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Land use decisions under uncertainty: optimal strategies to switch between agriculture and afforestation

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Abstract: When carbon pricing is part of the economical landscape, agricultural land has the extra option to sequester carbon through afforestation. There is a trade-off between the profits from traditional agricultural crops and from the afforestation income through carbon trading.

In this paper we study the optimal switching strategies between agricultural production and afforestation of agriculture land. The future commodity prices of agriculture products and carbon price are simulated via stochastic asset models. These commodity prices are the risk factors in evaluating the trade-offs between growing crops and afforestation. We model the value for the landholders to change land usage from agriculture production to afforestation, at a sequence of decision making time (annually), as a real option. We employ the least squares Monte Carlo algorithm to calculate the maximum expected value of this land use option, and more importantly, to determine the optimal time to switch land use (stopping rule) and the conditions of switching at each decision time (exercise regions). The valuation framework is based on finding the optimal switching time to maximise the expected discounted cash flow under the uncertainty of the multiple risk factors.

Keywords: *Optimal stopping, least squares Monte Carlo, land use, real options, American option, Bermudian option, afforestation*

1 INTRODUCTION

The Australian Carbon Farming Initiative scheme (CFI) provides farmers, forest growers and land managers with opportunities to gain income by sequestering carbon or reducing emissions through changes to agricultural land management practices. Recent research has been dedicated to the landholders decision making to switch between the agricultural production and the carbon afforestation, see, for example, Paterson and Bryan (2012), Polasky et al. (2008), Bryan et al. (2011), Dymond et al. (2012). However, few have considered the embedded real options in the land use such as options to defer investment, abandon options and options to extend.

In this paper, we consider land under agricultural production with potential use for carbon farming through afforestation. We quantify agriculture-carbon trade-off as real options of American style (American style option is an option which the holder can exercise at anytime during the tenor), by assuming the land use can be switched at anytime between carbon forestry and traditional agricultural production within a finite time horizon. Our objective is to find the optimal stopping time for land use switching in order to maximize the value of the potential land production.

We solve this optimal stopping problem using stochastic dynamic programming by working backwards recursively. In particular, the least squares Monte Carlo (LSM) approach is applied to find approximation of continuation functions for the Bellman equation. LSM is introduced initially by Longstaff and Schwartz (2001) as a numerical methodology to value American or Bermudian options by a least-squares regression. The option's optimal exercise boundaries provide the optimal decision rule for the portfolio as a function of values of the risk factors.

The prices of carbon allowance units and agricultural products are considered as uncertain risk factors. We construct multi-variable stochastic asset models for these risk factors based on historical and projected future price data. We use a 2km x 2km unit of land in the Lower Murray area of Australia as a case study in this paper.

The contribution from this paper is threefold: firstly, we introduced a land use model with embedded real options to start afforestation at a sequence of decision time; secondly, we demonstrate the use of LSM as a numerical technique to solve the optimal switching problem for carbon afforestation, and finally, through numerical results, we use a realistic case study to demonstrate effectiveness of the presented solution approach.

The remainder of this paper is organized as follows. Section 2 presents details of the land use model and the real option valuation model. Section 3 defines the stochastic asset models for the risk factors and discusses the model calibration procedure. Finally in Section 4, we present the numerical results of current methodology of relying on LSM to assist with optimal decisions. Summary is provided in Section 5.

2 LAND USE MODEL

Consider a finite time horizon $[0, T]$ with sequence of discrete time $0 = t_0 < t_1 < ... < t_m = T$. The agent (landholder or farmer) who owns a unit of land can use the land to produce agricultural products such as food crops or fibre products such as cotton, or they can reafforest the land for carbon sequestration. The unit of land is not subject to partial usage.

We assume that the agent makes his decisions at each discrete time $\{t_i, i = 0, 1, 2, ..., m\}$ to start afforestation or to continue producing agricultural commodities. An important assumption is that once the land is used as carbon forestation, the agent cannot switch the land use in following N years. Here, for ease of explanation, we use barley as the agriculture crop from which the agent needs to decide if and when to switch to forestation for carbon permit.

