How do we know macroeconomic time series are stationary?

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Abstract: It is generally accepted in macroeconomics, especially since the validation of Real Business Cycle Theory by the awarding of the Noble Prise in Economics to Edward Prescott in 2004, that macroeconomic time series data are stationary. Other time series empirical work on unit roots and co-integration indicates that most macro time series data are difference stationary. That is they follow a first order of integration (or I1) process.

This paper questions whether we can be assured that this is the case by generating artificial data designed to emulate the macroeconomic data from a simulation model of general purpose technology (GPT) driven growth based on Carlaw and Lipsey (2007). The data generating process is explicitly non-stationary (trend and difference). We analyse the business cycle properties of the data by matching growth rates to actual Canadian data from the period 1961-2007 and find that the growth properties of the simulated data are consistent with the Canadian data. We then filter the simulated data using the annualized Hodric-Prescott filter and examine the data's Real Business Cycle properties. We find that the business cycle properties are a close but not exact match with those of the Canadian.

We then perform a time series econometric analysis of the data to determine its time series properties and conclude that the data exhibit first order difference stationarity (i.e., the follow an I1 process). We conclude that it remains an open question as to whether real macroeconomic time series data is in fact trend or difference stationary and further empirical methodology may need to be developed to verify or refute the statement.

These findings have some important implications for testing and drawing inference from macroeconomic time series data. In particular as noted by Libania (2005) the original work by Nelson and Plosser (1982) presented an empirical case that macroeconomic time series are difference stationary. This was taken as evidence in support of the RBC hypothesis that once the trend is filtered from macro data the business cycle exhibits stationarity and therefore monetary shocks can only have temporary effect. Furthermore, although Nelson and Plosser (1982) state explicitly that they assumed that the classical dichotomy held for their analysis, it has come to be generally accepted that the business cycle is stationary and therefore the dichotomy does hold.

Our analysis shows that it may not be easy to determine whether macroeconomic time series are trend or difference stationary and a model that exhibits neither of these characteristics might appear to have the same empirical properties as models that do.

Keywords: Macroeconomic Time Series Data, Stationarity, Macroeconomic Policy

1. INTRODUCTION

It is generally accepted in macroeconomics, especially since the validation of Real Business Cycle Theory by the awarding of the Noble Prise in Economics to Edward Prescott in 2004, that macroeconomic time series data are stationary. Other time series empirical work on unit roots and co-integration indicates that most macro time series data are difference stationary. That is they follow a first order of integration (or I1) process.

This paper questions whether we can be assured that this is the case by generating artificial data designed to emulate the macroeconomic data from a simulation model of general purpose technology (GPT) driven growth based on Carlaw and Lipsey (2007). The data generating process is explicitly non-stationary (trend and difference) (i.e., it is a higher order than first difference stationary). We analyse the business cycle properties of the data by matching growth rates to actual Canadian data from the period 1961-2007 and find that the growth properties of the simulated data are consistent with the Canadian data. We then filter the simulated data using the annualized Hodric-Prescott filter and examine the data's Real Business Cycle properties. We find that the business cycle properties are a close but not exact match with those of the Canadian.

We then perform a time series econometric analysis of the simulated data to determine its time series properties. Our tentative finding from our preliminary analysis indicate that the data exhibit first order difference stationarity (i.e., the follow an I(1) process).

If confirmed by more extensive analysis, these findings have some important implications for testing and drawing inference from macroeconomic time series data. In particular as noted by Resenha (2005) the original work by Nelson and Plosser (1982) presented an empirical case that macroeconomic time series are difference stationary. This was taken as evidence in support of the RBC hypothesis that once the trend is filtered from macro data the business cycle exhibits stationarity and therefore monetary shocks can only have temporary effect. Furthermore, although Nelson and Plosser (1982) state explicitly that they assumed that the classical dichotomy held for their analysis, it has come to be generally accepted that the business cycle is stationary and therefore the dichotomy does hold.

