18th World IMACS / MODSIM Congress, Cairns, Australia 13-17 July 2009
http://mssanz.org.au/modsim09

An approach to modelling and inference based on Axelrod's Ising model of alliance formation

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Abstract: Alliances have critical value. This observation is especially pertinent given the dynamics of the most recent conflicts of our time. Traditional military modeling and simulation has focused on understanding the dynamics of the physical aspects of conflict. This is in contrast to role-playing games in which the formation of alliances is essential to victory (take Diplomacy as the perennial example and the modern game SuperPower as a recent example). It therefore seems reasonable to consider the formation of alliances as a subject that warrants modelling and simulation-based research.

This paper outlines and extends a political science approach to modelling the formation of alliances, termed landscape theory. Landscape theory was formulated by the political scientist Robert Axelrod and is based on the Ising model of statistical mechanics. Between each nation or group an interaction propensity is calculated measuring the level of inter-nation influence and support. This propensity can range from positive values, in which nations are affable to negative where hostility exists. It is this collection of propensities that determines the global alliance structure. In Axelrod's model, the nation or group is characterised by a measure of its size or influence along with its propensities. Alliances are specified by defining a distance between the group or nation. The collection of distances between the nations is termed the configuration. If the distance between two nations is zero then they are considered to be in the same alliance. If greater than zero, then the nations are in different alliances. Given a collection of nations and propensities, any given configuration of alliances induces an energy function, which is the Ising equation of statistical mechanics. This function is a measure of the collective tension between nations.

The principal of alliance prediction is based on the assumption that nations or groups seek to minimise this collective tension. Nations can alter their alliance structure by altering their configuration of distances. They seek to minimise the distance with nations that share a positive propensity and distance themselves to nations that have a negative propensity. Collectively, this process minimises the energy (Ising) function. Thus to predict a stable alliance configuration, given a set of nations and the corresponding propensities, the energy function is minimised and the resulting configuration is viewed.

Our contribution is to extend landscape theory by a number of methods for calculating propensity based on importance weighted attributes from national statistics or historical data. Such attributes could include import/export levels, religious commonality, or data concerning past conflicts. These methods are applied to a dataset derived from hypothetical international data to make predictions of alliance structures.

Methods for adjusting the importance of different attributes that make up the propensity values, in light of new alliance data are then briefly discussed. Here, a predicted structure does not match the outcome and therefore the adjustment of attribute importance is vital. Finally, we discuss future topics of research in the modelling of alliance formation.

Keywords: *Alliances, political modelling, statistical mechanics, energy, configuration.*

1. INTRODUCTION

History is replete with the value of military-political alliances. It is well known that Churchill went to great lengths to secure the United States as an ally in World War Two (Churchill, 1986). The North Atlantic Treaty Organisation is another example of a vital international alliance. With the advent of political-science based modelling techniques, alliance formation and dissolution may be modelled and applied to the military domain in ways complementary to the opinions of domain experts.

Traditionally, the application of simulation-based modelling to reasoning in military domains has focussed on either wargaming or training. Considerable effort has been placed in designing models that are as realistic as possible, in terms of the physical or kinetic dynamics of the warfighting elements across space and time. Though these models are indispensable to strategic reasoning, it may be argued that the psychology and international placement of a potential adversary is as important as numerical and kinetic strength. At the forefront of M&S attempts to understand the world is the incorporation of numerous psychological and social effects (Ball, 2004). These include cognitive models of intent or attrition and social models of networking.

Models that include specific networking are of great interest to wide range of planners from such domains as air combat to intelligence. Through network models, one may examine such system features as fuel or information flows, centrality or robustness. There exist a rich set of algorithms to aid a planner to comprehend and reason with such properties.

In this paper we extend the scope of network analysis to some understanding of the political dynamics of conflict and cooperation. We look at the simulation of alliance formation.

Before continuing, let us remind ourselves of the definition of an alliance. Generally an alliance is a close association of nations or other groups formed to advance common causes or interests. Alliances, be it at a local state or international level form for a number of reasons. Some have reasoned that an alliance forms when two groups have complementary resources that strengthen their mutual positions. Others have reasoned that an alliance forms between two groups when the interactions between them are sufficiently beneficial in comparison to other groups. Both of these views are complementary.