Given a probability space $(\Omega, \mathcal{F}_t, \mathbb{P})$, the expected excess value (EEV) by carbon afforestation over a N-year period is calculated as:

$$\phi_t(X_t, Y_t) = \mathbb{E}\left[\sum_{s=1}^{N} e^{-rs}\left(X_{t+s}q_{t+s} - k^c_{t+s} - Y_{t+s}p_{t+s} + k^b_{t+s}\right)|\mathcal{F}_t\right] - k^s_t, \tag{1}$$

where X_t and Y_t are the carbon price and the barley price at time t; r is the discount rate; the carbon sequestration rate q_t and the barley production rate p_t are known non-random functions; k^c_t, k^b_t are the maintenance costs for carbon forestation and agriculture production, k^s_t is an initial lump sum capital cost for starting carbon afforestation at time t, $\mathbb{E}[\cdot|\mathcal{F}_t]$ is the expectation conditional on the information available at time t.

When the expected excess value by carbon afforestation $\phi_{t_i},\ i = 0, 1, ...m$, is positive at any decision time t_i, switching land use to afforestation is expected to be profitable for the landholder.

Assume a class of stopping time $\mathcal{T}$ with values at $t_n,\ n = 0, 1, ..., m$, the real option value of the afforestation option for the landholder becomes:

$$V_t(X_t, Y_t) = \sup_{\tau \in \mathcal{T}} \mathbb{E}\left[e^{-r(\tau - t)} \max\{\phi_\tau(X_\tau, Y_\tau), 0\} | \mathcal{F}_t\right], \tag{2}$$

where $\mathbb{E}[\cdot|\mathcal{F}_t]$ is the expectation conditional on the information available at time t. We are interested in calculating the value of $V_0(X_0, Y_0)$ in formulating the optimal stopping time class $\mathcal{T}$. This can be determined recursively by dynamic programming.

3 A CASE STUDY

To solve the optimal stopping problem for switching the land use from agriculture production to forestation, we adopt the Least Squares Monte Carlo (LSM) algorithm discussed in Longstaff and Schwartz (2001).

For simplicity, we assume the landholder makes decisions on land usage on an annual basis. For demonstration purposes, as an example, we assume the first possible date for forestation is January 1st 2011. We set the last possible decision date as $T = 20$ year (2011-2031), the minimum carbon forestation term is set as $N = 60$ years. We use $100,000$ simulated economical scenarios, each scenario containing 80 years (T+N) trajectories of the future prices for the crop barley and the carbon commodity prices.

3.1 Stochastic Models

We use two types of stochastic asset models for carbon permit allowance price X_t and barley commodity prices Y_t. The model we used for carbon price X_t is a mean-reverting process (MR):

$$\mathrm{dln\ X}_t = (\theta_t - a\mathrm{lnX}_t)dt + \sigma_t^X \mathrm{dW}_1, \tag{3}$$

where W_1 is a Brownian motion, σ_t^X, θ_t and a are constants.

The model type for Y_t is a geometric Brownian motion(GBM) represented by the stochastic differential equation:

$$\mathrm{dY}_t = \mu \mathrm{Y}_t \mathrm{d}t + \sigma_Y \mathrm{Y}_t \mathrm{dW}_1, \tag{4}$$

where W_2 is a Brownian motion independent to W_1, μ and σ_Y are constants.

3.2 Carbon Allowance

Despite the growing interest on research of carbon price dynamics, the viability of carbon trading is still a hot topic, and it is difficult to estimate the price of carbon emissions in the long term. One of the reasons for such an uncertainty about carbon trading is due to the unpredictability of future political landscape across many countries, see Thomson Reuters Point Carbon (2013) for some interesting surveys on the EUA market and the development of emerging carbon markets. On August 28th 2012, the Australian Department of Climate Change announced a plan to link-up with the EU Emissions Trading Scheme (EU ETS) from July 1st 2015 onwards, Australian firms will be allow to use EUAs as domestic compliance with up to 50%. Australian Emission Trading Scheme (AETS) will then be fully link with the EU ETS from July 1st 2018.

