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A Robust Scheme for Spatio-Temporal Inverse Modeling of Oil Reservoirs

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Abstract: Numerical models used nowadays in oil reservoir characterization are increasing in complexity, due in part to the progressive decline of the world oil reserves. There is a greater need to model the complex spatial heterogeneity and fluid flow in the subsurface. With more involved models we expect that, once properly calibrated, better forecasts and depletion strategies can be made. This calibration process requires in essence the solving of an inverse problem.

In the oil industry there are two main trends within model inversion. The first one, based on filtering, can be efficient from a computational point of view, but it relies heavily on the linearity of the models. In order to deal with nonlinear cases, not infrequent in oil reservoir characterization, a purely optimization approach can be adopted. The inversion problem is formulated as minimizing the mismatch function between observations and the output of the numerical models. These observations are functions of space and time. If the level of uncertainty in the data acquisition is known, this information can be included in the mismatch function. The optimal search is carried out by adjusting model parameters, typically one or more for each of the grid-points of the reservoir discretization. The model inversion optimization problem is of a large-scale nature, with a nonlinear and nonconvex objective function, that often involves time-expensive simulations. Additionally, this problem is generally ill-conditioned, because the number of degrees of freedom usually is larger than the number of observations available.

In this work we present a robust and fairly efficient methodology to deal with these difficulties in the framework of oil reservoir characterization. The ill-conditioned character of the optimal search can be attenuated in two ways. By principal component analysis (PCA) the optimization search space can be projected to a subspace of much smaller dimension, while keeping consistency with prior spatial geological features already known for the reservoir under study. The number of optimal solutions can be reduced further by increasing the diversity of the data observed. In this research we combine flow production measurements (localized around wells and of high temporal periodicity) with seismic data (spatially distributed and of lower temporal periodicity). This data integration methodology can be extended to any observable for which a numerical model is available.

The numerical models in the optimization process frequently consist of complex simulators, and therefore, invasive techniques to extract analytic derivative information are either not possible or prone to a time-consuming implementation. Besides, cross-disciplinary data integration in the model inversion makes this difficulty more evident. The drastic reduction in the number of optimization variables obtained by PCA allows the use of numerical derivatives of the cost function. Within a distributed computing framework these approximate derivatives can be calculated efficiently. In our scheme we also consider several derivative-free algorithms. These methods, besides being mathematically sound and amenable to being parallelized, have been observed to perform robustly when noise is present in the objective function.

In this paper we will present the spatio-temporal model inversion scheme described above and illustrate its application to oil reservoir description by means of examples extracted from realistic cases.

Keywords: *Model inversion, Time-Lapse Seismic Reservoir Monitoring, Principal Component Analysis*

1. INTRODUCTION

One of the challenges in modeling reservoirs is the highly heterogeneous spatial distribution of reservoir properties, including variations in porosity, permeability, and facies, all of which can impact the flow. Integrating data of different types and scales can help to address this challenge.

Production data are measurements of pressure and flow rate in producer and injection wells. Well locations are limited in number, but the measurements there are repeated frequently. Because these data are only at a few locations, inverting for the model parameters can be a highly non-unique problem. Seismic data, on the other hand, has large spatial coverage and is one of the key technologies for characterizing reservoirs. Quantitative seismic interpretation involves using rock physics to predict reservoir parameters, such as lithologies, porosities, and pore fluids, from seismic attributes. While seismic measurements are available over a much larger spatial extent, they are repeated much less often than production data.

Combining flow measurements (localized around wells and of high temporal periodicity) with seismic data (spatially distributed and of lower temporal periodicity) helps to better constrain the inversion problem. Considerable literature exists on estimation of reservoir parameters based on only one kind of observable, typically either production history or seismic data. A very efficient methodology for assimilating production data that keeps geological consistency in the solution is presented in Sarma et al. (2006). However, that procedure is invasive with respect to the flow simulator (and thus not straightforward to implement) and is not robust with respect to being trapped in one of the multiple solutions. The approach in Maschio et al. (2008) is not invasive for the production simulation and, because of distributed computing and a parameter reduction, is fairly efficient. But this parameter reduction, unlike in Sarma et al. (2006), is not performed systematically. Integration of disparate data has been suggested in a number of publications (Huang et al., 1997, and Aanonsen et al., 2003). Though these clearly identify data integration as a means for better constraining the problem, they do not provide a clear and precise optimization methodology.

