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A stochastic runoff model incorporating spatial variability

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Abstract: The volume of catchment discharge that reaches a stream via the overland flow path is critical for water quality prediction, because it is via this pathway that most constituents are generated and transported to the stream channel. Two of the key properties determining this runoff volume are the rainfall rate and the infiltration rate. Both these rates are variable in both space and in time, but it is common to neglect the spatial variability. In this paper we propose a stochastic runoff model that deals explicitly with spatial variability, while neglecting the temporal variability.

We use an idealised model for the catchment terrain, which allows us to obtain analytical results. We consider a single hillslope, broken up into a series of parallel and independent strips, perpendicular to the stream edge. Each strip has width l_x and is divided into blocks of length l_y. Consider a single strip, and number the blocks $1, 2, \ldots, n$, starting at the top of the slope, and let X_k be the flow of runoff from block k to $k+1$, in m^3h^{-1}. If P_k is the flow of rain falling on block k, and I_k the maximum flow of water absorbed by block k (both in m^3h^{-1}), then the runoff-runon phenomena can be expressed as

$$X_k = \begin{cases} \max\{0, P_1 - I_1\}, & k = 1 \\ \max\{0, X_{k-1} + P_k - I_k\}, & k > 1. \end{cases} \tag{1}$$

If the $\{I_k\}_{k=1}^n$ and $\{P_k\}_{k=1}^n$ are i.i.d. sequences then (1) can be interpreted as a description of the waiting time for a single server queue, a classic topic in stochastic processes. In particular, if $\mathrm{E}P_k < \mathrm{E}I_k$ then it is well known that X_n has a limiting stationary (or equilibrium) distribution X. We interpret X as the discharge from the given strip into the stream, and can give general equations for its mean and variance, in terms of the moments of the I_k and P_k.

In addition to the runoff X_n, we are interested in the connected area contributing to X_n. That is, we wish to know $M_n = \sup\{m : X_n > 0, X_{n-1} > 0, \ldots, X_{n-m+1} > 0\}$. As for X, we can obtain the mean and variance of M, the limiting distribution of the M_n, though under somewhat stronger assumptions. The connected area Ml_xl_y is the area of land from which pollutants can be transported to the stream.

Aggregating the runoff from independent strips we obtain an analytic form for the volume of runoff from the hillslope, which is applicable to general precipitation and infiltration distributions. We also have an analytic form for the connected contributing area of the hillslope, though this makes more restrictive assumptions about the precipitation and infiltration distributions. From these we can immediately see the effect of the spatial variation of infiltration and precipitation. For example, we see that runoff increases approximately linearly with c_I^2 and c_P^2, the coefficients of variation for infiltration and precipitation.

We also see from the aggregated models the key role played by the spatial scale l_y, which can be interpreted as the natural spatial correlation scale of the infiltration and precipitation processes. Its importance is that both the aggregated runoff and connected contributing area scale linearly with l_y. That is, the spatial correlation scale is an important characteristic of the terrain when determining both the volume of runoff and the connected contributing area.

Keywords: *runoff, overland flow, spatial correlation, stochastic, queue.*

1. INTRODUCTION

The volume of catchment discharge that reaches a stream via the overland flow path is critical for water quality prediction, because it is via this pathway that most constituents are generated and transported to the stream channel. Two of the key properties determining this volume are the rainfall rate and the infiltration rate. In natural systems both these rates are variable in both space (Neilson et al. 1973; Price 1994) and in time (Green and Ampt 1911), however it is common to neglect spatial variability and model infiltration as a function of time only. This can be attributed to the early development of analytical expressions for the change in infiltration rate with time (Green and Ampt 1911). For catchment scale predictions these point-scale representations of infiltration have been scaled up, though in the process the parameters lose their physical meaning (Grayson et al. 1992).

The spatial distribution of saturated conductivity (K_{sat}) has been measured for many different soils and is widely reported as log-normal (Neilson et al. 1973; Price 1994). The partial interception of rainfall by vegetation can create spatial structure in the throughfall, which is commonly reported as normally distributed (Carlyle-Moses et al 2004). With the exception of Park and Cameron (2008), the coefficient of variation appears to decrease quickly with intensity, ranging from around 40-50% for storms <5mm, rapidly asymptoting to about 5-12 % for storms >5mm (Carlyle-Moses et al 2004; Mitchell unpub).

