

# A penalty minimisation resource (water) allocation model to simulate the effects of new infrastructure in the Goulburn irrigation system

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**Abstract:** The Goulburn Irrigation system is considered to be a large and complex irrigation system. As documented in Our Water Our Future (2007), the Goulburn system together with Victoria's Murray irrigation system account for over 70% of Victoria's stored water, and provide 3,500 GL of water for irrigation annually. As part of the Our Water Our Future plan, the Food Bowl Modernisation Project has been initiated at a cost of \$2 billion by the Government and water authorities, scheduled for completion in 2012. It is estimated up to 900 GL of water is lost annually due to inefficiencies in the system. The Project has the potential to capture 450 GL, expected to capture 425 GL of this previously lost water annually. The Sugarloaf Interconnector is currently under construction and will connect the Goulburn Murray water grid to the Melbourne water grid. The pipe is one way from the Goulburn River near Yea to the Sugarloaf Reservoir. Only water gained via the Food Bowl Modernisation project is to be used for this pipe, up to 75 GL of water per year. Water is expected to be available for Melbourne by 2010. The remainder of the savings is to be spread evenly with irrigators and rivers (175 GL each).

The Government is also investigating the feasibility of building a Murray-Goulburn Interconnector which would enable water to bypass the Barmah Choke. The Barmah Choke is a narrow section of the Murray River near the town of Barmah. The Choke limits the volume of water that can be moved along the river to supply peak demands downstream of the Choke. Water from the Murray Valley irrigation area can then be used within the Goulburn system. This Interconnector may increase activity in water markets previously constrained by the Barmah Choke.

The Goulburn irrigation system is modelled using REALM - REsource ALlocation Model (Diment, 1991), (Perera and James, 2003). The model determines allocations using a network linear program. This work introduces Mobi, a REALM stylised model, determining monthly water allocations and movement in a water network via flexible non-linear penalty minimisations. The model has as inputs and outputs:

## Inputs

- initial state of the system
- irrigational and urban demands
- preferred reservoir volume distributions
- system inflows
- evaporation data
- irrigational and urban restrictions data

## Outputs

- cumulative irrigation deliveries and revised allocations
- urban restrictions and deliveries
- system water movements for the month
- losses and spills
- updates the state of the system

To begin with the purpose of Mobi is to investigate the effects of different penalty functions and compare with those results obtained from REALM. A future aim is to represent these penalty functions as cost functions. A modelling benefit of this would be that cost is what is minimised regarding water movement in a time-step. Mobi will first be used to simulate the effects of the introduced infrastructure of the Food Bowl Modernisation Project.

**Keywords:** *Water allocation, network optimisation, REALM, scenario assessment*

# 1. INTRODUCTION

Climate change is now a global phenomenon, pressuring communities into adapting to new sustainable standards of living. Victoria is expected to become warmer and drier, with decreasing and more variable rainfall (DSE, 2007). This means reservoirs will receive less inflow and there will be less flow in rivers. Figure 1, generated by Goulburn-Murray Water (<http://www.g-mwater.com.au/water-resources/storage-levels>), shows storage levels of Lake Eildon for selected years. A downward trend in storage levels is evident.

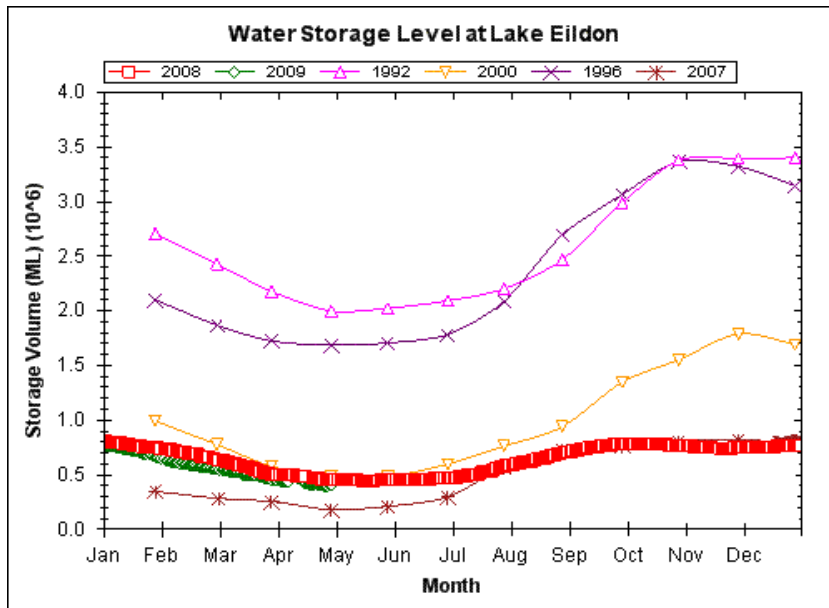


Figure 1. Historical storages for Lake Eildon (Goulburn-Murray Water)