This paper aims at presenting a robust, general and mathematically sound methodology for inverse modeling. We stress that this scheme is easy to implement and, if distributed computing resources are available, it can be fairly efficient. The integration of data of distinct nature provides the approach with robustness with respect to the natural ill-conditioning of the inversion problem. The parameter reduction step proposed accelerates the whole procedure and endows it with geological consistency. Though here applied to the estimation of oil reservoir parameters, the methodology can be easily adapted for other inversion problems.

2. METHODOLOGY: FORMULATION OF THE PROBLEM AND APPROACHES

We formulate the model inversion problem from an optimization point of view. A popular alternative approach by filtering techniques (Evensen, 2003) relies heavily on linearity assumptions that are not present explicitly in a formulation based on nonlinear optimization. This means that we are able to characterize the solution as satisfying the first order necessary conditions for local optimality.

We refer to the model (inversion parameters) by $\mathbf{m} \in \mathbf{M} \subset \mathbf{R}^n$, and $\mathbf{M}$ is the set of admissible models. The admissibility criteria can be formulated in terms of geological consistency, for example. The model $\mathbf{m}$ can be a physical property (permeability and/or porosity) or an indicator (facies) associated with every grid block. Thus, n is typically at least a few thousands. The optimization problem is stated as follows[1]

$$\mathbf{m}^* = \arg\min_{\mathbf{m}\in M} \| \mathbf{O}(\mathbf{m}) - \mathbf{O}_\mathbf{m} \|^2, \tag{1}$$

where $\mathbf{O}_\mathbf{m} \in \mathbf{R}^m$ comprises all the observables in the inversion, and $\mathbf{O}(\mathbf{m}) \in \mathbf{R}^m$ represents these same observables computed numerically for $\mathbf{m}$. In the norm (Euclidean in this work) we can take into account uncertainty in the data acquisition, and include weights/normalization for different components in the observables. Usually we have a much larger number of inversion parameters than of measurements ($n >> m$), and therefore the optimization problem in (1) is ill-conditioned. This work proposes different ways to reduce the number of optimal solutions.

Firstly, if several observables of disparate nature are considered, e.g. $\mathbf{O}(\mathbf{m}) = [\mathbf{O}_1(\mathbf{m}), \mathbf{O}_2(\mathbf{m})]$ with $\mathbf{O}_1(\mathbf{m}) \subset \mathbf{R}^{m_1}$, $\mathbf{O}_2(\mathbf{m}) \subset \mathbf{R}^{m_2}$ and $m_1 + m_2 = m$, then the ill-conditioned character of (1) is alleviated, because one observable acts as a regularization term for the inversion of the rest. Obviously, the observables

[1] We use the notation arg min for indicating the optimizer of the corresponding cost function. This optimizer is usually further denoted by a star super index.

should not be functions of each other. Such observables can be obtained by considering physical quantities intrinsically related to variations in space and time, respectively. Examples in reservoir model inversion are crosswell tomograms and flow production data.

Secondly, we can expect a better conditioned optimization problem if the number of parameters is decreased. Instead of searching in n dimensions, we consider a subspace of dimension $n_R < m$. This subspace selection is not arbitrary and essentially aims at reducing, by Principal Component Analysis, the correlation between inversion parameters. It can also be interpreted, from a data compression perspective, as the Karhunen-Loève Transform. The statistical information needed is obtained from a prior knowledge of the reservoir, and provides the inversion with geological consistency.

Local inversion schemes are more efficient than global ones; however, the solution from a local optimizer depends strongly on the initial guess (Nocedal and Wright, 2000). In multimodal problems, as is the case for inverse modeling, local optimization has to be used with care. As a third way to alleviate the impact of multiple optima, we present a relatively inexpensive strategy for obtaining a good initial guess. This procedure, thanks to the earlier mentioned parameter reduction, is based on solving approximately a few simple one-variable optimization problems. This computation can be seen as a global exploration stage that increases robustness with respect to local optima.

Every simulator related to the observables is considered as a black box. The motivation for this approach is to have ease of implementation and extension. Though some simulator-invasive techniques, such as adjoint sensitivity analysis (Cao et al., 2003), can accelerate significantly the search, their application is often not at all trivial (incorporation of new observables) or even not possible (source code unavailability). Optimization schemes using black-box simulators (in absence of parameter reduction) have a computational cost per iteration of roughly n observable simulation runs (e.g., gradient-based optimizers with numerical derivatives). This cost can be prohibitive with large n and complex simulators. By reducing the number of parameters to n_R and distributing simulations in a cluster with a number of nodes comparable to n_R, we can obtain a fairly efficient methodology.

