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Space and Time Analysis of Tourist Movements using Semi-Markov Processes

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Abstract: Tourist movement is a complex process, but it provides very useful information for park managers and tourist operators. This paper aims to establish a sound methodology for modelling the spatial and temporal movement of tourists, with the objectives of understanding, predicting, controlling and optimising the decisions made by them as they go about choosing the attractions they want to visit..

Tourist movements, in this paper, are modelled as discrete processes between specific tourist locations, which could be located some distance apart. A Semi-Markov process has a Markov chain and a renewal process embedded within its structure. Therefore, the Semi-Markov chain can be used to understand the interaction of tourists with attractions as a sequence of movements over time, rather than their interaction with individual attractions.

The following two assumptions, which underlie the Semi--Markov process, make it an especially ideal tool for modeling movements of tourists:

- The probability that a tourist will visit a particular attraction depends only on the most recent attraction that was visited by that tourist.
- The distribution of the time spent at each attraction is dependent on both that attraction and the next attraction that is visited..

One of the outcomes of this approach is a measure which assesses the attractiveness of particular tourist attractions based on spatial and temporal interactions between the attractions. Two assumptions, based on the assumptions of the Semi--Markov process, are derived for assessing attractiveness of tourist attractions

- The more tourists visit an attraction, the more attractive it is. A transition probability matrix is developed for estimating the probability that a tourist will visit a particular attraction based on the first assumption.
- The longer tourists stay at an attraction, the more attractive it is. A mean time transition matrix is calculated, based on the second assumption, to estimate the time spent at each attraction.

The attractiveness of each attraction can then be calculated based on these two matrixes. A case study conducted at Phillip Island Nature Park, Victoria, Australia is used to validate the model. The studies' results prove that model is efficient. They are also useful, in that knowing which attractions are the most popular, how long tourist will spend at any one site, and what the likely routes are that they will follow and how attractions associate with each other, can inform marketing decisions of park managers and tourist operators

Keywords: *Semi-Markov process, tourist movement, space and time analysis*

1. INTRODUCTION

Markov Chain models have been used in many disciplines for modelling the sequence of events related to each other by first-order dependency (Isaacson and Madsen 1976; Kemeny and Snell 1976; Stewart 1994; Cassandras and Lafortune 2008) . Xia et al. (2009) utilise Markov Chains to analyse the outcome and trend of events associated with spatio-temporal movement patterns. However, this method focuses on modeling movements of tourists spatially, not temporally, i.e. this method does not take the duration of each visit into consideration. In order to understand, predict, control and optimise decisions made by tourists regarding their decisions on which attractions to visit, this time information can be integrated into the Markov chain modeling process. To achieve these aims, we extend the Markov chain approach by modeling spatio--temporal movements using a Semi--Markov process (Cinlar 1975b; Janssen and Manca 2006) .

The following two assumptions, which underlie the Semi--Markov process, make it an especially ideal tool for modeling spatio—temporal movements of tourists:

- The probability that a tourist will visit a particular attraction depends only on the most recent attraction that was visited by that tourist.
- The distribution of the time spent in each attraction is dependent on both that attraction and the next attraction that is visited next.

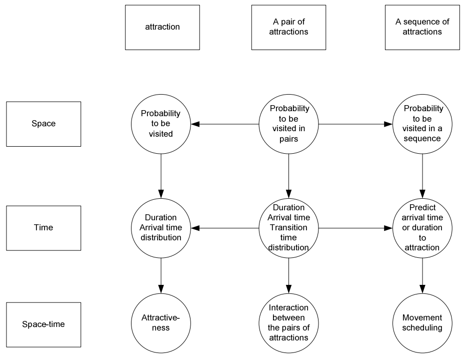

Figure 1. Markov tourist movement application matrix

Thus, a Semi--Markov process has a Markov chain and a renewal process embedded within its structure, and as such, can be used to provide a wide variety of practical models useful in applications (See Figure 1).