The Victorian Government commissioned the *Our Water Our Future Water Plan* in 2004, with stage two, \$4.9 billion the *Next Stage of the Government's Water Plan* released in 2007. The plan includes the Food Bowl Modernisation Project and the Sugarloaf Interconnector which will influence the Goulburn water network. Feasibility of a Murray-Goulburn Interconnector has also been discussed providing faster flows between the two rivers. Figure 2 (DSE, 2007) shows these proposed developments.



Figure 2. New/proposed infrastructure as part of *Our Water Our Future* (DSE, 2007).

In Victoria irrigation uses 77% of harvested water generating \$9 billion in production annually and \$1.53 billion in exports. Thus irrigated agriculture has a large share in the local economy. It is therefore imperative the resource is efficiently managed for all concerned.

Traditionally the REALM model is used in Victoria, South Australia and Western Australia and determines the state of a system in time-steps via a network linear optimisation. IQQM (Simons et al., 1996) is used in NSW and Queensland, operates on a daily time-step and determines states via operating rules rather than optimisation. The different modelling approaches adopted for the southern and northern water supply systems have their justification in significant differences in the physical system characteristics and water management priorities (CRC, 2004).

Mobi is REALM stylised in the sense that the state of the system at any time-step is determined by minimisation of penalties. Is is an aim investigate different penalty functions on the network. The authors incorporate the concept of target storage volumes and the methods for determining irrigation allocations and urban restrictions from REALM in the skeleton for Mobi. The Goulburn Simulation Model (GSM) (CRC, 2004) has been provided by the Department of Sustainability and Environment (DSE), data from which can be used to simulate the effects of new infrastructure.

## 2. WATER ALLOCATION MODEL - MOBI

Defining the physical elements and sets of the model, let

$$\begin{aligned}
\mathcal{R} &= \text{the set of reservoirs} & i \in \mathcal{R} &\Leftrightarrow i \text{ is a reservoir} \\
\mathcal{D} &= \text{the set of demand centres} & i \in \mathcal{D} &\Leftrightarrow i \text{ is a demand centre} \\
N_{ij} &= \text{the number of carriers from } i \text{ to } j \\
\mathcal{C} &= \text{the set of carriers} & (i, j, n) \in \mathcal{C} &\Leftrightarrow (i, j, n) \text{ is the } n\text{-th carrier from } i \text{ to } j \\
& & & n \in \{1, \dots, N_{ij}\}
\end{aligned}$$

Defining the variables of the model, let,

$$\begin{aligned}
x_{[i,j,n,t]} &= \text{the flow along } n\text{-th carrier from node } i \text{ to node } j \text{ during time-step } t \\
d_{[i,t]} &= \text{the volume of shortfall in demand for demand centre node } i \text{ in time-step } t \\
v_{[i,t]} &= \text{the volume in reservoir node } i \text{ in time-step } t
\end{aligned} \tag{1}$$

Finally defining the penalty functions, let

$$\begin{aligned}
c_{[i,j,n,t]}(x_{[i,j,n,t]}) &= \text{penalty for flow } x_{[i,j,n,t]} \text{ along carrier } (i, j, n) \text{ at time } t \\
s_{[i,t]}(d_{[i,t]}) &= \text{penalty for (restricted) demand shortfall } d_{[i,t]} \text{ in demand centre node } i \text{ at time } t \\
r_{[i,t]}(v_{[i,t]}) &= \text{penalty for end reservoir volume } v_{[i,t]} \text{ in reservoir node } i \text{ at time } t
\end{aligned} \tag{2}$$

So streamflows, shortfalls and reservoir volumes are chosen  $x_{[i,j,n,t]}$ ,  $d_{[i,t]}$ ,  $v_{[i,t]}$  so as to minimise the following,

$$\sum_{(i,j,n) \in \mathcal{C}} c_{[i,j,n,t]}(x_{[i,j,n,t]}) + \sum_{i \in \mathcal{D}} s_{[i,t]}(d_{[i,t]}) + \sum_{i \in \mathcal{R}} r_{[i,t]}(v_{[i,t]}) \tag{3}$$

