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MORE and POMORE sensitivity analysis of salt interception schemes in the River Murray

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Abstract: The MSM-BIGMOD model of the River Murray is a comprehensive flow and salinity routing model, used to assess the impacts of potential changes in river management on river flow and salinity levels. The modelling suite consists of a combination of two models (MSM and BIGMOD) that have been developed over a period of years. Sensitivity analysis of the model is particularly important, given that decisions are made about management of the River Murray based on outputs from the model. The large number of model inputs and parameters arising from the inclusion of the many tributaries, storages, drains, and diversions pose a challenge for traditional sensitivity analysis methods, such as one-at-a-time parameter perturbation methods. The Management Option Rank Equivalence (MORE) method of sensitivity analysis and the expanded Pareto Optimal Management Option Rank Equivalence (POMORE) are innovative methods of sensitivity analysis developed especially for use with complex models used for decision-making. The methods assess the sensitivity of management decisions based on model output, to changes in the model inputs, in order to provide a sensitivity analysis in the decision context. MORE searches the parameter space to find parameter combinations that result in an equal preference of two management options that are closest in Euclidean distance to the calibrated model parameters. POMORE searches similarly for parameter combinations that result in an equal preference of two management options; however, it uses a Pareto optimal search to determine the combinations which are the most similar to the original calibrated parameters. The difference in the search criteria, allows POMORE to find several solutions, allowing further categorisation of sensitivity throughout the parameter space. A sensitivity analysis of the MSM-BIGMOD model of the River Murray using MORE and POMORE is presented in this research. The analysis investigates the sensitivity of the decision to improve a salt interception scheme (SIS) based on the net present value of the savings due to a reduction in the salinity of water used for irrigation, domestic and industrial use, to changes in the cost parameters, the crop yield reduction parameters, and the salt removal parameters. The model is found to be reasonably robust, with a change of 37.5% of the maximum possible change in parameters required in order to alter the decision. However the sensitivity varies throughout the parameter space, indicated by a further 30.2% change of the maximum possible, required to ensure that the decision will change. This variation in sensitivity throughout the parameter space, has shown that there is a need for further sensitivity analysis, and as such POMORE is used to gain additional information about other sensitive regions. This research demonstrates a comprehensive use of the MORE and POMORE methods of sensitivity analysis on a case study which poses considerable problems for traditional sensitivity analysis methods. The results obtained in the case study demonstrate the value of the MORE and POMORE approaches and the extremely useful information that they provide to decision-makers.

Keywords: *MORE, POMORE, sensitivity analysis, multi-objective optimization, decision-making, genetic algorithm, pareto dominance*

1. INTRODUCTION

Integrated assessment models (IAM) used to assist with decision-making are often highly complex systems, with multiple sources of uncertainty. Many parameters may be uncertain due to difficulties with data collection, natural variability, or poor system understanding. At the same time, decisions made based on model output are likely to have considerable financial or environmental impact. Sensitivity analysis provides a means of understanding how changes to model inputs and parameters, which may occur due to the uncertainty surrounding those parameters, will affect the model output. By determining which parameters the model output is most sensitive to, future research can be more appropriately directed towards determination of those parameters.

Increased computing power in recent years has seen the increase in use of computer simulation models for environmental decision making, as well as rapid increases in their size and complexity. Integration of different models to determine outcomes in line with sustainability criteria further increases model size. Along with this, there is generally an increase in non-linearities and non-monotonicity within the model, as well as the presence of feedback loops, particularly in IAM. Thus model outputs may not be intuitive, creating a further need for reliable and informative sensitivity analysis.

There are currently several different methods of sensitivity analysis in use. Well established techniques, such as Fourier Amplitude Sensitivity Testing (FAST) (Cukier et al. 1978; Saltelli et al. 1998), and the method of Sobol' (Sobol' 1993, 2001), use analysis of variance to assess the contribution of the variance in a single parameter to the variance in the output, while methods such as Morris one-at-a-time factor screening (Morris 1991) or the Improved Elementary Effects method (Campolongo et al. 2007), attempt to identify unimportant parameters such that further analysis can be simplified.