To calibrate the stochastic model of carbon price (3), we use three trajectories ("CPRS-5", "CPRS-15" and "Garnaut-25") of the future carbon prices forecasts projected in Australian Government Treasury (2008). We assume the three scenarios appear with equal probability and use these carbon forecasts as approximations of the futures prices to calibrate the stochastic models following a framework described in detail by Zhu et al. (2009). The calibrated parameters are shown in Table 1, where σ_t^X and θ_t are piecewise linear term structures.

3.3 Market Price for Commodity Barley

To estimate the parameters of the stochastic asset model (4) for barley, we use the historical monthly barley spot price data from World Bank. The full range of monthly price data runs from June 1983 to April 2013, representing 359 end-of-month observations. In order to estimate the market price of risk, we follow the methodology introduced in Hull (2009). However, instead of using capital asset pricing model of the stock

Table 1. Calibrated parameters for the carbon price model.

Time t	σ_t^X	a	θ_t
1/01/2011	0.230431619	0.051025291	0.1576169
1/01/2016	0.27094448		0.181792469
1/01/2022	0.24228286		0.200496676
1/01/2027	0.225186198		0.208203645
1/01/2033	0.184822735		0.222013825
1/01/2038	0.22968032		0.232877684
1/01/2044	0.214909639		0.243155728
1/01/2050	0.24940403		0.255640355

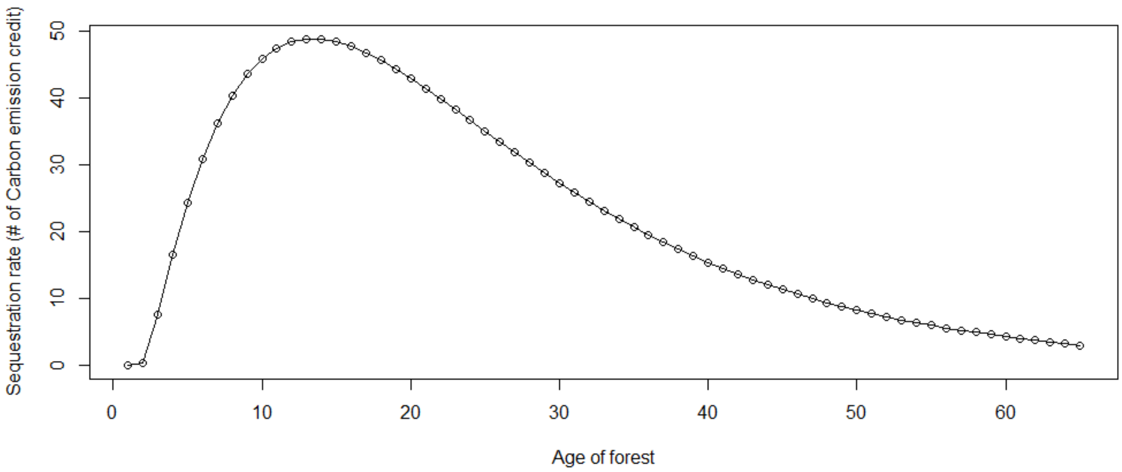

Figure 1. Sequestration rate.

market, we use Commodity Food Price Index from the International Monetary Fund as the market portfolio for barley. The calibrated parameters in Eq (4) are $\mu = 0.0324$ and $\sigma_Y = 0.2397$ with production rate $p_t = 3.5$ and maintenance cost $k_t^b = A\$600$.

3.4 Carbon Sequestration Rate and Discount Factor

Carbon sequestration rate varies over time by following the growth curves of individual trees. We use the carbon sequestration rate derived by Paterson and Bryan (2012) in which a von Bertalanffy-Chapman-Richards(vBCR) growth curve of Eucalyptus kochii is modelled with 3-PG using average monthly rainfall and temperature values, and representative soil parameters for the Lower Murray area in Australia. Figure 1 plots the carbon sequestration rate per unit of land subject to the age of the forest. We assume the landholder starts carbon afforestation with forest aged zero. The maintenance cost for carbon afforestation is $k_t^c = A\$33$ and initial cost $k_t^s = A\$2000$.

In this case study, we assume a constant bond yield, and use the Australia 15 Year bond yield which is 3.74% at the time of this study (May 2013). However, bond yields in term-structure form can also be readily used in the current implementation.

