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Diagnostic checking for Non-stationary ARMA Models: An Application to Financial Data

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Abstract: This paper first derives the limiting distributions of the residual and the squared residual autocorrelation functions of the nonstationary autoregressive moving-average model, respectively. We then use them to construct two portmanteau statistics for testing the adequacy of the fitted model. Simulation results show that the tests have reasonable empirical sizes and powers in the finite samples.

Keywords: *Portmanteau test, Nonstationary ARMA, Residual ACFs, Squared residual ACFs*

1 INTRODUCTION

Diagnostic checking is one of the most important steps in time series modeling. Box and Pierce (1970) proposed a portmanteau statistic based on the residual autocorrelation functions (ACFs) of the autoregressive moving-average (ARMA) model for testing the adequacy of the fitted model. Ljung and Box (1978) suggested a simple test statistic, which is a modification of Box and Pierce's statistic, and is easy to calculate. McLeod (1978) extended their results directly to more general situations. McLeod and Li (1983) proposed a new portmanteau statistic based on the squared residual ACFs. All these tests were developed for stationary time series models.

Shin and Lee (1996) proposed a portmanteau statistic for the nonstationary AR model. This statistic is asymptotically follows a χ^2 distribution. However, the corresponding result has not been established for the nonstationary ARMA (NARMA) model. Testing the unit root in the NARMA model is one of the most important issues in studies of times series and econometrics. The Dickey-Fuller tests have been extensively used in the literature. However, an adequate model is essential for the use of these tests. This paper first derives the limiting distributions of the residual ACFs and the squared residual ACFs of the NARMA model, respectively, and then uses them to construct two portmanteau statistics.

Throughout this paper, the following notation are commonly used. $O(1)(O_p(1))$ denotes a series of numbers (random numbers) that are bounded (in probability). $o(1)(o_p(1))$ denotes a series of numbers (random numbers) converging to zero (in probability). $\xrightarrow{D}$ and $\xrightarrow{p}$ denote convergence in distribution and in probability, respectively. V' denotes the transpose of vector V. $D[0,1]$ denotes the space of function $f(s)$ on $[0,1]$, which is defined and equipped with the Skorokhod topology in Billingsley (1968).

2 REVIEW OF NONSTATIONARY ARMA MODELS

In this section, we review some results on the estimation of the NARMA model.

2.1 MODELS

Let $\{Y_t\}_{t=1}^n$ be a set of observations generated by the NARMA(p,q) model,

$$\phi(L)\,Y_t = \Theta(L)\,\varepsilon_t\,, \tag{1}$$

where $\{\varepsilon_t\}$ is a sequence of unobservable independent and identically distributed (i.i.d.) errors with mean zero and unknown variance σ_ε^2, $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$. We assume that $\phi(L) = 0$ has one root that is equal to 1 and its other roots are outside the unit circle, and all the roots of $\theta(L) = 0$ lie outside the unit circle. Model (1) is nonstationary in this case.

Let $W_t = Y_t - Y_{t-1}$. Then, model (1) can be expressed in the error-correction form as

$$W_t = CY_{t-1} + \sum_{j=1}^{p-1} \phi_j^*\, W_{t-j} + \varepsilon_t - \sum_{j=1}^{q} \theta_j\, \varepsilon_{t-j}\,, \tag{2}$$

where $C = -\phi(1) = -(1 - \sum_{j=1}^p \phi_j)$ and $\phi_j^* = -\sum_{i=j+1}^p \phi_i$. We let $\phi^*(L) = 1 - \sum_{j=1}^{p-1} \phi_j^* L^j$. Since $C = -\phi(1) = -(1 - \sum_{j=1}^p \phi_j) = 0$, W_t has a stationary representation,

$$W_t = \psi_\theta(L)\varepsilon_t, \tag{3}$$

where $\psi_\theta(L) = \psi(L)\theta(L)$ and $\psi(L) = {\phi^*}^{-1}(L) = \left(1 - \sum_{j=1}^{p-1} \phi_j^* L^j\right)^{-1} = \sum_{k=0}^\infty \psi_k L^k, \psi_0 = 1$, and $\psi_k = O(\rho^k)$ with $\rho \in (0,1)$, see Hannan (1970, p.11).