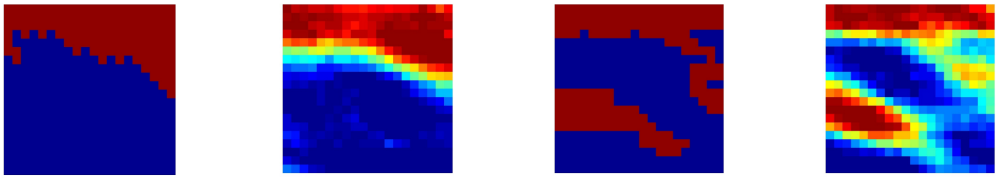
3. A ROBUST SCHEME FOR INVERSE MODELING

This section describes in more detail the three ways suggested above for coping with multimodal oil reservoir inversion problems. The model **m** to estimate is a facies indicator for every grid block in the reservoir.

3.1. Observables of disparate nature

The first observable $\mathbf{O}_1(\mathbf{m})$ refers to flow data, in particular the total water injection and oil production cumulative rates during 90 days of reservoir exploitation, resampled every 10 days (therefore, $m_1 = 10 + 10 = 20$). The production data is computed by solving the discretized reservoir flow equations for fluids in porous media (Aziz and Settari, 1979). For obtaining this observable we use Stanford's General Purpose Research Simulator (GPRS), but due to the general black-box approach, any other software could be employed. In our case, the unknown parameters in these equations are the permeability and porosity fields. Given a facies distribution, we compute the associated porosity field by regression with respect to well data. We assume that porosity and permeability are related by the Kozeny-Carman equation (Mavko et al., 1998) with parameters regressed against well data.

The second observable $\mathbf{O}_2(\mathbf{m})$ is associated with seismic tomography. Seismic tomography estimates subsurface properties (seismic velocity or attenuation) by analyzing elastic wavefield propagation from sources to receivers. Hence, tomographic reconstruction is itself an inversion. In crosswell tomography, sources and receivers are positioned at two wells. We consider two classes of tomography: diffraction and traveltime tomography. In diffraction tomography (Stewart, 1991) scattered wavefields are inverted to estimate seismic velocities. In our case we have two crosswell regions, each a 20x20 matrix of velocities (hence, $m_2 = 400 + 400 = 800$). Traveltime tomography is based on only the arrival times between source-receiver pairs, and velocities are then inverted from these times. Our observables are the traveltimes themselves, without inversion. This accelerates the computation, and also can reduce the ill-conditioning. In our case, the observable comprises two matrices corresponding to the traveltimes between two sets of 10 sources and 10 receivers. The entry (i,j) in each matrix is the traveltime from source i to receiver j. Therefore, for traveltime tomography $m_2 = 100 + 100 = 200$. The input for the seismic tomography simulator is obtained from the model **m**, the associated porosity, and the production observable, together with rock physics relations between porosity, saturations, and elastic velocities (Mavko et al., 1998).



(a) (b) (c) (d)

Figure 1. Layer 4 for the (a) true original, (b) true reconstructed, (c) random from $\{\mathbf{m}_k\}_{k=1}^N$ original, and (d) random from $\{\mathbf{m}_k\}_{k=1}^N$ reconstructed models. The original facies model is binary valued but in the PCA appears with continuous values.

3.2. Parameter reduction

The reduction in the number of parameters from n to $n_R << n$ (with $n_R < m$) is motivated by the need to alleviate the problem of multiple solutions, as well as reduce the overall computational cost. Principal Component Analysis (PCA) optimally selects a subspace from a larger space. Given N possible models $\{\mathbf{m}_k\}_{k=1}^N \subset \mathbf{M} \subset \mathbf{R}^n$, PCA looks for an affine transformation $\hat{\mathbf{m}}_k = \sum_{i=1}^{n_R} \left(\mathbf{s}_i^T (\mathbf{m}_k - \boldsymbol{\mu})\right)\mathbf{s}_i + \boldsymbol{\mu}$, where $\boldsymbol{\mu} \in \mathbf{R}^n$ and the set $\{\mathbf{s}_i\}_{i=1}^{n_R} \subset \mathbf{R}^n$ is orthonormal, that is optimal in the sense that the average Euclidean reconstruction error $\frac{1}{N}\sum_{k=1}^N \|\mathbf{m}_k - \hat{\mathbf{m}}_k\|_2^2$ is minimized. Equivalently, the average reconstruction energy $\frac{1}{N}\sum_{k=1}^N \hat{\mathbf{m}}_k^T \hat{\mathbf{m}}_k$ is maximized. The optimal solution (Miranda et al., 2008) implies that $\boldsymbol{\mu} = \sum_{k=1}^N \mathbf{m}_k$ and also that $\mathbf{C}\,\mathbf{s}_i = \lambda_i \mathbf{s}_i$, with