2. METHOD

A *random process* is formally defined as a family of random variables $\{X_t, t \in T\}$ defined on a given probability space and indexed by *t*. The set *T* is often used to represent the time sequence of the process and is usually discrete, i.e. $T = \{0,1,2\ldots\}$ or continuous, i.e. $T = [0,\infty)$. The set of values of X_t as *t* ranges over *T* is the *state space S* of the random process, which could again either be discrete or continuous. If *S* is discrete, the process is referred to as a *discrete random process*; otherwise, the process is a *continuous random process*. In this paper, a random process is used to represent the spatio--temporal movement of tourists; the temporal component is represented by t and the spatial component by the values of $ X_t for $t \in T$. The state space *S* contains the destinations and transit route locations traversed by the tourists.

A random process is referred to as a *Markov process* if it satisfies the following intrinsic property: given any fixed sequence of time points $t_0 < t_1 < t_2 < \ldots < t_n < t$, positive integer *n* and values $i_0, i_1, \ldots, i_n$, the following identity involving conditional probabilities holds:

$$\Pr(X_t = j \mid X_{t_n} = i_n, X_{t_{n-1}} = i_{n-1}, \ldots, X_{t_1} = i_1, X_{t_0} = i_0) = \Pr(X_t = j \mid X_{t_n} = i_n) \quad (1)$$

If *T* is discrete, say $T = \{0,1,2\ldots\}$, then the Markov process is known as a *Markov chain* (MC). The set of conditional probabilities

$$\Pr(X_{n+1} = j \mid X_n = i) = p_{ij}(n) \quad (2)$$

for $i, j \in S$ and n = 1, 2, … is called the set of one-step transition probabilities of the MC. From the rules of probability, one-step transition probabilities satisfy

(i) $p_{ij}(n) \geq 0$ for all pairs $i, j \in S$.

(ii) $\sum_{j \in S} p_{ij}(n) = 1$

A Semi-Markov process (or a Markov Renewal process) generalizes a Markov process, in that the time spent in each state is dependent not only on that state but also on the next state it transitions to. Given the focus of this paper, we deal exclusively with a discrete Semi--Markov random process and assume that the cardinality of the set *S* is equal to a finite number *N*. Let $X_0, X_1, X_2, \ldots,$ be the successive states in *S* visits by a random process starting in state X_0 and let $0 = T_0 < T_1 < T_2, \ldots$ be the times of transitions into each of these states. The random process is a Semi-Markov process if it satisfies the following fundamental property (Cinlar 1975b):

$$\begin{aligned} &\Pr(\mathrm{X}_{n+1} = j, T_{n+1} - T_n \le t) \mid X_0, X_1, \ldots, X_n; T_0, T_1, \ldots, T_n) \\ &= \Pr(\mathrm{X}_{n+1} = j, T_{n+1} - T_n \le t \mid X_n) \end{aligned} \tag{3}$$

for n = 0,1,2, … In words, given that we know the states that the process will be visiting and how long it spends in each of these states up to and including the nth epoch, the probability of where the process visits next, i.e. X_{n+1}, and the distribution of the time it will have moving from the most recent state to the next state, depends only on the most recent state X_n. Note that for a Markov process, the distribution of $T_{n+1} - T_n$ does not depend on X_{n+1} and is also restricted to the class of exponential distributions determined by the infinitesimal rates of the process (Isaacson and Madsen, 1976).

The probability on the right hand side of (4) is called the Semi--Markov kernel of the process and is denoted by

$$\Pr(\mathrm{X}_{n+1} = j, T_{n+1} - T_n \le t \mid X_n = i) = Q_{ij}(t) \tag{4}$$

Note that if we let $t \to \infty$*;* then

$$\begin{aligned} Q_{ij}(\infty) &= \Pr(X_{n+1} = j \mid X_n = i) \\ &= P_{ij} \end{aligned} \tag{5}$$

and is just the one{step transition probabilities associated with the underlying Markov chain.

