18th World IMACS / MODSIM Congress, Cairns, Australia 13-17 July 2009
http://mssanz.org.au/modsim09

Realized Volatility, Covariance and Hedging Coefficient of the Nikkei-225 Futures with Micro-Market Noise *

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Abstract

For the estimation problem of the realized volatility, covariance and hedging coefficient by using high frequency data with possibly micro-market noises, we use the Separating Information Maximum Likelihood (SIML) method, which was recently developed by Kunitomo and Sato (2008a,b). By analyzing the Nikkei 225 futures and spot index markets, we have found that the estimates of realized volatility, covariance and hedging coefficient have significant bias by the traditional method which should be corrected. Our method can handle the estimation bias and the tick-size effects of Nikkei 225 futures by removing the possible micro-market noise in multivariate high frequency data.

Key Words
Realized Volatility, Realized Covariance, Realized Hedging Coefficient, Micro-Market Noise, High-Frequency Data, Separating Information Maximum Likelihood (SIML), Nikkei 225 Futures Tick Size Effects.

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*This is KSII09-2-10, which is a summary version of Discussion Papers (CIRJE-F-581, 601), Graduate School of Economics, University of Tokyo. The papers were originally prepared for the International Conference organized by the Osaka Securities Exchange on September 2, 2008. The research was initiated while the first author was visiting the Center for the Study of Finance and Insurance (CSFI) of Osaka University as the Osaka-Securities-Exchange professor.

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1. Introduction

The Nikkei-225 futures at the Osaka Securities Exchange (OSE) are the futures contracts for the Nikkei-225 Index and they are the most important futures contracts in Japanese financial markets over the past 20 years. Because of their important role in financial markets, there have been basic questions to be answered on their performance and function as a hedging tool on the Nikkei-225 spot index as futures contracts. As the high frequency data of Nikkei-225 futures have become available, it may be natural to examine these problems because the majority of the past analyses are based on daily or monthly data. We shall demonstrate that the estimates obtained by the traditional realized variance, covariance and the hedging ratio are often not reliable and they should be corrected while the estimates we have obtained by our method give stable and reliable results on these key quantities. Then it is important to incorporate the micro-market noise when we estimate the realized volatility, covariance and the hedging ratio for practical purposes. We shall show that the new estimation method called the Separating Information Maximum Likelihood (SIML) approach recently proposed by Kunitomo and Sato (2008a,b) gives an easy way to handle these problems and construct reliable estimates for the realized variance, covariance, correlation and the hedging ratio. A considerable interest has been recently paid on the estimation problem of the realized volatility by using high-frequency data in financial econometrics. For instance, see Barndorff-Nielsen, O., P. Hansen, A. Lunde and N. Shepard (2008), for further discussions on the related topics.

2. High Frequency Data of Nikkei-225 Markets

The Nikkei-225 Futures have been the major financial tool in the financial industry because the Nikkei-225 is the major index in Japan and hence they have been actively traded. Since we have high frequency data less than 1 second of Nikkei-225 Futures, we use 1 second, 5 seconds, 10 seconds, 30 seconds and 60 seconds. Although we have high frequency data on the Nikkei-225 Futures within less than one second, we only have the Nikkei-225 Spot Index at every minute. Then we have an interesting new problem in the high frequency data analysis.

When we analyze the tick data over a day, there has been an observation that the volatility of asset price changes over time within a day. Thus it is important to develop the method of measurements on the realized volatility, the realized covariance and the realized hedging ratio, which are free from these movements within a day.

In the standard finance theory the continuous time stochastic processes are often assumed for dynamic behaviors of securities prices. The typical example is the Black-Scholes theory. On the other hand, the Nikkei-225 Futures have the minimum tick size and thus the observation of prices cannot be continuous over time. We may interpret the underlying price process as the efficient price and the tick seize effects as a kind of the micro-market noise.

It has been known in financial economics that the standard way of hedging is to use the covariance and variance. Thus it has been important to estimate the realized covariance and variance of the Nikkei-225 spot and Nikkei-225 futures for practical purposes.