The management option rank equivalence (MORE) (Ravalico et al. submitted) and Pareto optimal management option rank equivalence (POMORE) (Ravalico et al. 2009) methods of sensitivity analysis are new methods that have been developed specifically for use with IAMs used in decision-making. The methods take advantage of evolutionary optimization algorithms to locate the most sensitive areas of the parameter space, informing decision-makers of parameter combinations that may cause a change in the recommended management options, based on model output. The methods allow for model non-linearity and non-monotonicity, as well as providing an efficient search through the use of evolutionary algorithms, which have been shown to outperform traditional mathematical approaches for complex problems (Elbeltagi et al. 2005)

The MSM-BIGMOD modelling suite is a comprehensive flow and salinity model of the River Murray in South-Eastern Australia. The modeling suite is used to predict the effect of changes to river management on flow and salinity at various locations along the river. The modelling suite also comprises an economic component where the cost of increases or decreases in salinity is calculated based on crop yield reduction due to saline irrigation and increased costs associated with corrosion of water infrastructure due to salinity. The size of the model, as well as its integrated nature, make it an ideal test for the MORE and POMORE methods of sensitivity analysis. This research aims to demonstrate the efficacy of the MORE and POMORE methods of sensitivity analysis through application of the methods to the case study of the MSM-BIGMOD modelling suite.

1.1. MORE Overview

The MORE method of sensitivity analysis (Ravalico et al. submitted) can be considered a break even method of sensitivity analysis (Winterfeldt et al. 1986). The method is used for comparing decisions based on model output, and determining whether the decision that is being based on model output can be accurately made, given current parameter uncertainties. In a situation where management options are ranked, the parameter space is searched for the parameter combinations that cause equal ranking of different management options. These parameter combinations form a Rank-Equivalence Boundary, which is then searched for the combination which is closest in parameter space to the original or calibrated parameter vector used in the original assessment of the management options. This search can be a considerable task, particularly in the case where there are multiple parameters under question, increasing the dimensionality of the search space.

The search can be considered as an optimization problem, where the distance from the original parameter vector, $\mathbf{x}_A$, to the surface where two management options give the same output, $\mathbf{x}_B$, needs to be minimized. In the case of large and complicated search spaces, evolutionary algorithms have been shown to outperform traditional mathematical optimization techniques, (Elbeltagi et al. 2005) and as such a genetic algorithm (GA) (Goldberg 1989) is used for the search. The distance between the minimum point and the original calibrated parameters, D_{min}, can then be used as the radius of a hyperspherical area, *C*, within which there is certainty that the preferred management option will not change.

Finding the radius D_{min},, gives both a means of determining an area of parameter space where the preferred management option will not alter, as well an indication of the region of parameter space where the model is most sensitive to parameter changes. However, it gives little information about the sensitivity of other regions within parameter space. In order to determine whether the sensitivity changes considerably throughout parameter space, an optimization to find the point on the REB, $\mathbf{x}_C$, that is the maximum distance from the calibrated model parameters, $\mathbf{x}_A$, and the corresponding distance between them, D_{max}, is performed. This then allows categorization of the parameter space into three regions, the hyperspherical region, *C*, a region, *U*, which is bounded by *C*, and the hypersphere created with centre $\mathbf{x}_A$ and radius D_{max}, where we are uncertain whether the preferred management option will alter or not, and the remaining parameter space, *S*, where we are certain that the preferred management option will change. By calculating the relative volumes of these spheres it is possible to determine whether the choice of management option is sensitive to changes in the parameters, and whether the sensitivity changes considerably over the parameter space. Further distance measures rD_{min} and rD_{max} can be calculated by dividing D_{min} and D_{max} by the maximum distance from the calibrated parameter vector to the boundary of parameter space. The volume measures only provide useful information for problems with a small number of dimensions, due to the geometric properties of the hypersphere in high dimensions, however, in the case of high dimensionality, the distance measures D_{min}, D_{max}, rD_{min} and rD_{max} can be used to assess both the sensitivity and the variation in sensitivity.

1.2. POMORE Overview

In the case that the sensitivity changes considerably through the parameter space, indicated by a large volume of *U*, or a large value of $D_{max} – D_{min}$, it is important to be able to gain an overview of what may be several different locations in parameter space where there is high sensitivity to changes in model parameters. POMORE (Ravalico et al. 2009) uses a multi-objective Pareto optimization, rather than the single objective optimization used in MORE, in order to achieve this. A solution **a** is considered to be Pareto dominant over another solution **b**, if it is at least equal to **b** in all objectives, and better than **b** in at least one objective. Thus Pareto optimization finds a set of optimal solutions through a dominance relationship, where multiple potentially conflicting objectives are optimized simultaneously (Ringuest 1992). In this case we can consider the multiple objectives as the changes in the different parameters, and hence by minimizing all of these simultaneously, we can generate an approximate Pareto front, consisting of non-dominated Pareto solutions. The Pareto front can be constrained to the REB, such that it consists of several points along the REB that are minimal in their change from the original model parameters.