Based on objectives of the system, the decreasing severity of penalties is for failure to

- Environment: satisfy environmental flow requirements
- Consumer reliability: satisfy (restricted) demands
- Reservoir preferences: achieve given system water distribution preference across reservoirs

These are subject to domain constraints,

$$\begin{aligned}
\forall (i, j, n) \in \mathcal{C}, \quad 0 &\leq x_{[i,j,n,t]} \leq C_{[i,j,n,t]} \\
\forall i \in \mathcal{D}, \quad 0 &\leq d_{[i,t]} \leq D_{[i,t]} \\
\forall i \in \mathcal{R}, \quad 0 &\leq v_{[i,t]}
\end{aligned} \tag{4}$$

Where exogenous to the time-step parameters,

$$\begin{aligned}
C_{[i,j,n,t]} &= \text{maximum capacity of carrier } (i, j, n) \text{ in time-step } t \\
D_{[i,t]} &= \text{(restricted) demand for demand centre node } i \text{ in time-step } t
\end{aligned} \tag{5}$$

There are also water balance conditions at all nodes that must be satisfied. Firstly for reservoir nodes, time-step evaporation must be determined. Define  $W_{[i]}^{max}$  as the maximum volume of reservoir node  $i$ . Then,

$$\begin{aligned} v_{[i,t-1]} < W_{[i]}^{max} &\Rightarrow W_{[i,t]} = v_{[i,t-1]} \\ v_{[i,t-1]} \geq W_{[i]}^{max} &\Rightarrow W_{[i,t]} = W_{[i]}^{max} \end{aligned} \quad (6)$$

gives start volume  $W_{[i,t]}$  for reservoir  $i$  in time-step  $t$ . Given start volume, the surface area of each reservoir, then resultant expected evaporation can be determined. Define,

$$\begin{aligned} E_{[i,t]} &= \text{system evaporation to reservoir } i \text{ in time-step } t \\ I_{[i,t]} &= \text{system inflow to reservoir } i \text{ in time-step } t \\ \lambda_{[i,j,n]} &= \text{percentage of flow lost in transmission from carrier } (i, j, n) \end{aligned}$$

Then for each reservoir  $i$  water balance implies,

$$v_{[i,t]} = W_{[i,t]} - E_{[i,t]} + I_{[i,t]} - \sum_{j,n|(i,j,n) \in \mathcal{C}} x_{[i,j,n,t]} + \sum_{j,n|(j,i,n) \in \mathcal{C}} (1 - \lambda_{[j,i,n]}) x_{[j,i,n,t]} \quad (7)$$

In words (7) states,

end volume (inc. spills) = start volume - evaporation + system inflows - carrier outflows + carrier inflows

Secondly the water balance conditions for demand nodes are,

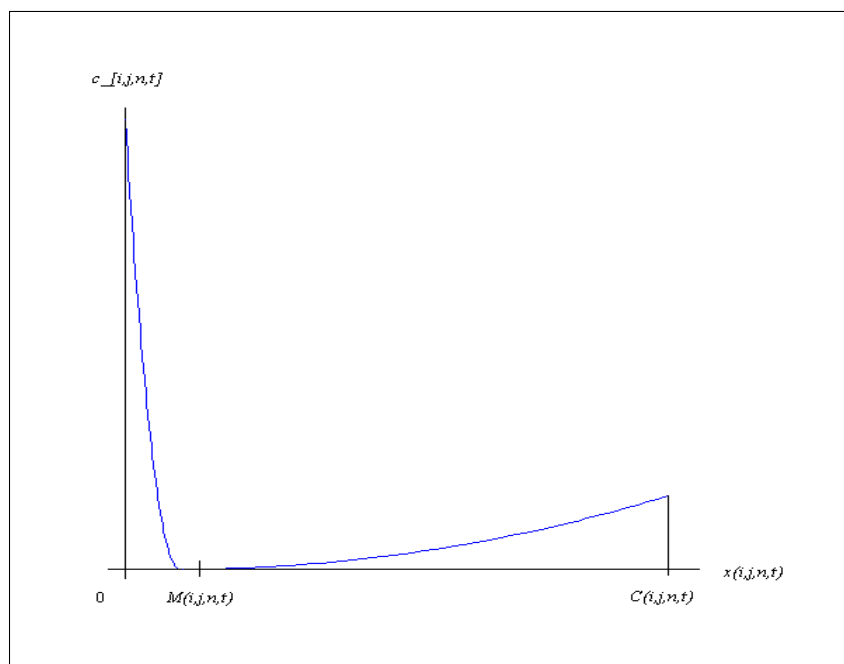
$$\sum_{j,n|(j,i,n) \in \mathcal{C}} (1 - \lambda_{[j,i,n]}) x_{[j,i,n,t]} + d_{[i,t]} = D_{[i,t]} \quad (8)$$