2.2 LIMITING DISTRIBUTION OF GAUSSIAN ESTIMATORS

Denote $\beta = [C, \alpha']'$ and β_0 be its true value, where $\alpha = [\phi^{*\prime}, \theta']'$, $\phi^* = [\phi_1^*, \cdots, \phi_{p-1}^*]'$ and $\theta = [\theta_1, \cdots, \theta_q]'$. Let $\hat\beta$ be the Gaussian estimator of β. Taking the first-order partial derivatives with respect to β in (2), we have

$$X_{t-1} \equiv -\frac{\partial \varepsilon_t}{\partial \beta} = [\,-\frac{\partial \varepsilon_t}{\partial C},\; U_{t-1}'\,]',$$

where $U_{t-1} = -\partial\varepsilon_t/\partial\alpha$. X_t satisfies the following recursive equation:

$$(1 - \sum_{j=1}^{q} \theta_j L^j)\, X_{t-1} = [Y_{t-1}, W_{t-1}, \cdots, W_{t-p+1}, -\varepsilon_{t-1}, \cdots, -\varepsilon_{t-q}]' \equiv X_{t-1}^{*}. \tag{4}$$

Similar to (4), the vector U_t satisfies the following recursive equation:

$$(1 - \sum_{j=1}^{q} \theta_j L^j)\, U_{t-1} = [W_{t-1}, \cdots, W_{t-p+1}, -\varepsilon_{t-1}, \cdots, -\varepsilon_{t-q}]' \equiv U_{t-1}^{*}. \tag{5}$$

Note that U_t^* is stationary. The following Lemma 2.1 is Proposition 17.3 in Hamilton (1994).

Lemma 2.1 *Let* $W_t = \sum_{j=0}^{\infty} \widetilde{\psi}_j \varepsilon_{t-j}$, *where* $\widetilde{\psi}_j = O(\rho^j)$ *with* $\rho \in (0,1)$. *Define* $\gamma_j = E(W_t W_{t-j}) = \sigma_\varepsilon^2 \sum_{s=0}^{\infty} \widetilde{\psi}_s \widetilde{\psi}_{s+j}$ *for* $j = 1, 2, \cdots, \lambda = \sigma_\varepsilon \sum_{j=0}^{\infty} \widetilde{\psi}_j$ *and* $Y_t = \sum_{s=1}^{t} W_s$ *for* $t = 1, \cdots, n$ *with* $Y_0 = 0$. *Then*

(i) $\frac{1}{\sqrt{n}} \sum_{t=1}^{n} W_t \xrightarrow{D} \lambda\omega(\tau)$ *in* $D[0,1]$,

(ii) $\frac{1}{\sqrt{n}} \sum_{t=1}^{n} W_{t-j}\varepsilon_t \xrightarrow{D} N(0, \sigma_\varepsilon^2 \gamma_0)$ *for* $j = 1, 2, \cdots$,

(iii) $\frac{1}{n} \sum_{t=1}^{n} Y_{t-1}\varepsilon_t \xrightarrow{D} \sigma_\varepsilon \lambda \int_0^1 \omega(\tau) d\omega(\tau)$,

(iv) $\frac{1}{n} \sum_{t=1}^{n} Y_{t-1} W_{t-j} \xrightarrow{D} \begin{cases} \frac{1}{2}\{\lambda^2\omega^2(1) - \gamma_0\} & \text{for } j = 0. \\ \frac{1}{2}\{\lambda^2\omega^2(1) - \gamma_0\} + \sum_{i=0}^{j-1} \gamma_i & \text{for } j = 1, 2, \cdots. \end{cases}$

The following Theorem 2.1 is Theorem 1 in Yap and Reinsel (1995).