With no analytic expressions for runoff generation as a function of spatial variability, it wasn't until the widespread availability of electronic computers that the uncertainty associated with neglecting the spatial dimension was revealed. Numerical methods have explored many aspects of runoff generation, including runoff-runon, rainfall variability, analytic infiltration and overland flow models, spatial correlation in infiltration and rainfall fields, and erosion and sediment transport (Nahar 2008). Many of these numerical investigations neglected the runoff-runon process, instead routing all runoff to the outlet. However, Nahar (2003) showed that for soils with moderate to high mean saturated conductivity relative to rainfall rate, routing all runoff to the outlet produced substantial errors in the outflow hydrograph, and in these cases runoff-runon processes should be incorporated in the runoff model. These conditions are typical in temperate forests, where saturated conductivity values are usually high, and are common in many other landscapes for the majority of rainfall events.

Contemplation of these and earlier results has led to the emergence of the notion of "hydrologic connectivity" (see Gomi et al. 2008 for a review), a recognition that (generally) only a fraction of the runoff generated in a catchment actually "connects" with the outlet during the rainfall event. Hawkins and Cundy (1987) were the first to propose an analytic solution to the runoff generation problem incorporating variability in the spatial dimension. These authors showed that for an area with spatially variable infiltration there exist maximum and minimum curves relating the net-plot infiltration rate to the precipitation rate. The curves are derived by assuming a downslope arrangement of point K_{sat} values from largest to smallest, or vice versa, that results in maximum or minimum net-plot infiltration rates, respectively. The true (but generally unknown) function relating precipitation rate to net-plot infiltration rate must lie within these envelope curves. The key features of this model are that i) the net-plot infiltration rate is a function of precipitation intensity, rather than a function of time as in temporal infiltration models, ii) that runoff is generated even when the precipitation rate is lower than the net infiltration capacity of the plot, and iii) that additional runoff is generated gradually as rainfall intensity increases.

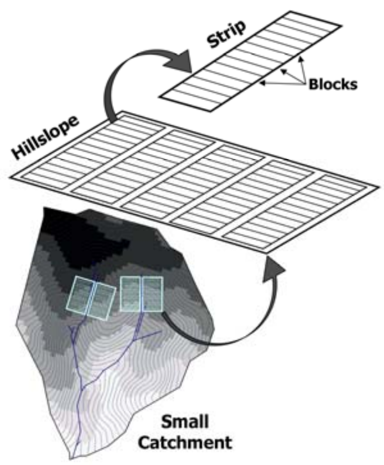

Figure 1. The two components of the Stochastic Runoff Connectivity model, a) the single strip model, and b) the hillslope or catchment model.

The Hawkins and Cundy (1987) model has not received the widespread attention it deserved, despite the fact that Yu et al. (1997; 1998) and others (Yu 1999; Fentie et al. 2002) have reported considerable success using the minimum infiltration curve as the basis of a rainfall-runoff model at the plot scale. The approach was found to perform better than the time-variant, space-invariant Green and Ampt (1911) model for the prediction of infiltration excess runoff at the plot scale (Yu 1999). The success of the approach indicates that the shapes of the net plot infiltration curves given by Hawkins and Cundy (1987) probably have some underlying physical basis, despite the ordering restriction.

In this paper we show how runoff-runon between adjacent downslope elements (pixels, blocks, gridcells), caused by the random arrangement of infiltration capacity, can be modelled using a stochastic queuing system.