Analysis of the Pareto optimal solutions enables the sensitivities of each of the parameters to be determined in conjunction with the smallest possible changes in all of the other parameters. Solutions corresponding to variation in a single parameter indicate the individual changes that are required to reach the REB. However, if there is joint parameter variation, the amount a given parameter has to change to reach the REB may be reduced or increased, depending on the joint effects of the parameter changes on the model output and the shape of the REB. By examining the statistical properties of the changes that are required to be made to a particular parameter to reach the REB, a true indication of the sensitivity of the decision to this parameter can be made, as sensitivity is considered in different directions of the parameter space and in conjunction with the minimal joint changes in all of the other parameters. In order to obtain this information for each parameter, only Pareto optimal solutions for which the parameter has the largest contribution are considered.

The smallest changes in a parameter to reach the REB indicate higher sensitivity of the decision to that parameter. Therefore, when considering the range of maximum parameter changes in minimum solutions, the parameter that has consistently smaller changes can be considered more sensitive than parameters with consistently larger changes. The median parameter change can be used to assess the sensitivity of the decision to that parameter. The range of variation of a parameter also gives important information regarding the sensitivity of the decision. Where the range of a parameter varies greatly over the Pareto solutions for which it provides the greatest contribution, the sensitivity to that parameter can be seen to vary in different directions of parameter space, even through the set of minimal solutions. The most critical parameters will be those with a small range of variation in combination with a small median change.

2. CASE STUDY: UPGRADE OF SALT INTERCEPTION SCHEME IN THE LOWER RIVER MURRAY

2.1. Model Outline

The MSM-BIGMOD modelling suite is a comprehensive flow and salinity model of the River Murray in South-Eastern Australia. Beginning with the inflows from Dartmouth Dam, the model incorporates tributaries, storages, weirs, irrigation and urban diversions, salt interception schemes, drainage diversions, wetlands and flood runners (MDBC 2002).

The modelling suite is a combination of five models: MSM, a monthly simulation model that computes irrigation demands, resources assessment and water accounting, MODFLW, which converts monthly values computed in MSM into daily input files for use in BIGMOD, GETDVM, which creates monthly inputs from MSM for BIGMOD, BIGMOD, which is a daily flow and salinity routing model from Hume Dam to Lake Alexandrina, and BIGARKW, which is used to analyse the results of MSM and BIGMOD. Of these, BIGMOD and MSM are the key calculation models, and can be run separately or sequentially using outputs from MSM as inputs to BIGMOD (MDBC 2002).

Changes to the Murray River are assessed by running the model over a benchmark period from 1891 to 2000 for flow modelling and 1975-2000 for salinity modelling, under current conditions, as well as under the proposed conditions. The flow and salinity outputs from the two different runs are compared to assess the impact that the proposed changes would have on the current condition of the river over a considerable period of time.

Proposed changes to the management of the river, particularly in relation to addressing high salinity problems, are also assessed on the basis of net-present value, with a cost in dollars per EC unit per year, based on data from Allen (2004) and outputs from the BIGMOD modelling suite. The reduction or increase in EC can then be equated to a financial cost, such that salinity management options can be implemented that maximise this value.

The large number of inputs and parameters in the modelling suite, and their potential interactions, prohibits standard one-at-a-time parameter variation as a method of sensitivity analysis. Further, use of the model in decision-making and the importance of the decisions made, make MORE and POMORE sensitivity analysis an ideal option. The modelling suite also provides a perfect test case for use of MORE and POMORE sensitivity in integrated modelling, since it combines several individual models with different purposes in order to create an overall output.

2.2. Analyses Conducted

The management decision that is the subject of this sensitivity analysis case study is the choice between upgrading the salt interception scheme (SIS) at Waikerie to increase annual salt removal by 20% (MO1) and maintaining the current SIS without upgrade (MO2). All other SIS schemes within the river are modelled at their current operating level. The choice of management option is based on the net present value (NPV) of the upgrade. Using the calibrated model parameters the NPV of MO1 is found to be $4,609,470, while the NPV of MO2 is found to be $3,520,640. Hence, with the calibrated model parameters, MO1, improving the SIS would be the preferred management option.

Three different groups of parameters were selected for the analysis. The first consisted of parameters that determine the salt removal at various flow levels for three SIS schemes of interest, one located at Noora, one at Waikerie and the other at Woolpunda, the second group of parameters governing the domestic urban and industrial costs due to salinity, and the third group of parameters governing yield reduction due to saline irrigation. The SIS parameter ranges are set as as $[x_i- 50, x_i+50]$, where x_i is the calibrated value of the parameter. The parameters governing the domestic, urban and industrial costs have the range [0, 0.8]. There are two different ranges considered for the agricultural parameters. For each crop grown in the region the yield reduction formula with 2 parameters, a and b is used to determine the reduction in yield. Parameter a is the threshold salinity, and b is the relative yield decline of the crop. For each crop, the range of a is [0 , 10] and the range of b is [1, 100].