4 RESULT AND DISCUSSION

We discuss the numerical results in the following three subsections: firstly, we consider the realistic afforestation decisions based on the continuation functions estimated through LSM. We then comment on the selection of basis orthogonal functions for LSM and numerical solution algorithms. Finally, we close with a discussion of some possible future studies.

Table 2. Results of LSM of afforestation options using Nominal orthogonal up to order 3.

Start Date	End Date	NPV	ErrorEst	Exercise Probability	Real Option Value
1/01/2011	31/12/2070	11741.71	167.713	0	19345.97
1/01/2012	31/12/2071	11785.92	172.7519	0.00309	
1/01/2013	31/12/2072	11830.93	178.4251	0.00428	
1/01/2014	31/12/2073	11869.22	182.82	0.0173	
1/01/2015	31/12/2074	11892.97	187.6787	0.01498	
1/01/2016	31/12/2075	11901.65	192.205	0.02009	
1/01/2017	31/12/2076	11891.19	196.7797	0.04098	
1/01/2018	31/12/2077	11864.21	201.7603	0.06217	
1/01/2019	31/12/2078	11824.29	206.9029	0.02084	
1/01/2020	31/12/2079	11771.92	211.8061	0.03126	
1/01/2021	31/12/2080	11710.22	216.6348	0.06191	
1/01/2022	31/12/2081	11641.54	221.1649	0.08456	
1/01/2023	31/12/2082	11569.75	226.2799	0.07232	
1/01/2024	31/12/2083	11497.75	231.3692	0.07086	
1/01/2025	31/12/2084	11425.24	237.3384	0.0362	
1/01/2026	31/12/2085	11353.99	244.1721	0.02505	
1/01/2027	31/12/2086	11281.56	253.1888	0.0493	
1/01/2028	31/12/2087	11210.67	261.9088	0.11856	
1/01/2029	31/12/2088	11137.15	271.16	0.12384	
1/01/2030	31/12/2089	11063	279.6139	0.14241	

4.1 Option of afforestation

Table 2 shows some numerical results for the LSM calculation on the optimal time to switch from crop barley to afforestation. Regarding the optimal stopping time for switching land use, we find that, $85,759$ of the $100,000$ simulations (85.76%) landholders would exercise the option by afforesting the land in 20 years, most of them would have exercised in the second 10 years period. In these cases, the payoff of real option on exercise, i.e. switching, is higher than the waiting value or the so-called continuation value at the optimal exercise time.

We can use the conventional decision making rule that is based on the expected future cash flow income (the expected investment value or net present value (NPV)) at each decision time: (Column "NPV" in Table 2) from 1/01/2011 to 1/01/2030. As year 2016 gives the highest NPV value of 11901.65, year 2016 is naturally the most profitable year to start afforestation. However the "Real Option value" we have calculated as the American style real option on 1/01/2011 is in fact 19345.97, which is much higher than the NPV value of starting afforestation at year 2016.

The NPV value for each future decision date is calculated as the accumulated sum of expected cash flows for the next 60 years from the decision date. The NPV calculation relies on all future economical scenarios stemming from the static carbon and barley prices at time $t = 0$. However, the Real-option value at $t = 0$ as listed in Table 2 is calculated by the LSM as the larger one of the NPV at $t = 0$ and the expected future continuation value if the switching decision is delayed to the next year $t = 1$. In this case, therefore, the Real-option value at $t = 0$ includes the waiting value, and it is much larger than the NPV at $t = 0$. By waiting for new information, the landholder is able to add value from using any new information until the decision time. Because the Real-option value is higher than the NPV value, i.e. the waiting value is positive, we should not exercise or switch the land use at $t = 0$, because if we do, the waiting value is lost.