Theorem 2.1 *Under the assumptions of model (1), it follows that*

$$\begin{aligned} (i) \quad n(\hat{C} - C) &= \Theta\Big(\frac{1}{n} \sum_{t=1}^{n} Y_{t-1}\varepsilon_t\Big) \Big(\frac{1}{n^2} \sum_{t=1}^{n} Y_{t-1}^2\Big)^{-1} + o_p(1) \\ &\xrightarrow{D} \Big\{ \int_0^1 B(u) dB(u) \Big\} \Big\{ \int_0^1 B^2(u) du \Big\}^{-1} \Psi^{-1}, \end{aligned} \tag{6}$$

where $\Theta = \theta(1) = 1 - \sum_{j=1}^{q} \theta_j, \Psi = \psi(1) = \phi^{*-1}(1)$ *and* $B(u)$ *is a standard Brownian motion;*

$$\begin{aligned} (ii) \quad \sqrt{n}(\hat{\alpha} - \alpha_0) &= \Big(\frac{1}{n} \sum_{t=1}^{n} \frac{U_{t-1}U_{t-1}'}{\sigma_\varepsilon^2}\Big)^{-1} \Big(\frac{1}{\sqrt{n}} \sum_{t=1}^{n} \frac{U_{t-1}\varepsilon_t}{\sigma_\varepsilon^2}\Big) + o_p(1) \\ &\xrightarrow{D} N(0, V^{-1}), \end{aligned} \tag{7}$$

where $V = E(U_{t-1}U_{t-1}')/\sigma_\varepsilon^2$.

3 MAIN RESULTS

This section discusses the main results of the paper. We first derive the limiting distributions of the residual ACFs and the squared residual ACFs of model (1) and then use them to construct the portmanteau statistics.

3.1 LIMITING DISTRIBUTION OF THE RESIDUAL ACFS

Let $\varepsilon_t(\beta)$ be defined as

$$\varepsilon_t(\beta) = \sum_{j=1}^{q} \theta_j\, \varepsilon_{t-j}(\beta) + W_t - C\, Y_{t-1} - \sum_{j=1}^{p-1} \phi_j^*\, W_{t-j}. \tag{8}$$

Denote $\hat{\varepsilon}_t = \varepsilon_t(\hat{\beta})$, where $\hat{\beta}$ is defined as in Section 2. Thus, $\hat{\varepsilon}_t$ is the residual of model (1) when β_0 is estimated by $\hat{\beta}$.

The residual autocovariance (ACV) is defined as

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=1}^{n-k} \hat{\varepsilon}_t \, \hat{\varepsilon}_{t+k} \,. \tag{9}$$

When $k = 0$, we can show that

$$\hat{\gamma}_0 = \frac{1}{n}\sum_{t=1}^{n} \hat{\varepsilon}_t^2 \stackrel{p}{\longrightarrow} \sigma_\varepsilon^2, \tag{10}$$

as $n \longrightarrow \infty$. The corresponding residual ACF is defined as

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k} \hat{\varepsilon}_t \hat{\varepsilon}_{t+k}}{\sum_{t=1}^{n} \hat{\varepsilon}_t^2}. \tag{11}$$

Denote $\hat{\rho} = (\hat{\rho}_1, \cdots, \hat{\rho}_M)'$. The aim of this section is to derive the limiting distribution of the vector $\sqrt{n}\hat{\rho}$. We first consider $\hat{\gamma}_k$. (8) can be rewritten as

$$\varepsilon_t(\beta) = W_t - \beta' X^*_{t-1}, \tag{12}$$

where $X^*_{t-1} = [Y_{t-1}, \widetilde{W}'_{t-1}, -\widetilde{\varepsilon}_{t-1}(\beta)']'$, $\widetilde{W}_{t-1} = [W_{t-1}, \cdots, W_{t-p+1}]'$ and $\widetilde{\varepsilon}_{t-1}(\beta) = [\,\varepsilon_{t-1}(\beta), \cdots, \varepsilon_{t-q}(\beta)]'$. Since $\varepsilon_t(\beta_0) = \varepsilon_t$, by Taylor's expansion, we have