For this analysis, the model was run for the period from April 1974 to May 2001 and the reaches of the BIGMOD model considered were between Lock 5 and Lock 1.

2.2.1 MORE Options

As mentioned previously, a genetic algorithm (GA) was used to locate the minimum and maximum points on the REB, which is an evolutionary algorithm, based on Darwinian principles of survival of the fittest (Goldberg 1989). The objective function for the genetic algorithm is the Euclidean distance from $\mathbf{x}_A$ to $\mathbf{x}_B$ or $\mathbf{x}_C$, and the single constraint that the solution must lie on the REB.

The fitness of each chromosome in a generation of the GA is based first on the amount that the constraint is violated, with those chromosomes with the smallest violation considered the fittest. For chromosomes without constraint violation, the distance is evaluated, and those with either the minimum or maximum distance (depending on which search is being performed) from the calibrated model parameter vector are considered the fittest. The GA is real coded, and uses the crossover method described by Gibbs et al. (2005), as well as incorporating elitism and string wise mutation with a probability of mutation of 0.5.

In order to determine the most appropriate population size and number of generations, the GA was run with population sizes varying between 100 and 300, and a number of generations varying between 100 and 300 while the random number seed was held constant, to ensure that any improvement in results was due to the changes in the GA parameters and not the random number seed. A population size of 200 was shown to converge in less than 200 generations, and provide good results, so these parameters were used for the analysis.

2.2.2 POMORE Options

The POMORE method was implemented using NSGA II (Deb et al. 2002). NSGA II is a multi-objective genetic algorithm, which searches for Pareto dominant solutions, and is known for its computational efficiency. In order to remain consistent with the MORE analysis, the decision variables for the GA are real coded. The probability of mutation was set at 1/k, where k is the number of parameters, as recommended by Deb (2000). The population size was selected to return a reasonable number of Pareto optimal solutions, while enabling a thorough search of the parameter space. The other parameters were obtained through repeated runs of NSGAII with the same random number seed and different combinations of the parameters. The population for the GA was set at 200, and run over 200 generations.

3. RESULTS

3.1. MORE Results

The results of the MORE method are shown in Table 1. The results show quite a small rD_{min} value of 0.063, corresponding to a radius which takes up 6.3% of the possible maximum distance to the parameter space boundary. This indicates that the model is quite sensitive to changes in the model parameters. The rD_{max} value of 0.953 indicates that there is considerable variation in sensitivity throughout the parameter space, with some small changes in parameters causing a change in the preferred management, while some larger changes will not.

The individual changes in parameters for the minimization are shown in Figure 1, while the individual changes in parameters for the maximization runs are shown in Figure 2. To allow a simpler visual comparison of the changes, they have been normalized by the parameter ranges.

From the figures it can be seen that there is little relationship between the minimum changes and maximum changes in individual parameters, indicating that there is considerable parameter interaction. Given the variation in sensitivity throughout the parameter space indicated by the size of $rD_{man} - rD_{min,,}$ further analysis into the variation in sensitivity can be provided by POMORE if necessary.

Table 1. MORE volume and distance measure results

Distance Measures					
D_{min}	rD_{min}	D_{max}	rD_{max}	D_{max}-D_{min}	rD_{max}-rD_{min}
0.434	0.063	6.516	0.953	6.072	0.890

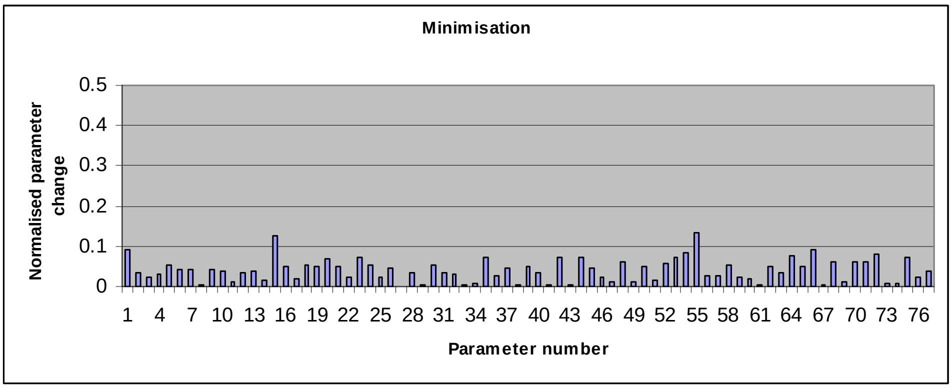

Figure 1. Individual parameter changes for minimisation.