$$\mathbf{C} = \frac{1}{N}\sum_{k=1}^N (\mathbf{m}_k - \boldsymbol{\mu})(\mathbf{m}_k - \boldsymbol{\mu})^T, \tag{2}$$

a symmetric, positive semidefinite matrix. The average reconstruction energy for this transformation satisfies

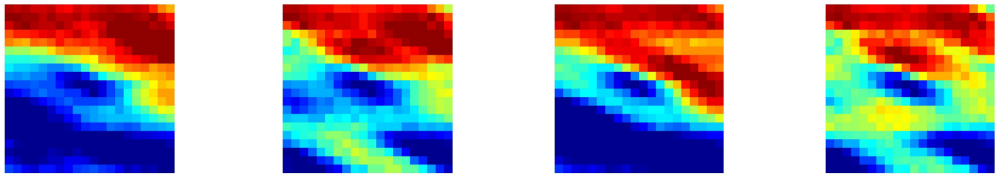
$$\frac{1}{N}\sum_{k=1}^N \hat{\mathbf{m}}_k^T \hat{\mathbf{m}}_k = \frac{1}{N}\sum_{k=1}^N \boldsymbol{\mu}^T \boldsymbol{\mu} + \sum_{i=1}^{n_R} \lambda_i . \tag{3}$$

The selection of the N models $\{\mathbf{m}_k\}_{k=1}^N$ is crucial and is done based on prior information. In this work, knowledge of the reservoir geology in the form of the so-called training image (Strebelle, 2002), together with facies obtained from the wells, allow the generation, by means of a multi-point geostatistical algorithm (Strebelle, 2002), of $N = 1000$ model realizations, all conditioned to the prior information. These realizations are used to compute the matrix $\mathbf{C}$. By PCA, the number of inversion parameters is reduced from $n = 4000$ to $n_R = 30$. We show in Fig. 1 two of these models (one of ten layers) and their corresponding reconstruction.

3.3. Local optimization, initial guess computation and distributed computing

For efficiency, we consider local optimizers as they are intrinsically faster than the global counterparts. However, the performance of local optimizers hinges on the initial guess selection. Inspired by the decorrelating properties of PCA we present a sound procedure for computing an initial guess. The matrix $\mathbf{C}$ in (2) is an approximation of the covariance matrix for the model. PCA diagonalizes $\mathbf{C}$, eliminating correlation between the new parameters. Moreover, from (3) we can see that the new parameters (the coefficients in the new basis $\{\mathbf{s}_i\}_{i=1}^{n_R}$) can be ranked: each eigenvalue λ_i indicates the contribution of each eigenvector $\mathbf{s}_i$ to the total variance. This suggests that a coordinate search could reduce the mismatch considerably in a few iterations. For efficiency, we optimize coarsely along only the three most important coordinates. This coarse search provides the local optimizers with some global search capabilities. From PCA we can determine the bounds for the optimization in (1) with the new parameters as optimization variables. The dynamic range (standard deviation) for the new parameters can be estimated at negligible computational cost. We use (+/-) twice the standard deviation of the first and most important parameter as its optimization bounds, while $\sqrt{\lambda_i/\lambda_1}$ scales these bounds for the other parameters.

We compare three different optimization methodologies. First, the gradient-based nonlinear optimizer SNOPT (Gill et al., 2005) is applied with a gradient estimated by first order finite differences. This approach



(a) (b) (c) (d)

Figure 2. Layer 4 after model inversion by the local optimizer SNOPT. Diffraction and traveltime tomographies are used in (a)-(b) and (c)-(d), respectively. The initial guess is as in Section 3.3 ((a) and (c)) or zero ((b) and (d)).

combined with parameter reduction and distributed computing can be fairly efficient. The dynamic range information indicated above can help in the delicate step of selecting the perturbation size in the gradient estimation. We also test two derivative-free optimization schemes: Generalized Pattern Search (Kolda et al., 2003) and Hooke-Jeeves Direct Search (Hooke and Jeeves, 1961). Derivative-free methods (Conn et al., 2009) do not employ any derivative information explicitly, and recent active research has strengthened their mathematical foundations. Since most derivative-free techniques are amenable to distributed computing, they can perform efficiently if a cluster is available. In absence of distributed computing resources, Hooke-Jeeves Direct Search, essentially a coordinate search, can still be reasonably efficient. Based again on the decorrelating properties of PCA, we expect it to be a good alternative when implemented serially. In addition, we also test a global optimizer, namely a genetic algorithm (Goldberg, 1989).