What constitutes complementary resources or sufficiently beneficial interactions forms a topic of debate in international relations. Nations trade, share common cultural beliefs, political, military ties and a mutual history ranging from cooperation to warfare and antagonism. Which of these factors has the greatest structural importance in forming an alliance is as yet unclear, though common culture has been espoused as having the greatest importance by at least one prominent author (Huntington, 1998). Undoubtedly, factors such as common culture or economic exchange each have prominent importance at different times.

In an era of conflict not bound to strict geographic borders or nation states, the impact of diplomatic or military operations on the alignment of different national or tribal groups will not be necessarily clear. Within one nation there will generally be different political power factions or groups. How each group cooperates or competes with others will be critical to the success of any military operation. Because of this inherent social complexity it is vital that researchers look for a set of principles or models through which changing alliances are analysed.

In this paper we discuss one such principle by which one might reason with the dynamics of alliances. This is the landscape model devised by the political scientist Robert Axelrod (Axelrod and Benett, 1993). We then discuss a number of approaches to modelling economic, political and cultural commonalities from various sources of national data. From this, we then apply these concepts to reasoning about the alliance structure derived from data of a group of eighteen nations. Following this, we discuss one extension of the model, that of parameter adjustment in light of the new data regarding current alliance structure. We finish with a brief discussion on future research.

2. MODELLING ALLIANCE FORMATION

There are a number of models developed to theorise and reason about alliance formation. N-person game theoretic methods may by applied to calculate potential alliance or coalition structures. One must calculate the so called coalition structure values (CSV) for each group in each possible alliance configuration (Owen, 1977). The CVS is the score or value given to that particular configuration. If there are n groups then there are 2^n such values. Generally, it is not practical to apply this method for pragmatic analysis due to the sheer number of values and the difficultly obtaining an objective value for each alliance structure.

Instead, we apply a practical method developed by the political scientists Robert Axelrod and D. Scott Bennet termed landscape theory (Axelrod and Benett, 1993). National statistics data are used to calculate the interaction propensity between two nations. Such propensity is calculated from a number of sources such as the level of trade, religious and political commonalities. Given the matrix of international propensities and an initial arbitrarily defined alliance structure (albeit bilateral, trilateral etc.) it is assumed that each nation calculates some measure termed its "frustration" of being in any one of the possible alliance states, given the current configuration of other nations. Each simulated group or nation then chooses to change to an alliance with minimum frustration, given the current configuration of other groups or nations. This process is iterated until a stable configuration is reached. Landscape theory also takes into account the size (for example the gross domestic product or military strength) of other nations in calculating frustration.

At this point it is worth defining our model formally. Suppose there are N groups or nations with indices $i, j \in \{1, 2 \cdots, N\}$. Each nation i has a size $s_i > 0$, and each pair of nations i, j possess a propensity to interact $p_{ij} \in \mathbb{R}$. It is assumed that the propensity reflects the general level of interaction between two nations and not any asymmetries such as trade imbalances. Thus

$$p_{ij} = p_{ji} \; \forall i, j \in \{1, 2, \cdots, N\}.$$

In order to model an alliance structure we define a distance $d_{ij} > 0$ between any two nations. If $d_{ij} = 0$ then two nations are in the same alliance. With this general distance matrix one may be flexible about the alliance structure. For example, to model a simple bilateral alliance structure, one simply sets $d_{ij} = 1$ if the two nations i, j are in differing alliances. This definition may be easily extended to multilateral alliances; however we will restrict our analysis to that of bilateral alliances in this paper.

We will label a particular configuration as X and describe the distance between two nations i and j induced by this configuration as $d_{ij}(X)$. With these definitions, the frustration of a nation i given alliance configuration X is

$$F_i(X) = \sum_{j \neq i} s_j p_{ij} d_{ij}(X).$$

The central idea behind this model of alliance formation is to assume each nation attempts to minimise its frustration. This implies that nations proportionally value other nations of greater size and attempt to minimise the distance (form alliances with) nations with positive propensity and distance themselves from nations with negative propensity. In all, nations attempt to minimise their respective frustration.