2. THE MODEL

The simulation performed in this paper utilizes the model of Carlaw and Lipsey (2009) which itself is based on the basic structure presented in Carlaw and Lipsey (2006).[1] It has three sectors. One sector produces pure research that occasionally discovers a new GPT; one sector produces applied research that develops applications for the GPT; one sector produces a consumption good using the results of applied research in its production function. Each sector has its own distinct aggregate production function. Thus, the intra-sector technology is flat, while a technology structure is imposed through the inter-sector relations among these three different production functions.

The economy has a fixed aggregate stock of a composite resource, R, which can be thought of as 'land' and 'labor.' The three aggregate production functions display diminishing marginal returns to this resource. The pure knowledge sector produces a flow of pure knowledge, g, which accumulates in a stock of potentially useful knowledge, Ω. Every once in a while a new GPT is invented. The existing stock of potentially useful pure knowledge is embodied in it and then its efficiency slowly evolves according to a logistic function to become increasingly useful in applied research. The applied R&D sector produces practical knowledge that is useful in both the consumption and the pure research sectors, the latter being a feedback that is well established in the

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[1] Because of space constraints here we leave the reader to look at Carlaw and Lipsey (2009) for the formal detail of the model, much of the description of parameters and the calibration details of the simulation. Here we provided some of the parameter calibration details since they influence the simulation results.

technology literature.[2] This knowledge is embodied in physical and human capital, which at this level of aggregation we do not need to treat separately.

Carlaw and Lipsey (2009) incorporates the following characteristics of these widely used technologies that have been established by historical research. Only point (1) and a few of the sources of uncertainty listed in point (6) below have been modeled so far.

1. The efficiency with which any new GPT delivers its services increases greatly over time.

2. The use of any new GPT spreads slowly through the economy and many decades are typically required for its full diffusion to many different sectors and uses.

3. GPTs occur in each of several "*classes*" of technology, such as materials, ICTs, power sources, transportation equipment, and organizational forms (e.g., the factory system) and at least one version of each class is in use at any one time.

4. Over time, many different "*versions*" of each class are invented. These often compete with each other and, as a result, there can be several versions of any one class in simultaneous use. For example, in 1900 some textile factories were shifting to electricity as a power source, while most were steam powered, and not a few were still using water wheels—three versions of GPTs all within the delivery-of-power class of technologies.[3]

5. In contrast, GPTs of different classes often complement each other, as when electricity enabled electronic computers and lasers.

6. The invention, innovation and diffusion of any major new technology, including GPTs, involves many uncertainties, most of which have not been modeled before. In particular, uncertainty pervades the following: (i) how much potentially useful pure knowledge will be discovered by any given amount of research activity; (ii) the timing of the discovery of new technologies; (iii) just how productive a newly innovated GPT will be over its lifetime; (iv) how well the new GPT will interact with GPTs of other classes that are also in use; (v) how long a new GPT will continue to evolve in usefulness; (vi) when it will begin to be replaced by a new superior version of a GPT of the same class (vii) how long that displacement will take and (viii) if the displacement will be more or less complete (as were mechanical calculators) or if the older technology will remain entrenched in particular niches (steam remains an important source of power for generating electricity).

We use Carlaw and Lipsey (2009) to simulate annual aggregate time series data on output, consumption, investment, labour and capital. The major modification to the simulation model is that we allow the arrival of GPTs to be dependent on endogenous allocation behaviour within the model in particular the allocation of resources to applied R&D increases the likelihood of GPTs arriving in every period, making the distribution of the random arrival process non-stationary.