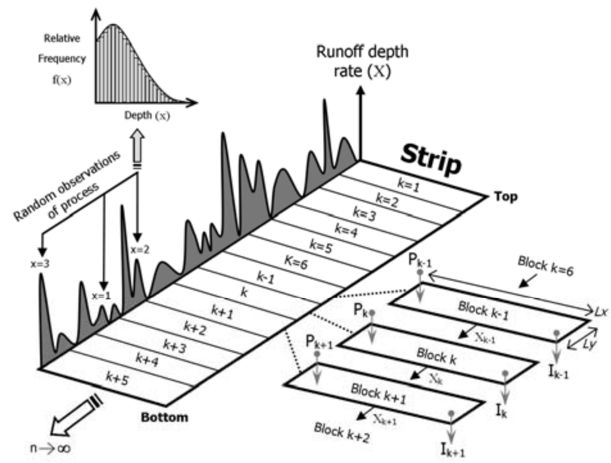

Figure 2. An illustration of the random fluctuation in runoff down a hypothetical slope, and a detailed illustration of the single strip model given by (1).

2. STOCHASTIC RUNOFF CONNECTIVITY (SRC) MODEL

We model the hillslope as a series of parallel and independent strips perpendicular to the stream edge (Figure 1). The model is constructed in two steps: firstly we consider the runoff generating properties of a single strip of land, perpendicular to the contour from the ridge to the stream edge, with a random arrangement of rainfall and infiltration capacity along the length. Secondly we consider the properties of the aggregated output from many such strips, analogous to a hillslope or catchment.

2.1. Single strip: equations for runoff flow

First we consider a single strip of land, width l_x, divided into blocks of length l_y. Number the blocks $1, 2, \ldots, n$, starting at the top of the slope. Let p_k be the precipitation (rainfall) rate and i_k the infiltration rate for block k (both are fluxes, measured in mmh^{-1}), assumed to be constant over time. Let $P_k = l_x l_y p_k/1000$ be the flow of rain falling on block k, and let $I_k = l_x l_y i_k/1000$ be the maximum flow of water absorbed by block k (both in m^3h^{-1}). P_k represents incident rainfall if there is no canopy or over-story, or through-fall if there is an over-story. Let X_k be the flow of water from block k to $k+1$, in m^3h^{-1}. If we assume that there is no significant runoff onto our given strip from neighbouring strips, then we have

$$X_k = \begin{cases} \max\{0, P_1 - I_1\}, & k = 1 \\ \max\{0, X_{k-1} + P_k - I_k\}, & k > 1. \end{cases} \qquad (1)$$

This is commonly referred to as the runoff-runon phenomena.

It turns out that (1) is exactly the equation governing the waiting time in a single server first-in first-out (FIFO) queue. If we let P_k be the service time for customer k and let I_k be the inter-arrival time between customers k and $k+1$, then X_k is the waiting time for customer $k+1$, that is, the time between arriving and service commencing.

We make the following assumptions:

1. Rainfall intensity and infiltration rate are time invariant, that is p_k and i_k, and thus P_k and I_k, are independent of time.
2. There is no spatial correlation in the infiltration capacity at the scale of blocks used in the model, that is the $\{I_k\}_{k=1}^n$ are independent of one another. Small-scale spatial correlation in the infiltration capacity i_k has been observed, thus the validity of this assumption requires l_x and l_y large enough that the correlation between I_k and I_{k+1} is negligible.
3. There is no spatial correlation in the rainfall at the scale of blocks used in the model, that is the $\{P_k\}_{k=1}^n$ are independent of one another. Again, small-scale spatial correlation in the throughfall p_k

has been observed, so to justify this assumption we need l_x and l_y to be large enough that there is negligible correlation between P_k and P_{k+1}.

4. Infiltration rate i_k is independent of surface water depth, that is I_k is independent of X_{k-1}, for all k.

Let $m_P = \mathrm{E}P_k$, $m_I = \mathrm{E}I_k$ and $\rho = m_P/m_I$, then it is readily shown (for example Asmussen 2003) that if $\rho < 1$ then X_n has a limiting stationary (or equilibrium) distribution X. That is, as $n \to \infty$, the cumulative distribution function (cdf) F_n of X_n approaches the cdf F of X:

$$F_n(x) = \mathrm{P}(X_n \le x) \to F(x) = \mathrm{P}(X \le x) \tag{2}$$

The distribution F characterizes the rate at which water runs from a single strip into the stream, see Figure 2.