Alliance configurations that are stable minimise the total energy of the system, this being the sum of the respective frustrations of each nation. The energy of the system is the same as the well known Ising model in statistical mechanics (Pathria, 2008).

$$\mathrm{H}(X) = 1/2 \sum_i \sum_{j \neq i} s_i s_j p_{ij} d_{ij}(X).$$

Thus the input to our model of alliance formations is the respective sizes and frustrations of the system. The output after energy minimisation is matrix of distances termed the configuration. If we define a vector of sizes

$$\mathrm{s} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{pmatrix}$$

and the symmetric matrices for propensity and distance as $\mathbf{P}$ and $\mathbf{D}$ respectively, then the energy of the system may be defined as

$$\mathrm{H}(X) = \tfrac{1}{2}\mathbf{s}^T \mathbf{P} \bullet \mathbf{D}\mathbf{s}.$$

Here $\mathbf{P} \bullet \mathbf{D}$ denotes the Hadamard entry by entry product of two matrices.

Finding the minimum energy of the system is a standard combinational optimisation problem that may be solved through a number of methods such as simulated annealing or genetic algorithms. It is important to point out that minimal solutions determined computationally will only be approximate and will in general depend on the initial conditions if the fitness structure of the configuration landscape is complex.

3. AN ILLUSTRATIVE EXAMPLE

We will now illustrate these ideas with an example. We modelled alliance formation in eleven hypothetical nations of differing sizes. The attributes that contributed to the calculation of the propensity matrix were the existence of substantial economic exchange, ideological alignment and a history of disputes. Our models for each factor were deliberately simple. If two nations possessed substantial economic exchange, the economic attribute $p_{ij}^{e} = 1$, otherwise its value was zero. Border disputes were modelled with $p_{ij}^{b} = -1$, if there is a past history of such a dispute, otherwise its value is zero for neutrality or one for close border cooperation. Ideological alignment was modelled either as good or bad, with $p_{ij}^{I} = 1$ if the alignment was good, or minus one otherwise. The overall propensity was calculated as

$$p_{ij} = p_{ji} = p_{ij}^{e} + p_{ij}^{b} + p_{ij}^{I}.$$

The following Figure 1 illustrates the hypothetical nation sizes and the structure of ideological alignment.

Given these propensity values we applied a standard simulated annealing algorithm to calculate a bilateral alliance configuration as is illustrated in Figure 2.

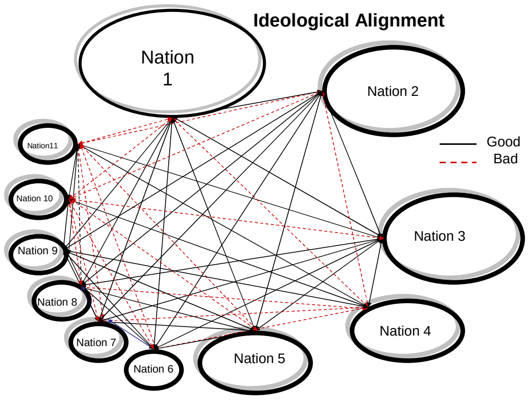

Figure 1. Diagram showing the hypothetical alignment of eleven nations. Here good indicates positive propensity and bad (red lines) indicates negative propensity.

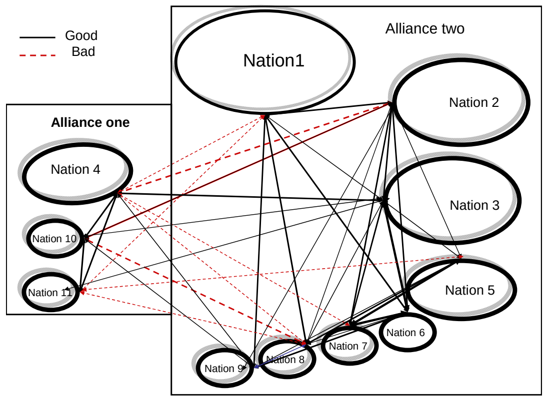

Figure 2. Diagram showing the hypothetical alliances given the propensities illustrated in Figure 1.

Collectively, nations one, two and three have the largest size, hence the greatest influence on calculating overall system energy. Nation four has significant negative propensity with nations one and two with nation eleven having negative propensity with one and another large nation, five. For this reason and the fact that nations four, ten and eleven have close ties, the bilateral structure has formed. Though nation ten does not have significant negative ties across many nations, in a sense one may interpret that this nation has been "coerced" to join alliance one, as its ties with nations four and eleven are substantial.

We thus see that this approach to modelling carries some, but not all of variegated phenomena associated with the formation of alliances. Politically, it could be argued that nations reticent to join conflicts have been coerced by powerful political allies.