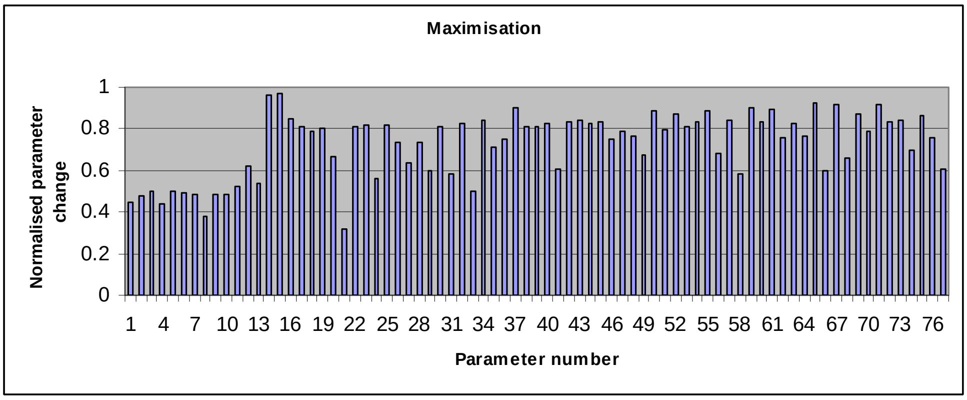

Figure 2 Individual parameter changes for maximization.

3.2. POMORE Results

Figures 3 is indicative of the results that can be obtained for each parameter, provided that its change from its original value is the largest of all the normalized parameter changes for at least four of the Pareto dominant solutions found. Parameter 14 is the parameter used to determine the cost of salinity in water cooling towers outside of Adelaide. Figure 3 focuses on the instance where parameter 14 is the parameter with the greatest change, and shows the minimum, maximum and median changes in the other parameter values, as well as the first and third quartile values. Since the change in parameter 14 is considerably higher than all the other parameters, this parameter can be considered to have a major contribution to the change in model output, which allows the other parameters to change less than they would have otherwise, hence making the model more sensitive to the other parameters. In this case however, there is still a considerable variation in parameters 15 and above, which may indicate that a smaller change in one parameter is being offset by a larger change in another parameter. Parameters 1-9, which are related to the SIS schemes, show less variation than the other parameters, indicating that the model is more sensitive to these than it is to other parameters.

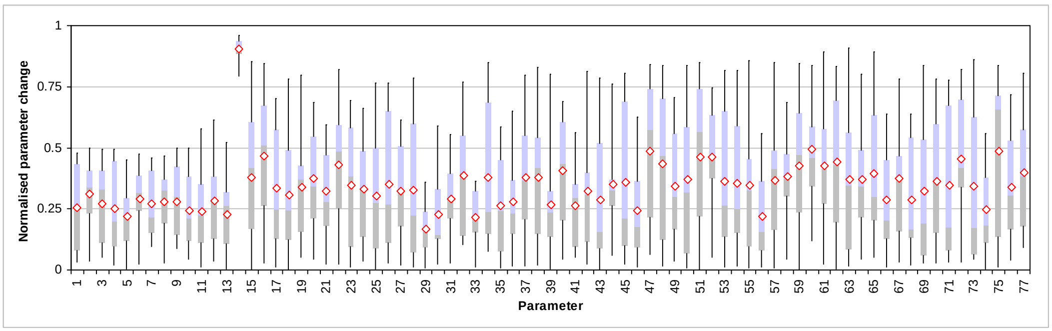

Figure 3. Variation of parameter changes where parameter 14 has the largest change.

4. CONCLUSIONS

The sensitivity analysis showed that the sensitivity of a 20% improvement in a single SIS scheme was not highly sensitive to changes in the 77 selected model parameters. There was however a reasonable variation in the sensitivity across the parameter space, shown by (rD_{max} – rD_{min}), which justified the application of the POMORE method, in order to further quantify the regions of interest on the REB, as well as the variation in parameter changes required to reach the REB. The results showed that for cases where the change in a particular parameter dominated, changes in other parameters varied considerably, with the exception of the parameters for the SIS (parameters 1-9) which had less variation than the remaining parameters.

The sensitivity analysis performed on the MSM-BIGMOD modeling suite demonstrates that the MORE and POMORE methods of sensitivity analysis are an effective method of analyzing the sensitivity of model output used to aid in decision-making.

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