For decision times $t > 0$, the Real-option values at various possible future carbon and barley prices are different, it is not possible to list these infinite number of Real-option values for all the possible future carbon and barley prices. Additionally, the absolute Real-option values are only helpful in formulating optimal decisions, it is more instructive to know the optimal decisions at future decision time $t > 0$. In Figure 2, we use scatter plots at each decision time to demonstrate the decisions at different carbon and barley prices at future decision times. In Figure 2 we show that, on each decision year, 5000 simulated carbon-barley price pairs are plotted in which green coloured ones represent cases of optimal time to start carbon forestation immediately, while the

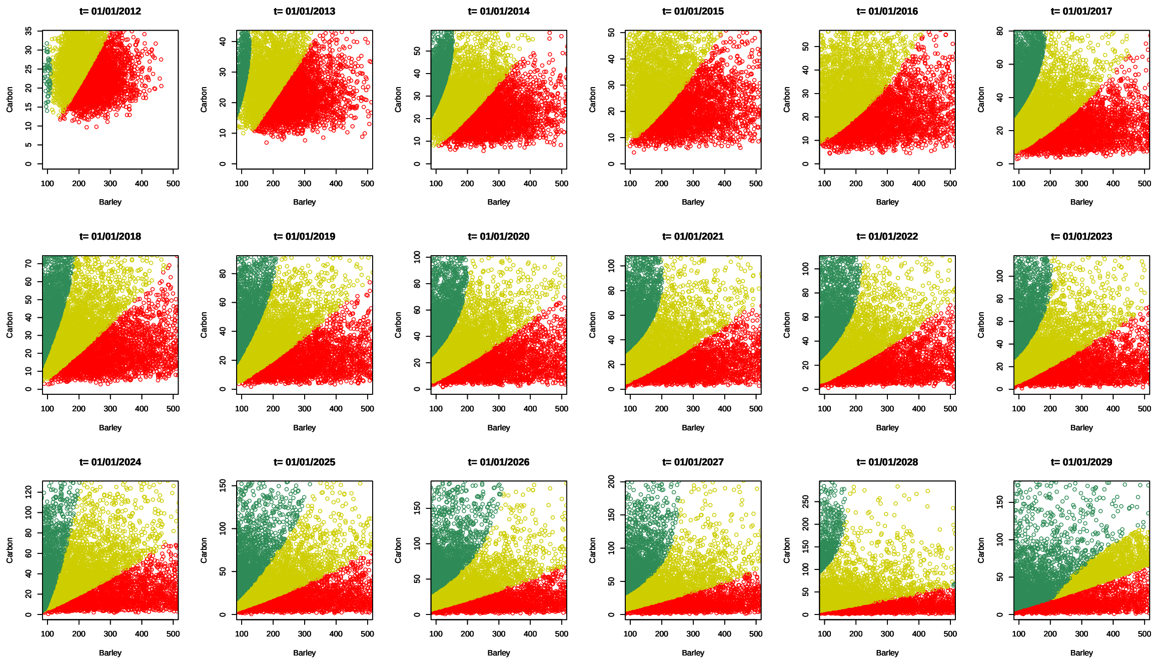

Figure 2. 5000 Monte Carlo generated samples of barley and carbon price pairs at time $t = 2, 3, , 19$. Points in green are decisions to start carbon farming immediately while yellow ones are decisions to defer investment and red ones are for not exercising the decision.

yellow coloured ones are cases of deferring the decision, whereas the red coloured plots are for not exercising the decision. It is interesting to see the optimal exercise (switching) boundary presented graphically as a function of future market prices for barley and carbon. The optimal exercise boundary varies across each future decision year. It is noted that the optimal exercise boundary in such a case study is two-dimensional, much more complicated than the conventional one-dimensional American options of financial markets.

4.2 LSM Implementation

One important issue when applying LSM algorithm is the choice of basis functions. The selection of basis function depends on the application in hand, this is the reason that LSM approach cannot be applied as a black-box algorithm. The work of Longstaff and Schwartz (2001) suggests Laguerre(weight) as the basis orthogonal function for single asset American put. Some studies of LSM algorithm have been focused on the robustness of LSM estimation and its convergence under different basis functions. For example, Stentoft (2004) shows that the LSM method is more efficient than finite difference or binomial model when valuing options on multiple assets, and Monomials are suggested as possible basis functions.