In words (8) states,

carrier inflows + demand shortfall = (restricted) demand

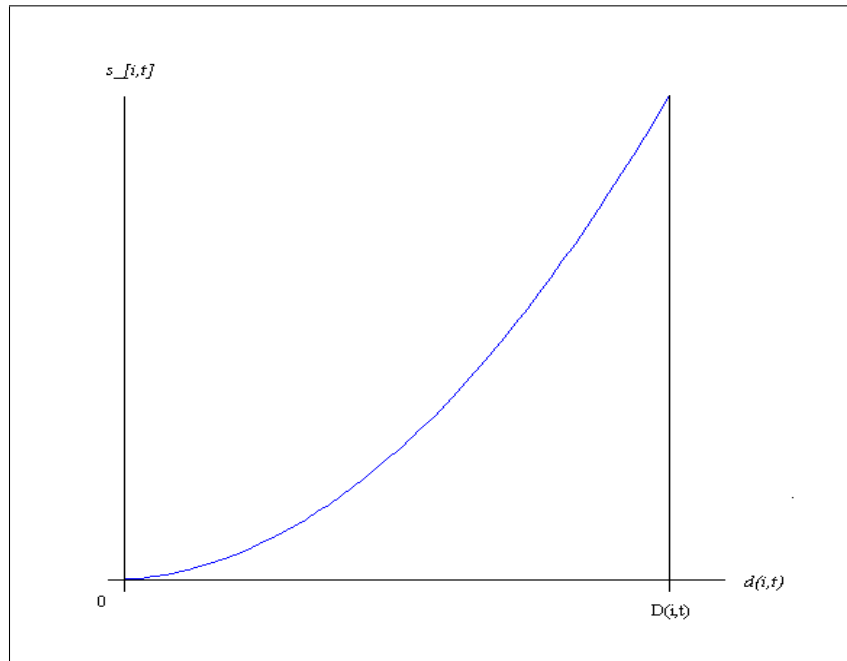
## 2.1. Penalty Functions

Penalty functions are  $c_{[i,j,n,t]}$ ,  $s_{[i,t]}$  and  $r_{[i,t]}$  for carriers, demand centres and reservoirs respectively. It is envisioned that Mobi uses polynomial penalty functions, however exponential functions may be explored. Figures 3,4 and 5 shows the conceptual style of the penalty functions. With regards to carrier penalties in Figure 3 it can be seen that minimum penalty occurs when environmental or minimum flow requirement  $M_{[i,j,n,t]}$  is met. The maximum penalty occurs when there is no stream flow.