4. PROPENSITIES FROM STATISTICAL DATA

If one accepts this approach to modelling alliances, its utility will only come from a cogent framework for the construction of international propensities. At present the modelling of propensities may be seen as more of an art than a science. In the example of Section 3, we assigned simple binary or ternary values to describe substantial economic exchange, border conflicts or ideological alignment. What is required is a framework for modelling propensities from data derived from either intelligence sources or national statistics such as gross domestic product, historical data and statistics about religion and culture.

From the outset, we assume that the propensity between any two nations i and j is modelled as

$$p_{ij} = p_{ij}(\mathbf{x}_{ij}, \boldsymbol{\theta}),$$

where $\mathbf{x}_{ij}$ is some vector of either statistical or intelligence data, and $\boldsymbol{\theta}$ is a collection of parameters that may be interpreted as weighting the relative importance of different attributes that contribute to the propensity. We make the assumption that the components of the vector $\boldsymbol{\theta}$ are positive and sum to one, making the space of parameters a convex set. The general form of the propensity will then be

$$p_{ij} = \sum_k f_k(\mathbf{x}_{ij})\theta_k, \left[\boldsymbol{\theta}\right]_k = \theta_k,$$

with the function f_k describes the value of an attribute from the data. Here we discuss the form of this mapping in terms of entries associated with national statistics databases.

In our modelling efforts, we took the size of a nation to be its gross domestic product. To model economic propensity we collected data from the World Trade Organisation on global trade imports and exports. We took as our measure of economic propensity to be the average export ranking across any pair of nations. This means that if we define r_{ij} to be the export trade ranking of nation j with respect to nation i then the economic propensity is taken as

$$f_e(r_{ij}, r_{ji}) = \frac{1}{2}\left(r_{ij} + r_{ji}\right).$$

This approach is one of many in modelling the level of mutual economic exchange. One could include import rankings and other such data. A deficiency of this approach is that it does not capture the non-linearities of export and import values. A nation whose rank is one may vastly import more goods than a nation who is of rank two, three or four.

To model cultural commonalities, we had as our starting point data describing the proportion of individuals belonging to a particular religious tradition. Our idea was to model some level of animosity or cooperation between differing nations due to a history of past religious conflict. Needless to say, this topic is highly speculative; nonetheless we approached the subject in the following way.

Let $\mathbf{v}_i$ and $\mathbf{v}_j$ be the proportion of individuals within nations i and j respectively of particular religious traditions. Now suppose there are two religious traditions, labelled religion k and religion l. We define a coefficient a_{kl} that measures the level of animosity or commonality of the two religious traditions. Clearly this coefficient is symmetric and we further assume that like begets like, by defining $a_{kk} = 1$. Our broad measure of cultural commonality is defined as

$$\sum_k \sum_l a_{kl} \left[\mathbf{v}_i\right]_k \left[\mathbf{v}_j\right]_l .$$

In matrix form this is simply

$$\mathbf{v}_i \mathbf{A} \mathbf{v}_j .$$

To make the analysis simple we focused only on the world's major religions, Christianity, Islam, Hinduism, Buddhism and Chinese Confucianism. We are aware that this ignores intra-religious conflict. Nonetheless, we broadly estimated the level of cooperation or impertinence through an analysis of current inter-religious conflict within or across nation states (Central Intelligence Agency, 2006).

We did not take into account national populations ideological sentiments when modelling the contribution of political philosophy or stance on the contribution to propensity. Instead, our model considered only the nature of the current political system in power at the present time. Though there are many political philosophies and systems, we focused on Democracy, Communism, and Plutocracy. Here again we assigned a coefficient q_{ce} for two political systems c and e.

Finally, we modelled the history of border conflicts in a way similar to that of the previous example; with the contribution to propensity of **-**1 if there is a considerable border dispute or national tensions over strategically important islands or boundaries at the present time (Central Intelligence Agency, 2006).

In order to illustrate the application of this modelling approach, we collected national statistics data from eighteen countries. We have labelled the countries numerically and presented only their internal religious composition, their GDP relative to the smallest nation and the political system in the following Table 1.

Table 1. Table showing eighteen nations, labelled one through to eighteen, with associated relative GDP, religious composition and current political system. Border conflicts and trade rankings are not displayed. Here D stands for democracy, P for plutocracy and C for communist.