Various exact and approximate forms of F are available from the queuing literature, depending on the distributions of P_k and I_k. For our purposes it is sufficient to know the mean and variance. Put

$$\sigma_I^2 = \mathrm{Var} I_k,\ \sigma_P^2 = \mathrm{Var} P_k,\ c_I^2 = \sigma_I^2/m_I^2 \text{ and } c_P^2 = \sigma_P^2/m_P^2.$$

For the mean we use an approximation due to Kramer & Lagenbach-Belz (1976, see Bhat 1993 Eqn. 1)

$$\mathrm{E}X \approx \frac{m_P^2(c_I^2 + c_P^2)g}{2(m_I - m_P)} \text{ where } \log g = \begin{cases} \frac{-2(1-\rho)(1-c_I^2)^2}{3\rho(c_I^2+c_P^2)} & \text{if } c_I^2 < 1 \\ \frac{-(1-\rho)(c_I^2-1)}{c_I^2+4c_P^2} & \text{if } c_I^2 \ge 1 \end{cases} \tag{3}$$

This approximation is exact in the case where the I_k have an exponential distribution. Alternative approximations to $\mathrm{E}X$ have been given by Marchal (1976, see Kleinrock Vol. II §2.3) and Whitt (1993, see Rao & Feldman 2001 Eqn. 12).

Approximations for $\mathrm{Var}X$ have been proposed Bhat (1993), Shanthikumar (1983), and Whit (1993). We found that the following minor modification of the approximation of Bhat (Bhat 1993 Eqn. 6) gave the best results.

$$\mathrm{Var}X \approx \left(\frac{c_I^2 m_P^2 + \sigma_P^2}{2(m_I - m_P)}\right)^2 + \frac{\mathrm{E}(P-m_P)^3 + m_P^3(3\sigma_I^4 - m_I\mathrm{E}(I-m_I)^3)_+/m_I^4 + 3c_I^2 m_P \sigma_P^2}{3(m_I - m_P)} \tag{4}$$

Compared to the original result, the factor $3\sigma_I^4 - m_I\mathrm{E}(I-m_I)^3$ has been replaced by its positive part. This was found to improve the approximation when the skewness of I was large. This approximation is also exact in the case where the I_k have an exponential distribution.

2.2. Single strip: equations for connected length

In addition to X_n, the runoff at the bottom of the slope, we are interested in the connected area contributing to X_n. That is, we wish to know

$$M_n = \sup\{m : X_n > 0, X_{n-1} > 0, \ldots, X_{n-m+1} > 0\}. \tag{5}$$

M_n can also be interpreted in terms of a single server FIFO queue, with service times P_k and inter-arrival times I_k. Let Q_k be the number of customers in the system just before the arrival of customer $k+1$, then $X_k = 0 \iff Q_k = 0$. That is, the waiting time for customer $k+1$ is zero iff there is no-one in the system when he arrives. Thus $M_n = \sup\{m : Q_n > 0, Q_{n-1} > 0, \ldots, Q_{n-m+1} > 0\}$, that is, M_n is the number of customers who arrived during the current busy period, observed just before the arrival of customer $n+1$.

The busy period of a queue is much less tractable than the waiting time, so to obtain results we need to make some relatively strong assumptions. In addition to our independence assumptions above, suppose that $I \sim \exp(\lambda)$, $P \sim \exp(\mu)$ and $\rho < 1$. Let M be the limiting distribution of M_n then

$$\mathrm{E}M = \rho\frac{1-\rho+\rho^2}{(1-\rho)^2} \text{ and } \mathrm{Var} = \frac{\rho(1-2\rho+6\rho^2-4\rho^3+3\rho^4-\rho^5)}{(1-\rho)^4} \tag{6}$$

A proof is given in the Appendix.

2.3. Hillslope model

We represent a hillslope as a collection of adjacent strips extending upslope perpendicular to the stream boundary. We assume that the runoff from adjacent strips are independent and identically distributed (there are no lateral inflows or outflows from a strip), and that there are a sufficient number of strips m for the asymptotic properties of the central limit theorem to be valid.