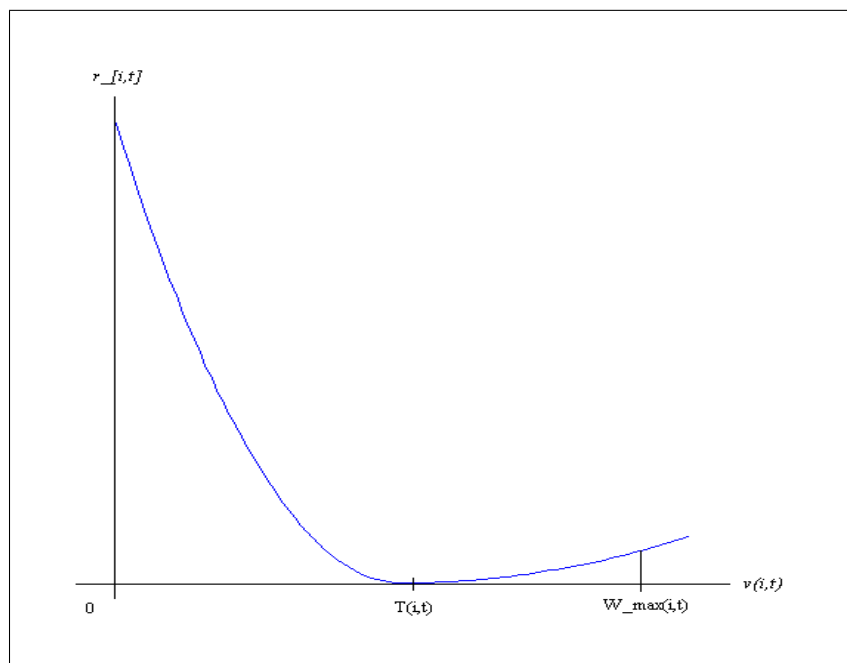


**Figure 3.** Carrier penalty function (equations (1) and (2)).

Regarding demand penalties in Figure 4 it can be seen there is no penalty if (restricted) demands are met and maximum penalty if none are met. From Figure 5 maximum penalty is when the reservoir is empty and the minimum penalty is when reservoir volume is equal to what is defined in REALM as the target storage volume,  $T_{[i,t]}$ . Reservoirs within a system can be separated into target groups or stand alone. Given the total end system storage for any reservoir group there may be a defined preference for the distribution of this water across reservoirs. This preference volume is the target storage volume  $T_{[i,t]}$ . Within the Goulburn Simulation Model there are seven target groups.



**Figure 4.** Demand centre shortfall penalty function (equations (1) and (2)).



**Figure 5.** Reservoir penalty function (equations (1) and (2)).

## 2.2. Time-Step Operations

In order to minimise (3) the individual penalty functions for carriers, demand centres and reservoirs must be determined. Exogenous data for carriers is read in and capacity is determined, then the penalty function is constructed. For demand centres the demand (possibly restricted) must be computed then the penalty

function constructed. Finally for reservoirs the target storage must be computed then penalty function constructed.

Demand restrictions for both irrigators and urban demand centres are implemented based on the total end system storage. At the beginning of the time-step this end system storage is unknown but is estimated by adding at the start storage and system inflows and subtracting from that evaporation and unrestricted demand. The estimate determines whether demand restrictions should be implemented and the nature of these restrictions. If demand is restricted then end system storage is re-estimated as above. The estimated end system storage is then used to determine target storages. Penalty functions are constructed and (3) is minimised. Following this simulation run an improved estimate for the end system storage is found by adding the end volume of reservoirs. Target storages are again computed, penalty functions are constructed and optimised. This procedure will continue in the time-step until the end system storage (including spills) converges. Then reservoir volumes, flows, deliveries, etc. are stored and the simulation of the next time-step may begin.

### 3. GOULBURN WATER NETWORK

The GSM was created in REALM and is used to simulate the major water supply systems in the Goulburn, Broken, Campaspe and Loddon valleys. Around 2 GL is supplied to irrigators and urban demand centres within this system per year (Perera et al., 2003). Within GSM, 20 storages and 58 demand centres are represented. The model can be used to forecast likely sales for water authorities as well as give irrigators updates on the likelihood of various allocations. There is uncertainty in water allocations due to the uncertainty of the volume and timing of future inflows.

Figure 6 shows a picture of system file for GSM created in REALM and is displayed to indicate the complexity of the system. Triangles represent reservoirs, green grids represent irrigator demand centres and black grids represent urban demand centres. Blue lines represent river carriers and black lines represent pipe carriers.