To simulate the model, we restrict it to three industries within the consumption sector, three facilities in the applied R&D sector and three labs in the pure knowledge sector ($I = Y = X = 3$). We impose symmetry across sectors and specific activities within sectors (i.e., industries, facilities and labs). We choose values of the parameters and initial conditions with the overall objective that the model will replicate the accepted stylized facts of economic growth. We set the values of most of the initial conditions at unity because none of the long run characteristics of the model are influenced by initial conditions.[4] Some values are chosen to ensure consistency with observed data in the following ways: $\alpha_{Y+1} = \beta_{X+1} = \sigma_{Y+1} = 0.3$ ensures diminishing returns to the composite resource in all lines of activity; $\varepsilon = \delta = 0.025$ produces an average annual growth rate between 1.5% and 2%; $\lambda^* = 0.66$ allows a GPT within each class to arrive on average every 35 years, but with a large variance in arrival dates. We choose $\gamma = 0.07$ and $\tau = -7$ to so that 90% of a GPT's potential is translated into its

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[2] See, for example, Nathan Rosenberg (1982: Chapter 7).

[3] Some informal models that are expressed in verbal terms do deal with multiple GPTs.

[4] The initial value of $t_{n_x} = 2$ is chosen because we have lagged variables indexed on it and MatLab does not allow zero as an index value.

actual efficiency over the first 120 years of its life. We set $\alpha_y = \beta_x = \sigma_y = 1$ to ensure that knowledge has constant returns.

Table 2.1 gives the parameter values and initial conditions used to simulate the results of the multi-GPT model as reported in the text and shown in the figures. The set of θ^x are random variables distributed uniformly with support [0.9, 1.1], mean 1, and variance $(0.4)^2/12$, which sets modest bounds on the uncertainty concerning the productivity of pure research.

TABLE 1: NUMERICAL SIMULATION OF THE MULTI-GPT MODEL

Parameter values:

$\alpha_y = 1 \quad \forall i \in [1, Y]$ $\alpha_{Y+1} = 0.3$ $\beta_x = 1 \quad \forall x \in [1, X]$ $\beta_{X+1} = 0.3$

$\sigma_y = 1 \quad \forall y \in [1, Y]$ $\sigma_{Y+1} = 0.3$ $\gamma = 0.07$ $\tau = -7$

$\phi_i = 1 \quad \forall i \in [1, I]$ $\varepsilon = 0.025$ $\delta = 0.025$ $\mu = 0.95$

$I = 3$ $Y = 3$ $X = 3$ $\lambda^* = f(r_t^y)$

$R = \sum_{i=1}^{I} r_t^i + \sum_{y=1}^{Y} r_t^y + \sum_{x=1}^{X} r_t^x = 1000$

Initial conditions:

$(G_{n_x})_0 = 1 \quad \forall x \in (1, X)$ $A_0^y = 1 \quad \forall y \in [1, Y]$ $n_x = 1$ $\theta_0^x = 1$

$t_{n_x} = 2$ $P_0^i = 1 \forall i \in [1, I]$ $d_{y,x} = 1$ $w_0 = 1$

3. BUSINESS CYCLE PROPERTIES OF THE SIMULATED DATA

We run a simulation of the model based on the above parameterizations and generate artificial time series data for output, consumption, labour, investment and capital. The simulated times series represents 151 annual observations.

We compare the growth properties of the simulated data with those of Canadian aggregate data for the period 1961 – 2007. The Canadian data is all aggregate values. Output is GDP, consumption is consumption of non-durables, semi-durables and services, investment is gross investment in non-residential capital, and labour is total hours worked. We find that the growth properties of the simulated data exhibit average first moment properties that are similar to the Canadian data. In Canada investment has experience very rapid growth over the last 25 years. In comparison, our data exhibits balanced growth. However, that may be what one would find in Canada too if we had access to a longer data (e.g., from the year 1900 to 2007).

Table 3.1: Growth Properties

Average Growth Rate	Simulated Data (151 annual periods)	Canada (1961-2007)
Output	3.06	3.58
Consumption	2.68	3.03
Investment	3.34	5.29
Labour	1.54	1.58
Capital	3.12	

We then filter each of the simulated time series using a Hodric-Prescott filter set for annual data and compare their properties to the filtered Canadian data. In the Canadian data we find that investment should be about 2.5 times more volatile than output. Consumption is less volatile than output and labour exhibits about the same volatility as output. We also find from the Canadian data that all variables except capital is highly correlated with output.