Another relevant distribution associated with Semi-Markov processes is

$$F_{ij}(t) = \Pr(T_{n+1} - T_n \le t \mid X_{n+1} = j, X_n = i) \tag{6}$$

which is the conditional distribution of the time spent in state X_n before the next transition, given $X_n = i$ and $X_{n+1} = j$. Using the product rule of probability, it follows that

$$\begin{aligned} &\Pr(\mathrm{X}_{n+1} = j, T_{n+1} - T_n \le t \mid X_n = i) \\ &= \Pr(T_{n+1} - T_n \le t \mid X_{n+1} = j, X_n = i) \Pr(X_{n+1} = j \mid X_n = i) \end{aligned} \tag{7}$$

hence

$$F_{ij}(t) = \frac{Q_{ij}(t)}{P_{ij}} \tag{8}$$

if $P_{ij} > 0$; otherwise, we define

$$F_{ij}(t) = H(x) \tag{9}$$

where

$$H(x) = \begin{cases} 0 \text{ if } & x < 0 \\ 1 \text{ if } & x \ge 0 \end{cases}$$

Using (4), the *joint distribution* of sojourn times in states $X_0, X_1, \ldots, X_n$ respectively, given $X_0 = x_0, X_1 = x_1, \ldots, X_n = x_n$, can be computed using

$$\begin{aligned} &\Pr(T_1 - T_0 \le t_1, T_2 - T_1 \le t_2, \ldots, T_n - T_{n+1} \le t_1 \mid X_0 = x_0, X_1 = x_1, \ldots, X_n = x_n) \\ &= F_{x_0 x_1}(t_1) F_{x_1 x_2}(t_2) \ldots F_{x_{n-1} x_n}(t_n) \end{aligned} \tag{10}$$

Based on (10), if *S* consists of a single state, then a Semi-Markov process also reduces to a *renewal process.*

2.1. Mean Values and Attractiveness of a Site

Let T_{ij} represent the time a tourist spends in state *I, given that the process starts in i before moving to state j.* To find the mean value of the random variable T_{ij} , we compute

$$E(T_{ij}) = \int_0^{\infty} tF'_{ij}(t)dt - t_{ij} \tag{11}$$

where $F'_{ij}(t)$ is density function of T_{ij} and t_{ij} is the expected travel time between destination *I* and *j*. Note that the first term on the right hand side of (11) is the expected time it takes for a tourist to move between state *i* and *j*. Therefore $F'_{ij}(t)$ also *includes* the time spent in state *i*. Subtracting t_{ij} from this term gives the expected time spent in state *i*. t_{ij} can be estimated, since the travel times between various destinations are known.

The attractiveness of state *i*, denoted by $A(i)$ will be defined as

$$A(i) = \sum_{j=1}^{N} P_{ij} E(T_{ij}) \tag{12}$$

where N is the total number of states. We remark that equation (12) is the expected duration of time spent in state i irrespective of which state the process moves to next; hence, it is not an unreasonable measure of the attractiveness of that particular site..

2.2. Case study area and sampling techniques

Phillip Island, located at the mouth of Westernport Bay, is 140 kilometres south-east of Melbourne. There are a large number of penguins, koalas, seals, and shearwaters living in the mangroves and wetlands, and on the sandy beaches and rugged rocky cliff faces. The major attractions are the Penguin Parade, the Koala Conservation Centre, Cowes, Churchill Island, Rhyll Inlet, Woolamai and the Nobbies (Figure 1), where visitors can experience wildlife in its natural environment (Phillip Island Nature Park 2001-02).

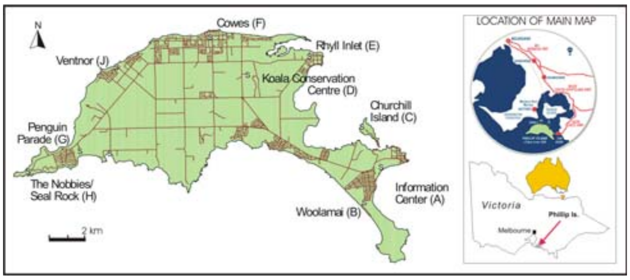

Figure 2. Map of Phillip Island (Phillip Island Nature Park 2005)