Nation	*GDP*	*Christian*	*Muslim*	*Hindu*	*Buddhist*	*Politics*
1	262	0.8	0.015	0.01	0.02	D
2	2	0.8	0.015	0	0	D
3	107	0.08	0.88	0.02	0	D
4	44	0.2	0.15	0.2	0.42	P
5	49	0.1	0.88	0.01	0	D
6	21	0.1	0.67	0.05	0.13	P
7	36	0.9	0.05	0	0	D
8	19	0.1	0.05	0	0.089	C
9	2	0.01	0.01	0	0.95	D
10	68	0	0.05	0	0.95	D
11	1	0	0.05	0	0.6	C
12	684	0.05	0.05	0	0.5	C
13	282	0.26	0	0	0.26	D
14	1,917	0.05	0	0	0.5	D
15	286	0.05	0.13	0.8	0.05	D
16	127	0.05	0.05	0	0.5	D
17	4,837	0.95	0.01	0	0.01	D
18	5,261	0.95	0.01	0	0.01	D

We ran our alliance modelling algorithm, assuming that trade ranking, cultural alignment, political ideology and national disputes each had an equal weighting of 0.25. The theorised alliance structure, assuming the restriction to a bipartite structure brings nations 17 and 18 together along with nation 14. The smaller nations 1, 13, 15, 16, 3, 7, 10 and 2 follow. In turn nation 12 anchors the alliance with nations 4, 5, 6, 8, 9 and 11. Broadly, our observation is that the first set of nations forms a democratic alliance, denominated by the Christian faith, with two notable exceptions, nation 3, which is Islamic, has significant trade ties in this alliance. Nation 15 is Hindu-Islamic and also has significant trade ties with the other nations of the alliance.

The second alliance may be broadly termed a Communist-Islamic alliance with some notable exceptions. Nation 5 is Democratic and Islamic whilst nation 9 is Democratic and mainly Buddhist with significant trade ties to the other nations within the alliance.

5. ADJUSTING ATTRIBUTE IMPORTANCE

In the previous section we took an empirical approach to the modelling of propensities from national statistics data. Now suppose we were presented with a dynamic sequence of alliance structures represented as $X(t), t = 1,\cdots,T$ and a corresponding sequence of attributes as calculated from data $\mathbf{a}_{ij}(t), i, j \in 1,\cdots,N,\ t \in 1,\cdots,T$. Now the propensities are time dependent and are assumed to take the form

$$p_{ij}(t) = \mathbf{a}_{ij}(t) \cdot \boldsymbol{\theta}.$$

That is we take the dot product between the time dependent attribute vector and the time independent parameter vector $\boldsymbol{\theta}$. It is assumed that there are $|\boldsymbol{\theta}| = K$ parameters. Given this framework we present an algorithm for updating the parameter vector $\boldsymbol{\theta}$. Suppose our predicted alliance structure, as calculated from propensities or from parameter weighted attributes is termed X_P whilst the observed structure is in fact different. The observed alliance structure we will label X_{Obs} with $d_{ij}(X_P) \neq d_{ij}(X_{Obs})$ for at least some nations i and j.

One approach is to assume that a new parameter $\boldsymbol{\theta}'$ has in combination with the new attributes has lower energy than the old parameter $\boldsymbol{\theta}$. So defining

$$H(X, \boldsymbol{\theta}) = \sum_{i,j} s_i s_j a_{ij}(t).\boldsymbol{\theta} d_{ij}(X),$$

we seek

$$H(X_{Obs}, \boldsymbol{\theta}') < H(X_{Obs}, \boldsymbol{\theta}).$$

This new value of the parameter can be found by gradient descent. This is done by choosing

$$\boldsymbol{\theta}' = \boldsymbol{\theta} - \alpha \nabla \mathrm{H}(X_{Obs}, \boldsymbol{\theta}),$$

where α is an adjustment parameter whose value is between zero and one.

6. DISCUSSION AND CONCLUSION

Alliances should be modelled to provide a level of strategic situational awareness complementary to that of domain experts, such as that from diplomats, liaison officers and the like. Still in its infancy, the approach we have outlined in this paper derives some general conclusions from widely available data. Modelling alliances in a systematic way may help us to make decisions regarding the placement of financial aid, or the appropriate third party best placed to secure some deal or pact. Such models may help in strategic-level military and political planning, involving decisions regarding the structure and nature of forces to be placed in a major campaign.

Modelling alliances is a rich avenue for research. Here we have looked a group models but there is the possibility that richer individual-based models could be pursued. There will also be other principles different from energy minimisation that might lead to modelling predictions.

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