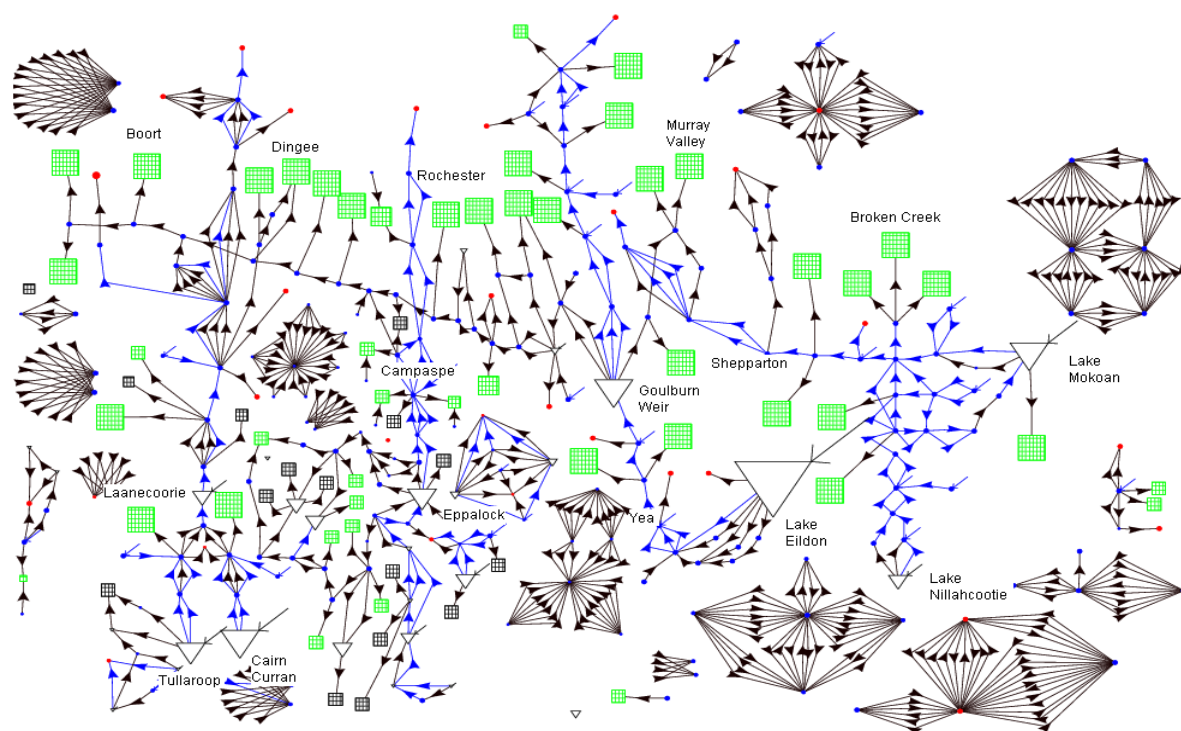


Figure 6. Goulburn Simulation Model - REALM.

Data from GSM, that is reservoir, demand and carrier information as well as historical system inflows, is used to construct and simulate the network with Mobi.

#### 4. DISCUSSION

The introduced REALM stylised model, Mobi, can be used to simulate networks in Victoria. In particular the model will be used to simulate the Goulburn, Broken, Campaspe and Loddon valleys' water network. It is the aim that various penalty functions be investigated which is not possible within REALM. Penalty functions of polynomial type and exponential type may be trialed. Furthermore rules governing irrigation allocations can be explored as well as urban restrictions.

After constructing the network with Mobi from information provided by the GSM, various scenarios can be implemented. DSE have provided system inflow and demand data from 1891 to 2007 for the GSM. Effects of reduced system inflows, increased demands and new infrastructure can be simulated.

The potential Murray-Goulburn Interconnector will also be modelled. Currently the Barmah Choke limits the flow of water between the Murray and Goulburn Rivers. This limits the volume of water that can be traded in the area. Dixon et al. (2008) discuss water trading in the Murray-Darling Basin using analysis from TERM-H20, a flexible regional computable general equilibrium model. During the drought of 2006/2007 with limited water trading results showed real GDP reduced by 1.45%. With water trading water is moved to higher value uses, results showing real GDP reduced by only 1.27%. This difference of around \$1.3 billion highlights the benefits of water trading.

#### ACKNOWLEDGEMENTS

The authors would like to thank Barry James from the Department of Sustainability and environment for access to and assistance with the GSM model. The authors also thank Peter Dixon and Glyn Wittwer from the Centre of Policy Studies, Monash University for economic insights, in particular, water trading.

#### REFERENCES

- CRC for Catchment Hydrology (2004), Sustainable water allocation, integration of water balance, climate and economic models. <http://www.catchment.crc.org.au/projects/projects9902/outcomes/3.1.pdf>
- Department of Sustainability and Environment (2007), Our water our future: the next stage of the Government's water plan. <http://www.ourwater.vic.gov.au/>
- Diment, G.A. (1991), Wide use of a generalised headworks and resources model: REALM. *Proceedings of International Hydrology and Water Resources Symposium I.E. (Aust.)*, Perth, Australia, NCP No. 91/22, 579-583.
- Dixon, P.B., T. Rimmer, T.M. and Wittwer, G. (2008), The benefits of water trading and the effects of the 2006-07 drought in Australia, Paper presented at 36th annual Conference of Economists, Hobart, 24-26 September.
- Perera, B.J.C. and James, B. (2003), A generalised water supply simulation computer software package - REALM. *Journal of Hydrology*, 26(1-2) March-June 2003, 67-83.
- Perera, B.J.C., James, B. and Kularathna, M.D.U. (2003), Computer software tool for sustainable water allocation and management - REALM. *Proceedings of MODSIM 2003*, Townsville, Australia, 4:1627-1632.
- Simons, M., Podger, G. and Cooke, R. (1996), IQQMa hydrologic modelling tool for water resource and salinity management, *Environmental Software*, 11/1-3, 185-192.