Let $X^{(i)}$ be the runoff flow from the i-th strip, with mean μ_X and variance σ_X^2. By the central limit theorem

$$Z := \sum_{i=1}^{m} X^{(i)} \approx N(m\mu_X, m\sigma_X^2) \text{ in } m^3h^{-1}. \tag{7}$$

Note that, because $X^{(i)} \propto l_x$ and l_y, we have that

$$\begin{aligned} \mathrm{E}Z &\propto ml_x = \text{stream length} \\ \mathrm{Std}Z &\propto \frac{\text{stream length}}{\sqrt{m}} = \sqrt{l_x}\sqrt{\text{stream length}} \end{aligned}$$

and that $\mathrm{E}Z$ and $\mathrm{Std}Z \propto l_y$.

We can think of l_y as a system parameter that measures the spatial correlation of rainfall and infiltration. It should be just large enough that the P_k and I_k appear to be uncorrelated. Thus, in a system where $\rho < 1$, *the run-off flow scales linearly with the spatial correlation scale of the precipitation and infiltration.*

The scale l_x has a second order influence on $\mathrm{Var}Z$. As l_y and l_x are primarily determined by the spatial correlation of rainfall and infiltration, they should be the same order of magnitude. They need not be the same however, as the strip width l_x also needs to be large enough that the lateral flow from one strip to another is negligible.

Let $M^{(i)}$ be the connected length for strip i, and let $A^{(i)} = M^{(i)}l_xl_y$ be the connected area along strip i, then the connected area for a catchment or hillslope consisting of m strips is

$$C := \sum_{i=1}^{m} A^{(i)} \approx N(m\mu_A, m\sigma_A^2) \text{ in } m^2, \tag{8}$$

where μ_A and σ_A^2 are the mean and variance of $A^{(i)}$. Since $A^{(i)} \propto l_x$ and l_y, we have that

$$\begin{aligned} \mathrm{E}C &\propto ml_x = \text{stream length} \\ \mathrm{Std}C &\propto \frac{\text{stream length}}{\sqrt{m}} = \sqrt{l_x}\sqrt{\text{stream length}} \end{aligned}$$

and that $\mathrm{E}Z$ and $\mathrm{Std}C \propto l_y$.

Thus l_x and l_y play the same scaling role for C as they did for Z. In particular we see that *the connected area scales linearly with the spatial correlation scale of the precipitation and infiltration.*

3. DISCUSSION

Combining (3) and (4) with (7) we have an analytic form for the volume of runoff from a hillslope, that is applicable to general precipitation and infiltration distributions. Combining (6) with (8) we have an analytic form for the connected area for a hillslope, though this makes more restrictive assumptions about the precipitation and infiltration distributions. From these we can immediately see the effect of the spatial variation of infiltration and precipitation. For example, from (3) we see that (roughly) $\mathrm{E}Z$ increases linearly with c_I^2 and c_P^2, the coefficients of variation for infiltration and precipitation.

The key calibration parameters required to fit the model are the spatial scales l_x and l_y. There should be negligible spatial correlation in the infiltration and rainfall at these scales, but if they are too large then you will mask some of the variability of the infiltration and rainfall. Because the mean and standard deviation of $Z$and C both scale linearly with l_y, we see that the spatial correlation scale is an important characteristic of the landscape when determining both the volume of runoff and the connected area.

The greatest limitation of the SRC model is the neglect of temporal variability. Another key limitation is that the asymptotic requirements of the model constrain the domain of the model to the case $\mathrm{E}P_k < \mathrm{E}I_k$. The dynamics of (1) are also of interest when mean rainfall $\mathrm{E}P_k$ is greater than mean infiltration capacity $\mathrm{E}I_k$. Under these conditions the process does not reach a limit (that is (2) does not hold), and the runoff down a strip of infinite length would tend to infinity. None-the-less, the queuing literature does provide some theory in this case, in particular we can describe the rate at which $X_n \to \infty$.

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APPENDIX

Here we derive (6) for the mean and variance of M. Let Q be the limiting distribution of the Q_n. We have that $\mathrm{P}(Q \geq q) = \rho^q$ (Kleinrock Vol 1 Eqn. 3.27) and $\mathrm{P}(M = 0) = \mathrm{P}(Q = 0) = 1 - \rho$.