For this paper, we have tested LSM with a set of different basis polynomial functions including Laguerre, Nominal, Hermite, Hyperbolic, Legendre. We use the total standard deviation of the least squares residuals as a measure of goodness-of-fit. We observe that essentially all the orthogonal functions provide comparable results. For this particular example, Hyperbolic polynomials provide the lowest error among the 5 basis polynomial functions. Note that, in theory, the higher order orthogonal fitting should give us a better fitting. However, it is not the case in this example. One reason may be that the least square fitting algorithm for multi-dimensional function is much less robust and higher order interpolation can potentially introduce more interpolating errors. As a standard approach for selecting a numerical approximation method, we suggest testing multiple possible orthogonal functions for each application before choosing the appropriate basis functions.

4.3 Possible Future Research

It would be of interest to study optimal land use strategies when the landholder can switch back from carbon forestation to agricultural production. To study this problem, one needs to make assumption on the cost to withdraw from the carbon farming initiative. One possibility is to assume a lump sum penalty payment when closing down a carbon forestation. Once the land is registered for carbon sequestration afforestation, it is locked for such purpose for a fixed time period (say 60 or 100 years), and any earlier change of land use will result in a penalty to the landholder. The landholder might be required to buy back all the carbon emission sold in the carbon market.

So far the model we've considered is for a single unit of land, and we assume that the market prices of carbon and barley are not impacted by individual land production. In practice, with limited land resources, over production of one product and the lack of others may lead to price fluctuation due to substantial change in supply and demand. In addition, it will be valuable to consider decision rules of a portfolio, when multiple units of land with embedded options form part of the portfolio.

5 CONCLUSIONS

This paper studies the agriculture-carbon farming trade-offs using a real options approach. When carbon price is high, it is often profitable to switch from agriculture land use to the afforestation of land to sell carbon emission permits. In this paper, we have presented a real-option methodology for selecting the optimal time to switch the land use to carbon afforestation. The method can also be used to indicate optimal decision times for switching at different future carbon and agriculture crop prices. It should be noted that the uncertainty of future market prices for carbon and agriculture commodities are represented by stochastic models calibrated to historical and estimated futures prices. The optimal investment time can be computed through the Least Square Monte-Carlo (LSM) method by estimating a continuation function against an immediate exercise payoff value. We have also discussed the robustness issues of the LSM implementation.

The case study in this paper shows that the waiting value through Real-option modelling can be significant in decision making to maximise carbon farming investment. The conventional decision rules relying on NPV can lead to investment decisions that are not optimal. We have also suggested some interesting future research directions on making optimal decisions in land use.

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REFERENCES

Australian Government Treasury (2008). *Australia's low pollution future : the economics of climate change mitigation : summary*. Commonwealth of Australia Barton, A.C.T.

Bryan, B. A., D. King, and J. R. Ward (2011). Modelling and mapping agricultural opportunity costs to guide landscape planning for natural resource management. *Ecological Indicators 11*, 199–208.

Dymond, J., A. A.-G. E., E. J. C., and K. M. U. F. (2012). Tradeoffs between soil, water, and carbon - a national scale analysis from new zealand. *Journal of Environmental Management 95*, 124–131.

Longstaff, F. and E. Schwartz (2001). Valuing American options by simulation: A simple least-squares approach. *Review of Financial studies 14*(I), 113–147.

Paterson, S. and B. A. Bryan (2012). Food-Carbon Trade-offs between Agriculture and Reforestation Land Uses under Alternate Market-based Policies. *Ecology and Society 17*(3), 21.

Polasky, S., E. Nelson, J. Camm, B. Csuti, P. Fackler, E. Lonsdorf, C. Montgomery, D. White, J. Arthur, B. Garber-Yonts, R. Haight, J. Kagan, A. Starfield, and C. Tobalske (2008). Where to put things? spatial land management to sustain biodiversity and economic returns. *Biological Conservation 141*, 1505–1524.

Stentoft, L. (2004). Assessing the Least Squares Monte-Carlo Approach. *Review of Derivatives Research* (7), 129–168.

Thomson Reuters Point Carbon (2013). Annual report. carbon 2013.

Zhu, Z., P. Graham, L. Reedman, and T. Lo (2009, March). A scenario-based integrated approach for modeling carbon price risk. *Decisions in Economics and Finance 32*(1), 35–48.