13), the α-cut of FLPM of FEA is defined as:

$$^{\alpha}\tilde{L}^{\delta}_{_{EA}\tilde{V}_k} = {}^{\alpha}\left(\sum_{i=1}^{m}\left(\left(\max(0, \tilde{\Delta}_{_{EA}\tilde{V}_{ki}})\right)^{\delta} \times \tilde{P}_i\right)\right) = \left[\sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\underline{\Delta_{_{EA}\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\underline{P_i}\right), \sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\overline{\Delta_{_{EA}\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\overline{P_i}\right)\right] = \left[^{\alpha}\underline{L^{\delta}_{_{EA}\tilde{V}_k}}, ^{\alpha}\overline{L^{\delta}_{_{EA}\tilde{V}_k}}\right]. \quad (23)$$

By Eq. (12), the EAI of fuzzy target return is:

$$^{\alpha}{}_{EAI}\tilde{T}_{ki} = \frac{^{\alpha}{}_{EA}\tilde{T}_{ki}}{^{\alpha}\tilde{R}^{'}_{ki}} = \left[\min\left(\frac{^{\alpha}\underline{_{EA}T_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\underline{_{EA}T_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EA}T_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EA}T_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}\right), \max\left(\frac{^{\alpha}\underline{_{EA}T_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\underline{_{EA}T_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EA}T_{ki}}}{^{\alpha}\underline{R}^{'}_{ki}}, \frac{^{\alpha}\overline{_{EA}T_{ki}}}{^{\alpha}\overline{R}^{'}_{ki}}\right)\right] = \left[^{\alpha}\underline{_{EAI}T_{ki}}, ^{\alpha}\overline{_{EAI}T_{ki}}\right]. \quad (24)$$

Let $\tilde{\Delta}_{_{EAI}\tilde{V}_{ki}} = {}_{EAI}\tilde{T}_{ki} - {}_{EAI}\tilde{V}_{ki}$. By Eq. (5), $^{\alpha}\tilde{\Delta}_{_{EAI}\tilde{V}_{ki}} = \left[^{\alpha}\underline{_{EAI}T_{ki}} - {}^{\alpha}\overline{_{EAI}V_{ki}}, {}^{\alpha}\overline{_{EAI}T_{ki}} - {}^{\alpha}\underline{_{EAI}V_{ki}}\right] = \left[^{\alpha}\underline{\Delta_{_{EAI}\tilde{V}_{ki}}}, ^{\alpha}\overline{\Delta_{_{EAI}\tilde{V}_{ki}}}\right]$. By Eqs. (13), the α-cut of FLPM of FEAI is defined as:

$$^{\alpha}\tilde{L}^{\delta}_{_{EAI}\tilde{V}_k} = {}^{\alpha}\left(\sum_{i=1}^{m}\left(\left(\max(0, \tilde{\Delta}_{_{EAI}\tilde{V}_{ki}})\right)^{\delta} \times \tilde{P}_i\right)\right) = \left[\sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\underline{\Delta_{_{EAI}\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\underline{P_i}\right), \sum_{i=1}^{m}\left(\left(\max(0, {}^{\alpha}\overline{\Delta_{_{EAI}\tilde{V}_{ki}}})\right)^{\delta}\cdot{}^{\alpha}\overline{P_i}\right)\right] = \left[^{\alpha}\underline{L^{\delta}_{_{EAI}\tilde{V}_k}}, ^{\alpha}\overline{L^{\delta}_{_{EAI}\tilde{V}_k}}\right]. \quad (25)$$

In order to rank the order of these fuzzy parameters, Eq. (8) is applied to get the defuzzifications – $S(\tilde{\mu}_{\tilde{V}_k})$, $S(\tilde{\mu}_{_{EA}\tilde{V}_k})$, $S(\tilde{\mu}_{_{EAI}\tilde{V}_k})$, $S(\tilde{L}^{\delta}_{\tilde{V}_k})$, $S(\tilde{L}^{\delta}_{_{EA}\tilde{V}_k})$, and $S(\tilde{L}^{\delta}_{_{EAI}\tilde{V}_k})$.

3.3. The Performance Ratio

Roy [17] in 1952 proposed a "performance index", the reward to variability ratio, which can be regarded as the predecessor of the later famous Sharpe ratio [19]. Here, we modify Roy's by substituting LPM for the standard deviation to a "performance ratio:"

$$I^{\delta}_{\tilde{V}_k-\tilde{T}_k} = S(\tilde{\mu}_{\tilde{V}_k-\tilde{T}_k}) \big/ S(\tilde{L}^{\delta}_{\tilde{V}_k-\tilde{T}_k}) \cdot \quad (26)$$

$I^{\delta}_{\tilde{V}_k-\tilde{T}_k}$ measures the expected excess-target profit carried by the expected below-target loss. It shows the trade off between premium and risk. The performance ratios for EA and EAI are: $I^{\delta}_{EA\tilde{V}_k-EA\tilde{T}_k} = S(\tilde{\mu}_{EA\tilde{V}_k-EA\tilde{T}_k}) \big/ S(\tilde{L}^{\delta}_{EA\tilde{V}_k})$; $I^{\delta}_{EAI\tilde{V}_k-EAI\tilde{T}_k} = S(\tilde{\mu}_{EAI\tilde{V}_k-EAI\tilde{T}_k}) \big/ S(\tilde{L}^{\delta}_{EAI\tilde{V}_k}) \cdot$