Tourists' daily movement data for the Phillip Island Nature Park were collected via a self-administered questionnaire. The questionnaire was designed to address three different areas. The first section aimed to acquire socio-demographic data or profiles of the tourists. The second section aimed to collect information regarding travel mode, length of stay and with whom the tourists were travelling. The final section gathered information on tourist movement. Here, tourists were asked to write down their approximate arrival time and duration of stay at each attraction visited for the entire day. Tourists were also asked to draw the route of travel to each attraction on a street map of Phillip Island. Eight hundred questionnaires were distributed from the 6-8th of March 2004 and from 17-20th Jan 2005 at the Phillip Island Nature Park Information Centre, Churchill Island, Koala Conservation Centre and Penguin Parade. Penguin Parade was the major sample site. As park managers estimate that more than 90% of tourists visit these attractions, it is therefore unlikely that significant movement patterns were missed by the survey. Five hundred questionnaires were returned with 456 entered into the database. The remaining 44 questionnaires were incomplete and discarded.

2.3. Results

3.2.1 Transition probability matrix

In our case study, we considered the movement of tourists on Phillip Island between the nine attractions listed above. A stationary discrete Markov Chain is used to model the movements of the tourists between each attraction, from the moment they entered the park until they completed their visits. The states of the chain are the nine attractions visited with an additional absorbing state labelled , "OUT", which signaled the completion of their tour. Movement of tourists on Phillip Island, therefore, is broken down into ninety one-step transitions in a transition probability matrix (See Table 1). The transition probability, for example from Cowes (*F*) to Penguin Parade (*G*), is calculated based on a conditional probability as follows:

Table 2. Transition Probability Matrix

	A	B	C	D	E	F	G	H	J	OUT
A	0.000	0.000	0.000	0.200	0.000	0.200	0.200	0.400	0.000	0.000
B	0.000	0.000	0.043	0.217	0.043	0.283	0.087	0.239	0.043	0.043
C	0.000	0.103	0.000	0.333	0.026	0.308	0.103	0.026	0.000	0.103
D	0.000	0.022	0.030	0.000	0.030	0.481	0.193	0.185	0.007	0.052
E	0.000	0.077	0.000	0.115	0.000	0.577	0.192	0.038	0.000	0.000
F	0.000	0.017	0.010	0.031	0.010	0.000	0.568	0.339	0.010	0.014
G	0.000	0.002	0.002	0.000	0.002	0.022	0.000	0.026	0.000	0.946
H	0.000	0.005	0.005	0.019	0.010	0.191	0.746	0.000	0.010	0.014
J	0.000	0.133	0.000	0.000	0.133	0.200	0.400	0.133	0.000	0.000

1. Count the number of movements that satisfy the profile $F(n) \cap G(n+1)$ for n = 1,2,…,m-1 where m is the maximum number of possible visits. For example, $N(F(2) \cap G(3))$ is the number of movements where tourists visit F as the second destination of their trip and then next move to G. The '*n*' is the number of times FG pairs occur along the movement sequence.

2. Sum these frequencies, i.e.

$$\sum_{n=1}^{m-1} N(F(n) \cap G(n+1)) = N(F(1) \cap G(2)) + N(F(2) \cap G(3)) + N(F(3) \cap G(4)) + N(F(4) \cap G(5)) + N(F(5) \cap G(6))$$

$$= 71+61+22+10+2 = 166$$

3. Divide the sum in (2) by the total number of one-step movement patterns from F to the other eight attractions A_i, such s A, B and C etc

$$\sum_{n=1}^{m-1} N(F(n) \cap G(n+1)) \Big/ \sum_{i=1}^{8} \left(\sum_{n=1}^{m-1} N(F(n) \cap A_i(n+1)) \right) = 166 / 2 = 0.568$$

3.2.2 *Mean time transition matrix*

The mean time transition matrix shows the mean time spent at attraction *I*, given that the movement starts in *i* and then moves to attraction *j*. The size of the mean time transition matrix is the same as the transition probability matrix (see Table 1) including nine attractions and one absorbing state labelled as "OUT".