3. The SIML Estimation of Realized Volatility, Covariance and Hedging Coefficient with Micro-Market Noise

Let y_{is} and y_{if} be the $i-$th observation of the $j-$th (log) spot price and the $j-$th (log) futures price at t_i^n for $j = 1, \cdots, p$; $0 = t_0^n \leq t_1^n \leq \cdots \leq t_n^n = 1$. We set $\mathbf{y}_i = (y_{is}, y_{if})$ be a 2×1 vector and $\mathbf{Y}_n = (\mathbf{y}_i^{'})$ be an $n \times 2$ matrix of observations. The underlying continuous process $\mathbf{x}_i = (x_{is}, x_{if})^{'}$ is not necessarily the same as the observed prices and let $\mathbf{v}_i^{'} = (v_{is}, v_{if})$ be the vector of the micro-market noise. Then we have

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i \tag{3.1}$$

where $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and

$$\mathcal{E}(\mathbf{v}_i\mathbf{v}_i^{'}) = \mathbf{\Sigma}_v = \begin{pmatrix} \sigma_{ss}^{(v)} & \sigma_{sf}^{(v)} \\ \sigma_{fs}^{(v)} & \sigma_{ff}^{(v)} \end{pmatrix} .$$

We assume that

$$\mathbf{x}_t = \mathbf{x}_0 + \int_0^t \mathbf{C}_x(s) d\mathbf{B}_s \ \ (0 \le t \le 1), \tag{3.2}$$

where $\mathbf{B}_s$ is a $p \times 1$ vector of the standard Brownian motions and we write $\mathbf{\Sigma}_x(s) = \mathbf{C}_x(s)\mathbf{C}_x(s)^{'}$. Then the main statistical problem is to estimate the quadratic variations and co-variations

$$\mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s) ds = \begin{pmatrix} \sigma_{ss}^{(x)} & \sigma_{sf}^{(x)} \\ \sigma_{fs}^{(x)} & \sigma_{ff}^{(x)} \end{pmatrix} \tag{3.3}$$

of the underlying continuous process $\{\mathbf{x}_t\}$ and also the variance-covariance $\mathbf{\Sigma}_v = (\sigma_{ij}^{(v)})$ of the noises from the observed $\mathbf{y}_i$ $(i = 1, \cdots, n)$. Although we assume the Gaussian processes in order to derive the SIML estimation in this section, the asymptotic results do not depend on the Gaussianity of the underlying processes as we have discussed in Kunitmo and Sato (2008a).

We consider the standard situation when $\mathbf{\Sigma}(s) = \mathbf{\Sigma}_x$ and $\mathbf{v}_i$ $(i = 1, \cdots, n)$ are independently and identically distributed with $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and $\mathcal{E}(\mathbf{v}_i\mathbf{v}_i^{'}) = \mathbf{\Sigma}_v$. We transform $\mathbf{Y}_n$ to $\mathbf{Z}_n$ $(= (\mathbf{z}_k^{'}))$ $(k = 1, \cdots, n)$ by

$$\mathbf{Z}_n = h_n^{-1/2} \mathbf{P}_n^{'} \mathbf{C}_n^{-1} \left(\mathbf{Y}_n - \bar{\mathbf{Y}}_0 \right) \tag{3.4}$$

where

$$\mathbf{C}_n^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ 1 & \cdots & 1 & 1 & 0 \\ 1 & \cdots & 1 & 1 & 1 \end{pmatrix}^{-1} . \tag{3.5}$$

and

$$\mathbf{P}_n = (p_{jk}) \ , \ p_{jk} = \sqrt{\frac{2}{n+\frac{1}{2}}} \cos\left[\pi(\frac{2k-1}{2n+1})(j-\frac{1}{2})\right] \ , \tag{3.6}$$

$$\bar{\mathbf{Y}}_0 = \mathbf{1}_n \cdot \mathbf{y}_0^{'} \ . \tag{3.7}$$

By considering the information on $\mathbf{\Sigma}_x$ and $\mathbf{\Sigma}_v$ in the Gaussian-likelihood function, Kunitomo and Sato (2008a) defined the SIML estimator of $\hat{\mathbf{\Sigma}}_v$ by

$$\hat{\mathbf{\Sigma}}_x = \frac{1}{m_n} \sum_{k=1}^{m_n} \mathbf{z}_k \mathbf{z}_k^{'} \tag{3.8}$$

and also they defined the SIML estimator of $\hat{\mathbf{\Sigma}}_v$ by

$$\hat{\mathbf{\Sigma}}_v = \frac{1}{l_n} \sum_{k=n+1-l_n}^{n} a_{kn}^{-1} \mathbf{z}_k \mathbf{z}_k^{'} \ , \tag{3.9}$$

where

$$a_{kn} = 4n \sin^2\left[\frac{\pi}{2}\left(\frac{2k-1}{2n+1}\right)\right] \ . \tag{3.10}$$

For both $\hat{\mathbf{\Sigma}}_v$ and $\hat{\mathbf{\Sigma}}_x$, the number of terms m and l should be dependent on n. Then we only need the order requirements that $m_n = O(n^{\alpha})$ $(0 < \alpha < \frac{1}{2})$ and $l_n = O(n^{\beta})$ $(0 < \beta < 1)$ for $\mathbf{\Sigma}_x$ and $\mathbf{\Sigma}_v$, respectively.