3.4. The Decision Criteria

When considering return and risk at the same time, the decision rule is to maximize return in terms of the same risk and minimize risk in terms of the same return. Therefore, a project with a high defuzzified expected return ($S(\tilde{\mu}_{\tilde{V}_k})$, $S(\tilde{\mu}_{EA\tilde{V}_k})$, or $S(\tilde{\mu}_{EAI\tilde{V}_k})$), low defuzzified LPM ($S(\tilde{L}^{\delta}_{\tilde{V}_k})$, $S(\tilde{L}^{\delta}_{EA\tilde{V}_k})$, or $S(\tilde{L}^{\delta}_{EAI\tilde{V}_k})$), and high performance ratio ($I^{\delta}_{\tilde{V}_k-\tilde{T}_k}$, $I^{\delta}_{EA\tilde{V}_k-EA\tilde{T}_k}$, or $I^{\delta}_{EAI\tilde{V}_k-EAI\tilde{T}_k}$) is suggested as the priority. Different criteria and risk tolerance may cause different ranking order among the candidates.

4. CONCLUSIONS

This work presents a set of algorithms that are able to model the expected fuzzy return and the fuzzy downside risk for evaluating capital investments. The application of the parameters of random variables and fuzzy sets respectively report the randomness of future outcomes and vagueness of data estimation. The definition of downside risk here is the absolute value of the expected below-target loss in terms of the capitalist's risk tolerance, not including the above-target profit. In addition to the expected return and downside risk, a performance ratio defined as the expected excess-target return carried by the downside risk is suggested to be the third decision criterion.

Linguistic terms, which can better reflect human intuitive thought than do quantitative numbers, are adopted to forecast economic prospects and possibilities. These linguistic judgments are then converted to fuzzy numbers. The cash flows and costs of capital for the investment candidates are estimated in fuzzy numbers so as to take into account the flexibility of estimation. The algorithms allow the operation of fuzzy random variables in multiple powers and the implied information to be recorded, integrated, and concluded systematically. They are anticipated to be practical in use and can be inferred to other financial problems, such as portfolio selection and mergers and acquisitions.

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REFERENCES

Bawa, V. S. (1975), Optimal Rules For Ordering Uncertain Prospects, *Journal of Financial Economics*, 2(1), 5-121.

Buckley, J.J. (1987), The Fuzzy Mathematics of Finance, *Fuzzy Sets and Systems*, 21, 257-273.

Chansa-ngavej, Ch. and C.A. Mount-Campbell (1991), Decision criteria in capital budgeting under uncertainties: implications for future research, *International Journal of Prod. Economics*, 23, 25-35.

Chiu, C. Y. and C. S. Park (1994), Fuzzy cash flow analysis using present worth criterion, *Eng. Economist*, 39 (2), 113-138.

Chu, T.C. and C.T. Tsao (2002), Ranking Fuzzy Numbers with an Area between the Centroid Point and Original Point, *Computers and Mathematics with Applications*, 43, 111-117.

Dubois, D. and H. Prade (1978), Operations on fuzzy numbers, *International Journal of Systems Science*, 9(6), 613-626.

Fishburn, P.C. (1977), Mean-risk analysis associated with below-target returns, *American Economic Review*, 67(2), 116-126.

Huang, X., Mean-variance model for fuzzy capital budgeting (2008), *Computers and Industrial Engineering*, 55(1), 34-47.

Kahraman, C., D. Ruan, and E. Tolga (2002), Capital budgeting techniques using discounted fuzzy versus probabilistic cash flows, *Information Science*, 142, 57-76.

Kaufmann, A. and M.M. Gupta (1991), *Introduction to Fuzzy Arithmetic: Theory and Application*, VanNostrand Reinhold, New York.

Kuchta, D. (2000), Fuzzy capital budgeting, *Fuzzy Sets and Systems*, 111, 367-385.

Kwakernaak, H. (1978), Fuzzy random variables – I, Definitions and theorems, *Journal of Information Sciences* 15, 1-29.

Markowitz, H.M. (1952), Portfolio selection, *Journal of Finance*, 7(1), 77-91.

Markowitz, H.M. (1959), *Portfolio selection* (First Edition), New York: John Wiley and Sons.

Puri, M.L. and D.A. Ralescu (1986), Fuzzy Random Variables, *Journal of Mathematical Analysis and Applications*, 114, 409-422.

Rebiasz, B. (2007), Fuzziness and randomness in investment project risk appraisal, *Computers & Operations Research*, 34, 199-210.

Roy, A.D. (1952), Safety first and the holding of assets, *Econometrica*, 20(3), 431-449.

Seitz, N. and M. Ellison (1999), *Capital Budgeting and Long-Term Financial Decisions,* Dryden. USA.

Sharpe, W.F. (1966), Mutual fund performance, *Journal of Business*, 39(1), Part II, 119-138.

Smith, D.J. (1994), Incorporating risk into capital budgeting decisions using simulation, *Management Decision,* 32(9), 20-26.

Tsao, C.T. (2005), Assessing the Probabilistic Fuzzy Net Present Value for a Capital Investment Choice Using Fuzzy Arithmetic, *Journal of the Chinese Institute of Industrial Engineers*, 22(2).,106-118.

Tsao, C.T. and T.C. Chu (2002), An improved fuzzy MCDM model based on ideal and anti-ideal concepts, *Journal of the Operations Research Society of Japan*, 45(2), 185-197.

Zadeh, L.A. (1965), Fuzzy sets, *Information and Control*, 8, 338-353.

Zadeh, L.A. (1975), The concept of a linguistic variable and its application to approximate reasoning, *Information Sciences,* 8(3), 199-249.

Zadeh, L.A. (1978), Fuzzy Sets as a Basis for a Theory of Possibility, *Fuzzy Sets and System*, 1, 3-28.