Table 3.2: Basic Business Cycle Properties

Simulated Data	Standard Deviation	Correlation with Output
Output	0.7	1
Consumption	0.89	0.87
Investment	1.17	0.93
Labour	0.81	0.83
Capital	0.5	0.15

Table 3.2 shows the simulated data properties. We find that investment is indeed more volatile than output but only 1.7 times as volatile. We also find that consumption and labour are just slightly more volatile than output. So in summary are simulated data matches the volatility properties of the Canadian data though not exactly. Investment is not volatile enough and consumption is too volatile.

We find that the correlation properties of the data match well with the Canadian business cycle facts. Investment, consumption and labour are all highly correlated with output and capital is not.

4. TIME SERIES PROPERTIES OF THE SIMULATED DATA

To test the time series properties of the data we first took logs of the simulated time series data. We then ran a series of augmented Dickey-Fuller (ADF) tests on each individual time series. We found that each one exhibited a unit root. We also ran panel test for Levin, Lin and Chu's t*, Breitung's t-stat, Im, Pesaran and Shin's W-stat, the ADF – Fisher Chi-square and the PP –Fisher Chi-square. In all cases the tests rejected the null hypothesis of an I(2) process in favour or an I(1) process.

These same tests were run on two sub sets of the data, a period from 1 – 60 and another from 61 – 151 and again the tests rejected the null hypothesis of an I(2) process in favour of an I(1) process.

Johansen maximum likelihood-based cointegration test yield supporting results that the data is difference stationary. These tests are run on the simulated data in log form with a number of lags for the vector autoregression (VAR). Table 4.1 shows a sample of these results.

Table 4.1: Johansen Maximum Likelihood-based cointegration tests (variables in logs) Linear deterministic trend

Variable	# cointegrating vectors	# common trends	VAR
Full sample period			
LOUT	4	1	2
LCONS			
LINV			
LCAP			
LLAB			

5. IMPLICATIONS AND CONCLUSIONS

We have generated macroeconomic time series data from the Carlaw Lipsey (2007) endogenous growth model of GPT driven growth in which we have deliberately set a non-stationary process in motion. That is we have made the arrival process of GPT in the system dependent on some of the endogenous choice variables. This implies that the data generated by the system are of a higher order than first difference stationary.

Our analysis thus far is preliminary and requires much further testing. However, the preliminary findings are that the business cycle properties of the Canadian data for the period 1961 – 2007 can be replicated by simulated data that has these properties and that standard empirical time series analysis implies that the data is difference stationary.

We conclude that it remains an open question as to whether real macroeconomic time series data is in fact trend or difference stationary and further empirical methodology may need to be developed to verify or refute the statement.

If verified by further analysis, these findings have some important implications for testing and drawing inference from macroeconomic time series data. In particular as noted by Resenha (2005) the original work by Nelson and Plosser (1982) presented an empirical case that macroeconomic time series are difference stationary. This was taken as evidence in support of the RBC hypothesis that once the trend is filtered from macro data the business cycle exhibits stationarity and therefore monetary shocks can only have temporary effect. Furthermore, although Nelson and Plosser (1982) state explicitly that they assumed that the classical dichotomy held for their analysis, it has come to be generally accepted that the business cycle is stationary and therefore the dichotomy does hold.

Further analysis will entail generating a number of simulated data sets that have various non-stationarity properties in them explicitly to see under what conditions RBC and time series analysis will detect their non-stationary properties. For example, one stylised fact them emerges out of the historical analysis of general purpose technologies and economic growth is that sometimes transforming GPTs arrive and cause structural disruptions to the economy that lead to economic slowdowns for a period while the gestate and mature in the economy to which they have been introduced. This can be modelled explicitly within the Carlaw and Lipsey (2007) framework and can provide another source of non-stationarity (in terms of first difference) in the simulated data. Further analysis will reveal if the RBC and time series econometric techniques will reveal these sources of non- stationarity in the data.

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