Two steps are used to derive the mean time transition matrix.

- Identify time spent at attraction *i* for each pair of transitions.

This step is to identify time spent at an attraction before moving onto the next attraction. For example, a maximum six transitions are identified in the survey data, which means that tourists visit, at the most, seven attractions per day. Firstly, we list time spent in F attraction before moving to *G* in the initial transition. Here, *F* is the first destination to be visited on Phillip Island before moving to second destination *G*. Then, the same procedure is repeated to identify the duration at F before moving to *G* at the other five transition points. In total, one hundred and sixty five tourists travelled from F to *G*. Therefore, 165 durations are available for the next step in the analysis.

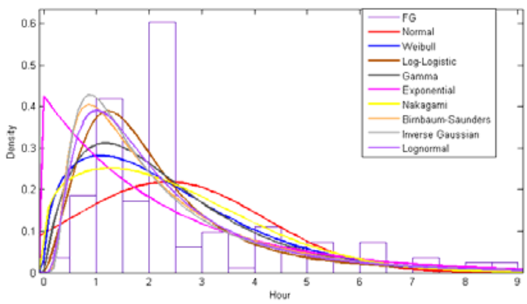

Figure 2. Probability density functions (PDF) for time spent at attraction F before moving to G

- Fitting data to distribution function

The Distribution Fitting Tool in the Matlab software package (The Mathworks, 2008) was used to fit duration data to different distribution functions. Twenty one distribution functions are available in Matlab. And we explored all these distribution functions to fit our data. Figure 2 illustrates possible good fit probability density functions (PDF) for time spent at attraction *F* before moving to *G*. However, in order to choose the best fit distribution, more accurate criteria had to be adopted. Akaike Information criterion (AIC), which is the loglikelihood penalized by the number of parameters was chosen to evaluate the goodness of fit of the distribution.

$$\text{AIC} = 2\times(\text{number of parameters in the model}) - 2\times\text{Log Likelihood}$$

The smallest value indicates the best fit (see Table 3). Therefore, the lognormal distribution function is the best fit distribution for time spent at attraction F before moving to *G*.

Table 3. AIC for each distribution function

FG	Lognormal	Log-logistic	Birnbaum-Saunders	Inverse Gaussian	Gamma	Weibull	Nakagami	Exponential	Normal	Lognormal
Log likelihood	-279.492	-280.331	-280.416	-281.78	-282	-286.1	-291.84	-300.5	-327	-279.492
Number of Parameters	2	2	2	2	2	2	2	1	2	2
AIC	562.984	564.662	564.832	567.562	568	576.1	587.676	603	657	562.984

The same procedure is repeated 90 times to identify the best fit distribution for each pair of transitions (see Table 4). Zero means no tourists in the survey data did this transition; e.g. from A to B, because the majority of tourists visiting Woolamai (B) are domestic visitors. Most of them are familiar with Phillip Island, and so, there is no need to visit the Information Centre. In Table 4, "?" indicates that there are not enough data to get a distribution fit..

Table 4. Distribution fit

	A	B	C	D	E	F	G	H	J	OUT
A	0	0	0	?	0	?	?	?	0	0
B	0	Gamma	Lognormal	Birnbaum-Saunders	Gamma	Inverse Gaussian	Gamma	Weibull	Gamma	Gamma
C	0	?	0	Nakagami	?	Logistic	?	?	0	Inverse Gaussian
D	0	?	Inverse Gaussian	0	Inverse Gaussian	Lognormal	Lognormal	Weibull	?	Log-Logistic
E	0	?	0	?	0	Inverse Gaussian	Birnbaum-Saunders	Weibull	0	0
F	0	?	?	Weibull	?	0	Lognormal	Log-Logistic	?	Weibull
G	0	?	?	0	?	Nakagami	0	Nakagami	0	Gamma
H	0	?	?	Weibull	?	Nakagami	Log-Logistic	0	?	?
J	0	Birnbaum-Saunders	0	0	?	?	Weibull	?	0	0

After selecting the best fit distribution, mean time is calculated for each pair of transitions (See Table 1). We have used the sample mean for the calculation when the best distribution can not be identified because of the small sample size.