Although the SIML estimation was introduced under the Gaussian processes and the standard model, it has reasonable finite sample properties as well as asymptotic properties under the non-Gaussian processes and the volatility models. Let the conditional covariance matrix of the (underlying) returns without noise be

$$\mathbf{\Sigma}_i = \mathcal{E}\left[n \ \mathbf{r}_i \mathbf{r}_i^{'} | \mathcal{F}_{n,i-1}\right] \ , \tag{3.11}$$

where $\mathbf{r}_i = \mathbf{x}_i - \mathbf{x}_{i-1}$ is a sequence of martingale differences and $\mathcal{F}_{n,i-1}$ is the $\sigma-$field generated by $\mathbf{x}_s$ $(s \leq t_{i-1})$ and $\mathbf{v}_s$ $(s \leq t_{i-1})$. In this setting it is natural to impose the condition

$$\frac{1}{n} \sum_{i=1}^{n} \mathbf{\Sigma}_i \xrightarrow{p} \mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s) ds \ . \tag{3.12}$$

When the realized volatility and covariance $\mathbf{\Sigma}_x = (\sigma_{ij}^{(x)})$ is a constant (positive definite) matrix, Kunitomo and Sato (2008a) have shown that the SIML estimator is consisntent and it has the asymptotic normality under some regularity conditions. Then it is possible to evaluate the SIML estimators of the realized volatility, covariance, correlation and the hedging ratio, which are useful for empirical analysis.

Hedging Ratio

The SIML estimator of the hedging ratio $H = \sigma_{sf}^{(x)}/\sigma_{ff}^{(x)}$ can be defined by

$$\hat{H} = \frac{\hat{\sigma}_{sf}^{(x)}}{\hat{\sigma}_{ff}^{(x)}} \ . \tag{3.13}$$

Then we can derive the limiting distribution of the hedging ratio estimator, which is given by

$$\sqrt{m_n}\left[\hat{H} - H\right] \xrightarrow{w} N(0, \omega_H) \tag{3.14}$$

as $m_n^5/n^2 \to 0$, where

$$\omega_H = \frac{\sigma_{ss}^{(x)}}{\sigma_{ff}^{(x)}}\left[1 - \frac{\sigma_{sf}^{(x)2}}{\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}}\right] \ . \tag{3.15}$$

4. Estimation Results and Concluding Remarks

As an illustration we have picked one day in April 2007 and estimated the realized volatility with different time intervals in Table 1 by both the traditional historical volatility estimation and the SIML estimation as a typical example. Then we found that the estimated HI heavily depends on the observation intervals while our estimation does not depend on them very much. The problem of significant biases of the estimated HI has been pointed out recently by several researchers.

Table 1 : Estimation of Realized Volatility :

	Σ_x	Σ_v	HI
1s	5.252E-05	9.853E-09	4.946E-04
10s	4.513E-05	4.168E-08	1.764E-04
30s	5.099E-05	7.217E-08	9.449E-05
60s	6.151E-05	8.976E-08	6.964E-05

We also have conducted a number of simulations and some details of our results of simulations are summarized in Kunitomo ans Sato (2008a,b).

Our analysis of high frequency data on the Nikkei-225 futures and Nikkei-225 spot suggest that the SIML estimation can handle the high-frequency data problem and the micro-market noise easily and properly. On the other hand, there are significant bias of the estimates obtained by the traditional historical method. Also we can treat the tick size effect as the micro-market noise and the data length problem properly by the SIML estimation method. These findings may have a number of implications on the derivative pricing and the risk management in practice.

References

[1] Barndorff-Nielsen, O., P. Hansen, A. Lunde and N. Shephard (2008), "Designing realized kernels to measure the ex-post variation of equity prices in the presence of noise, Econometrica, 355-380.

[2] Kunitomo, N. and S. Sato (2008a), "Separating Information Maximum Likelihood Estimation of Realized Volatility and Covariance with Micro-Market Noise," CIRJE Discussion Paper F-581, University of Tokyo.

[3] Kunitomo, N. and S. Sato (2008b), "Realized Volatility, Covariance and Hedging Coefficient of the Nikkei-225 Futures with Micro-Market Noise Separating Information Maximum Likelihood Estimation of Realized Volatility and Covariance with Micro-Market Noise," CIRJE Discussion Paper F-601, University of Tokyo.