Let B be the length of a busy period, and B^* the length of a randomly sampled busy period, where the probability of choosing a busy period is proportional to its length. Then $\mathrm{P}(B^* = n) = n\mathrm{P}(B = n)/\mathrm{E}B$, and we have (Kleinrock Vol 1 Eqn. 5.157) $\mathrm{P}(B = n) = \frac{1}{n}\binom{2n-2}{n-1}\rho^{n-1}(1+\rho)^{1-2n}$.

Given that $M > 0$, let B^* be the length of the corresponding sampled busy period, then $M|(M > 0, B^*) \sim U\{1, \ldots, B^*\}$. Thus

$$\begin{aligned}
\mathrm{P}(M = x | M > 0, B^* = n) &= \frac{1}{n}1_{\{x \leq n\}} \\
\mathrm{P}(M = x | M > 0) &= \sum_{n=1}^{\infty} \mathrm{P}(B^* = n)\frac{1}{n}1_{\{x \leq n\}} = \sum_{n=1}^{\infty} \frac{\mathrm{P}(B = n)}{\mathrm{E}B}1_{\{x \leq n\}} = \frac{\mathrm{P}(B \geq x)}{\mathrm{E}B}
\end{aligned}$$

We can use this to calculate the mean and variance of M. It is known (Kleinrock Vol 1 Eqn. 5.156) that B has probability generating function

$$F_B(z) = \mathrm{E}z^B = \frac{1+\rho}{2\rho}\left(1 - \left(1 - \frac{4\rho z}{(1+\rho)^2}\right)^{1/2}\right)$$

Taking derivatives at 1 we get, after a little algebra,

$$\mathrm{E}B = \frac{1}{1-\rho} \quad \mathrm{E}B^2 = \frac{1+\rho^2}{(1-\rho)^3} \quad \mathrm{E}B^3 = \frac{1 + 2\rho + 6\rho^2 + 2\rho^3 + \rho^4}{(1-\rho)^5}$$

Hence we have for the expected value of M

$$\begin{aligned}
\mathrm{E}(M | M > 0) &= \sum_{x=1}^{\infty} x\frac{\mathrm{P}(B \geq x)}{\mathrm{E}B} = \sum_{x=1}^{\infty} \frac{1}{2}x(x+1)\frac{\mathrm{P}(B = x)}{\mathrm{E}B} = \frac{\mathrm{E}B^2}{2\mathrm{E}B} + \frac{1}{2} \\
&= \frac{1 - \rho + \rho^2}{(1-\rho)^2} \\
\mathrm{E}M &= \rho\frac{1 - \rho + \rho^2}{(1-\rho)^2}
\end{aligned}$$

and for the variance

$$\begin{aligned}
\mathrm{E}(M^2 | M > 0) &= \sum_{x=1}^{\infty} x^2\frac{\mathrm{P}(B \geq x)}{\mathrm{E}B} = \sum_{x=1}^{\infty} \frac{1}{6}x(2x+1)(x+1)\frac{\mathrm{P}(B = x)}{\mathrm{E}B} \\
&= \frac{\mathrm{E}B^3}{3\mathrm{E}B} + \frac{\mathrm{E}B^2}{2\mathrm{E}B} + \frac{1}{6} = \frac{1 - \rho + 4\rho^2 - \rho^3 + \rho^4}{(1-\rho)^4} \\
\mathrm{E}M^2 &= \rho\frac{1 - \rho + 4\rho^2 - \rho^3 + \rho^4}{(1-\rho)^4} \\
\mathrm{Var}M &= \frac{\rho(1 - 2\rho + 6\rho^2 - 4\rho^3 + 3\rho^4 - \rho^5)}{(1-\rho)^4}
\end{aligned}$$

REFERENCES

Asmussen, S. (2008), *Applied Probability and Queues*, Second Edition. Springer.

Bhat, B.N. (1993), Approximation for the variance of the waiting time in a GI/G/1 queue, *Microelectron. Reliab.* 33, 1997-2002.

Carlyle-Moses, D.E., J.S. Flores Laureano, and A.G. Price (2004), Throughfall and throughfall spatial variability in Madrean oak forest communities of northeastern Mexico, *Journal of Hydrology*, 297, 124-135.