3.3.3 Attractiveness of an attraction

The final step is to calculate the attractiveness of each attraction using equation (9) (See Table 5). Attractiveness of attraction is the sum of the product of transition probability and mean time. For example, attractiveness of attraction *F* (i = 6 and N = 10) is calculated as:

Table 5. Mean time transition Matrix

	A	B	C	D	E	F	G	H	J	OUT
A	0	0	0	0.2	0	0.5	0.3	0.35	0	0
B	0	0	2.3	1.849	0.75	1.54	2	0.84	2	0.75
C	0	2.5	0	1.289	2	2.07	2	4	0	2.5
D	0	1	1.3	0	2.13	1.29	1.29	1.41	1	1.098
E	0	1	0	1	0	0.92	1.29	1	0	0
F	0	1.8	2.5	2.001	1.75	0	2.39	2.04	2.67	1.767
G	0	4	1.5	0	0	1.36	0	0.89	0	2.533
H	0	1.5	1	0.704	1.5	1.19	1.14	0	0.75	1.5
J	0	1.3	0	0	1	0.6	1.42	0.75	0	0

$$A(6) = \sum_{j=1}^{10} P_{6j} E(T_{6j}) = 0\times 0 + 0.017\times 1.75 + 0.01\times 2.5 + \ldots + 0.014\times 1.767 = 2.238$$

Table 5 reveals attractiveness of pairs and attractions. Higher values indicate higher attractiveness; for example, G is the most attractive attraction on Phillip Island. There is a higher probability that tourists will visit G and will stay there longer as well. In addition, the high value of pairs of attractions such as G-OUT, F-H, F-G, H-G, and C-F indicates high association between the pairs. Therefore, park managers can combine these attractions when designing tour packages.

3. CONCLUSION AND FUTURE WORK

This paper applies a Semi-Markov process to measure the attractiveness of tourist attractions and the association of pairs of attractions, based on spatial and temporal interactions between the attractions. A case study conducted at Phillip Island Nature Park, Victoria, Australia is used to validate the model.

Semi-Markov chains were used to break down movement processes into one-step transitions to identify how pairs of attractions associate with each other spatially and temporally. For example, tourists who visit attraction F are most likely to visit attraction G afterwards. Similarly tourists spend a longer time at attraction F before moving to attraction G than they do when moving to the other attractions. Tourists who spend a longer time at attraction F are intending to visit attraction G.

Table 5. Attractiveness of attractions

	A	B	C	D	E	F	G	H	J	OUT	A(i)
A	0	0	0	0.04	0	0.1	0.06	0.14	0	0	0.34
B	0	0	0.1	0.402	0.03	0.43	0.17	0.2	0.09	0.033	1.46
C	0	0.3	0	0.43	0.05	0.64	0.21	0.1	0	0.256	1.94
D	0	0	0	0	0.06	0.62	0.25	0.26	0.01	0.057	1.32
E	0	0.1	0	0.115	0	0.53	0.25	0.04	0	0	1.01
F	0	0	0	0.062	0.02	0	1.36	0.69	0.03	0.024	2.24
G	0	0	0	0	0	0.03	0	0.02	0	2.396	2.46
H	0	0	0	0.013	0.01	0.23	0.85	0	0.01	0.022	1.14
J	0	0.2	0	0	0.13	0.12	0.57	0.1	0	0	1.09

In conclusion, one-step transitions of tourist movements can be examined to give useful information on the relationship between time and movement Previous research has focused on either space or time perspectives rather than space-time interactions, or concentrated on one single attraction instead of the association between a pair or a sequence of attractions (see Figure 1). This research fills a gap by investigating space and time interactions between pairs of attractions. A measure of the attractiveness of attractions is possible using Semi-Markov chains. In the future, further exploration of Markov methods may predict arrival times, duration at attractions, or scheduling of movement between sites.

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