The optimization schemes considered in this work, at the expense of ease of implementation, use and extension, often require a large number of cost function evaluations. However, these computations can be distributed. Additionally, with parameter reduction the whole approach can be efficient. We can also assume that parameter reduction strategies exist in most disciplines of applied science.

4. CASE STUDY: A SUBSET OF STANFORD VI SYNTHETIC RESERVOIR

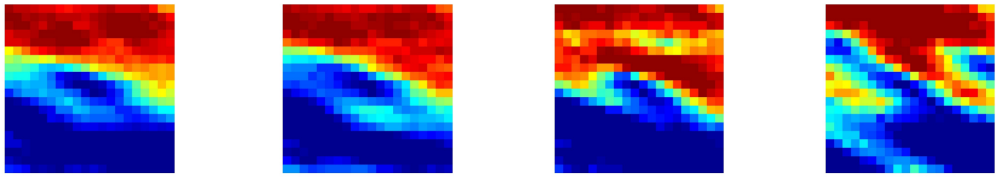
4.1. Case description

The case study is based on a synthetic dataset (with 20 x 20 x 10 = 4000 cells; i.e., it has ten layers) extracted from the Stanford VI reservoir (Castro, 2007). This realistic reservoir is a very good framework for comparing model inversion methodologies since the solution $\mathbf{m}^*$ is known. We simulate the operation of a so-called five-spot well pattern (four injectors in the corners, and one producer in the center of the domain) first during 90 days. The optimization variable is a facies indicator (i.e., sand or shale) at every grid block. The production observable consists of the total cumulative oil production and water injection, obtained at intervals of ten days. We study two different types of seismic observables: diffraction and traveltime tomography. Both of them are computed along the two perpendicular crosswell sections from the injectors, and only at the end of production. Principal Component Analysis (PCA), based on 1000 model realizations and constrained to well data, reduces the number of inversion parameters to 30. The initial guess for the local optimizers is computed as outlined in Section 3.3. We can see in Fig. 2 some of the inversion results (one layer) for the local optimizer SNOPT starting from this initial guess and from the model average $\boldsymbol{\mu}$ (i.e., all the PCA coefficients equal to zero). In this case, because only the effect of taking different initial guesses is studied, no noise is added to the observables. For both types of seismic observables the initial guess proposed yields acc-eptable results (as can be noticed if the results in Fig. 2 are compared with those for the true model in Fig. 1).

4.2. Inversion results and prediction

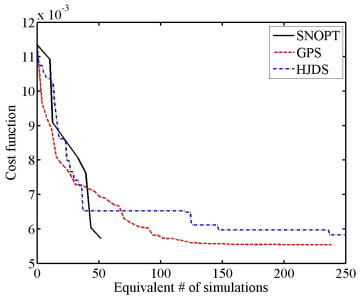
The inversion problem in (1) is solved by the methodology described in this work. We have tested three local optimizers: the gradient based SNOPT, Generalized Pattern Search (GPS) and Hooke-Jeeves Direct Search (HJDS). All of them take as initial guess the one introduced in Section 3.3. We have also considered a genetic algorithm (GA). The distributed computing environment consists of a SPARC cluster with 48 nodes, and it is incorporated in SNOPT, GPS and GA. Each observable data presents noise, with amplitude 5 per cent the standard deviation of the corresponding observable.

We show in Fig. 3 some of the inversion results for the same layer as in previous figures. These results are representative for all the layers in the model. We notice that the model inversion with diffraction tomography as observable appears to be closer to the true model $\mathbf{m}^*$ than with traveltime tomography. This is in accordance with the fact that, unlike traveltime tomography, diffraction tomography takes into account both time and waveform data.