Fentie, B., B. Yu, M.D. Silburn, and C.A.A. Ciesiolka (2002), Evaluation of eight different methods to predict hillslope runoff flows for a grazing catchment in Australia, *Journal of Hydrology*, 261, 102-114.

Freeze, R.A. (1980), A stochastic-conceptual analysis of rainfall-runoff processes on a hillslope, *Water Resources Research*, 16(2), 391-408.

Gomi, T., R.C. Sidle, S. Miyata, and K. Kosugi (2008), Dynamic runoff connectivity of overland flow on steep forested hillslopes: scale effects and runoff transfer, *Water Resources Research*, 44.

Govindaraju, R.S., C. Corradini, and R. Morbidelli (2006), A semi-analytical model of expected areal-average infiltration under spatial heterogeneity of rainfall and soil saturated conductivity, *Journal of Hydrology*, 316, 184-194.

Grayson, R.B., I.D. Moore, and T.A. McMahon (1992), Physically based hydrologic modelling 2. Is the Concept Realistic, *Water Resources Research*, 26(10), 2659-2666.

Green, W.H., and G.A. Ampt (1911), Studies on soil physics: I. Flow of air and water through soils, *J. Agr. Sci.*, 4, 1-24.

Hawkins, R.H., and T.W. Cundy (1987), Steady-state analysis of infiltration and overland flow for spatially varied hillslopes, *Water Resources Bulletin*, 23(2), 251-256.

Kleinrock, L. (1975), Queueing Systems, Volume 1: Theory, *Wiley Interscience*, New York.

Kleinrock, L. (1975), Queueing Systems Volume II: Computer Applications. *Wiley Interscience*, New York.

Kramer, W., and M. Lagenbach-Belz (1976), Approximate formula for the delay in the queueing system GI/G/1. Proceedings ITC 8, Melbourne.

Marchal, W.G. (1976), An approximate formula for waiting time in single server queues, *IIE Trans.*, 8(4), 473-474.

Nahar, N. (2003), Influence of run-on on field scale surface and subsurface water and contaminant movement over spatially variable hillslopes, PhD Thesis.

Nahar, N.R. S. Govindaraju, C. Corradini, and R. Morbidelli (2008), Numerical evaluation of the role of run-on on sediment transport over heterogeneous hillslopes, *Journal of Hydrologic Engineering*, 13(4), 215-225.

Nielsen, D.R., J.W. Biggar, and K.T. Erh (1973), Spatial variability of field measured soil-water properties, *Hilgardia*, 42, 215-259.

Pacheco, A., and N.U. Prabhu (1996), A markovian storage model, *The Annals of Applied Probability*, 6(1), 76-91.

Park, A., and J.L. Cameron (2008), The influence of canopy traits on throughfall and stemflow in five tropical trees growing in a Panamanian plantation, *Forest Ecology and Management*, 255, 1915-1925.

Price, A.G. (1994), Measurement and variability of physical properties and soil water distribution in a forest podzol, *Journal of Hydrology*, 161, 347-364.

Rao, B.V. and R.M. Feldman (2001), Approximations and bounds for the variance of steady-state waiting times in a GI/G/1 queue, *Oper. Res. Lett.*, 28, 51-62.

Shanthikumar, J.G. (1983), Bounds and an approximation for single server queues, *J. Oper. Res. Soc. Japan*, 26 118-134.

Whitt, W. (1993), Approximations for the GI/G/m queue. *Production and Operations Management,* 2, 114-161.

Yu, B. (1999), A comparison of the Green-Ampt and a spatially variable infiltration model for natural storm events, *Transactions of the ASAE*, 42(1), 89-97.

Yu, B., C.W. Rose, K.J. Coughlan, and B. Fentie (1997), Plot-scale rainfall runoff characteristics and modelling at six sites in Australia and South East Asia, *Transactions of the ASAE*, 40(5), 1295-1303.

Yu, B., U. Cakurs, and C.W. Rose (1998), An assessment of methods for estimating runoff flows at the plot scale, *Transactions of the ASAE*, 41(3), 653-661.