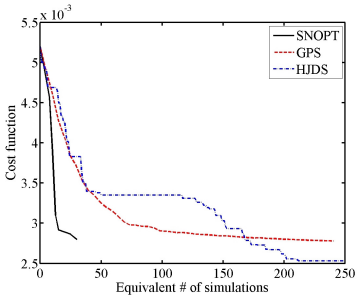


(a) (b) (c) (d)

Figure 3. Layer 4 after model inversion with diffraction tomography by SNOPT (a) and GPS (b), and with traveltime tomography by HJDS (c) and GA (d). The genetic algorithm, because of its global nature, does not require initial guess.



(a)



(b)

%	diff.	trav.
SNOPT	88.7	81.4
GPS	87.6	83.9
HJDS	87.1	81.9
GA	82.2	76.7

(c)

Figure 4. Performance results for the local optimizers (SNOPT, GPS and HJDS) in the model inversion with diffraction and traveltime tomography, (a) and (b), respectively. In both cases one complete observable simulation takes 43 seconds; thus, 250 equivalent simulations require approximately 3 hours of computing time. In the table in (c) we present the percentage of cells where the model is inverted correctly, both with diffraction (diff.) and traveltime (trav.) tomography.

In Fig. 4 we illustrate the performance of the local optimizers used. In these plots the horizontal axis (number of equivalent simulations) is proportional to the total computing time. One equivalent simulation comprises the distributed computation of all observables, and in all cases here it takes around 43 seconds. The initial guess computation needs roughly 5 equivalent simulations. SNOPT yields efficiently a good solution. For this level of noise we have appreciated some minor deterioration in the performance of this gradient-based method. GPS and HJDS can be alternatives to SNOPT in noisier situations. We stress that HJDS may be a good option in absence of distributed computing resources. The use of genetic algorithms is recommended when the other strategies fail. The inversion results by GA, as seen in Fig. 3 (d), demonstrate clear discrepancies with the true model. The performance of GA is slower[2] and more gradual, as far as cost function reduction is concerned, than those in Fig. 4.

Since the model **m** is a binary indicator, the model dissimilarity can be quantified by computing the percentage of cells where the indicator is determined correctly (something which is proportional to the zero-norm of the model error). We can check again in the table in Fig. 4 (c) that diffraction tomography yields solutions with less errors than traveltime tomography, and also that the quality of the model estimated by GA is not as good as those obtained by the other optimizers.

In Fig 5 (a) we can see, for the solution obtained by SNOPT and with diffraction tomography as seismic observable, the oil production and water injection forecast for 360 days. When the prediction interval is extended we have observed an increasing mismatch in water production. We can recalibrate the previous solution by simply scaling the model (one parameter to adjust) and considering as observables oil and water production, together with seismic data at the end of the new interval. In Fig. 5 (b) we show a prediction of oil and water production during 2000 days, based on a recalibration for the first 1000 days. We notice that the associated one-dimensional optimization step required approximately two additional equivalent simulations.

5. CONCLUSIONS

In this paper we have presented a robust, easy to use, and mathematically sound methodology for inverse modeling of oil reservoirs. A parameter reduction step combined with local optimizers within a distributed computing framework endows the scheme with efficiency. By means of a realistic case study we have demonstrated the applicability of the procedure to oil reservoir modeling. Because the observables are seen as black boxes, the approach can be adapted for other inversion problems.

[2] In the case of diffraction (traveltime) tomography, a cost function value of 0.006 (0.0031) is obtained after 207 (250) equivalent simulations.

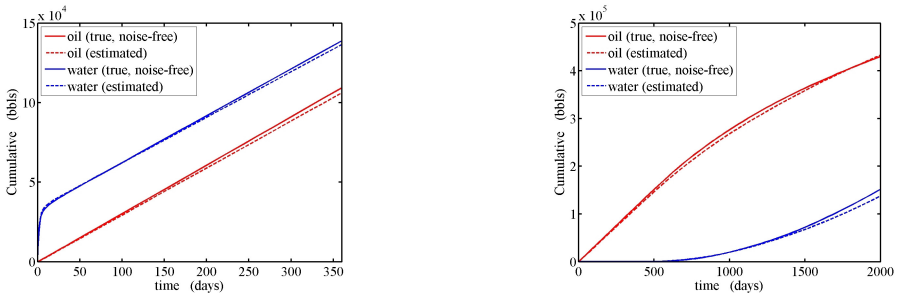


Figure 5. (a) Oil production and water injection forecast (360 days) for the solution obtained by SNOPT and diffraction tomography. (b) Oil and water production forecast (2000 days) for the recalibrated (first 1000 days) solution. In both cases the results are compared with those obtained for the true model $\mathbf{m}^*$.

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