$$\begin{aligned}
\frac{1}{\sqrt{n}}\sum_{t=1}^{n-k} \hat{\varepsilon}_t \, \hat{\varepsilon}_{t+k} &= \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k} \varepsilon_t(\hat{\beta}) \, \varepsilon_{t+k}(\hat{\beta}) \\
&= \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k} \varepsilon_t \, \varepsilon_{t+k} + \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k} (\,\hat{\beta} - \beta_0\,)' \frac{\partial \big[\, \varepsilon_t(\beta) \, \varepsilon_{t+k}(\beta) \,\big]}{\partial \beta}\Big|_{\beta=\beta_0} \\
&\quad + \frac{1}{2\sqrt{n}}\sum_{t=1}^{n-k} (\,\hat{\beta} - \beta_0\,)' \frac{\partial^2 \big[\, \varepsilon_t(\beta) \, \varepsilon_{t+k}(\beta) \,\big]}{\partial \beta \, \partial \beta'}\Big|_{\beta=\beta^*} (\,\hat{\beta} - \beta_0\,) \\
&\equiv A_1 + A_2 + A_3 \,,
\end{aligned} \tag{13}$$

where $\beta^* = \beta_0 + \rho\,(\,\hat{\beta} - \beta_0\,)$ with $|\rho| < 1$. Then, we give two lemmas and their proofs are given in the Appendix.

Lemma 3.1 *Under the assumption of model (1), it follows that*

(i) $\frac{1}{n\sqrt{n}} \sum_{t=1}^{n-k} \big[\, Y_{t+k-h-1} \, \varepsilon_t \,\big] = o_p(1)$,

(ii) $\frac{1}{n\sqrt{n}} \sum_{t=1}^{n-k} \big[\, Y_{t-h-1} \, \varepsilon_{t+k} \,\big] = o_p(1)$,

(iii) $\frac{1}{n} \sum_{t=1}^{n-k} \big[\, \theta^{-1}(L) \, \widetilde{W}_{t+k-1} \,\big] \, \varepsilon_t \stackrel{p}{\longrightarrow} E[\, \theta^{-1}(L) \, \widetilde{W}_{t+k-1} \, \varepsilon_t \,]$,

(iv) $\frac{1}{n} \sum_{t=1}^{n-k} \big[\, \theta^{-1}(L) \, \widetilde{W}_{t-1} \,\big] \, \varepsilon_{t+k} = o_p(1)$,

(v) $\frac{1}{n} \sum_{t=1}^{n-k} \big[\, \theta^{-1}(L) \, \widetilde{\varepsilon}_{t+k-1} \,\big] \, \varepsilon_t \stackrel{p}{\longrightarrow} E[\, \theta^{-1}(L) \, \widetilde{\varepsilon}_{t+k-1} \, \varepsilon_t \,]$,

(vi) $\frac{1}{n} \sum_{t=1}^{n-k} \big[\, \theta^{-1}(L) \, \widetilde{\varepsilon}_{t-1} \,\big] \, \varepsilon_{t+k} = o_p(1)$.

Lemma 3.2 *Under the assumption of model (1),*

(i) $A_2 = -\Big\{ E\Big(\frac{U_{t-1}U'_{t-1}}{\sigma_\varepsilon^2}\Big)^{-1} \Big(\frac{1}{\sqrt{n}} \sum_{t=1}^{n-k} \frac{U_{t+k-1}\varepsilon_{t+k}}{\sigma_\varepsilon^2}\Big) \Big\}' E(U_{t+k-1}\varepsilon_t) + o_p(1)$,

(ii) $A_3 = o_p(1)$.

Thus, we have the following theorem.

Theorem 3.1 *Under the assumption of model (1) or (2),*

$$\sqrt{n}\hat{\rho} = \sqrt{n}(\hat{\rho}_1, \cdots, \hat{\rho}_M)' \xrightarrow{D} N(0, \Sigma),$$

where $\Sigma = I_M - D^{'}\Omega^{-1}D, \Omega^{-1} = E(U_{t-1}U_{t-1}')^{-1}, D_k = E(U_{t+k-1}\varepsilon_t)$ *and* $D = (D_1, \cdots, D_M)/\sigma_\varepsilon$.

We consider the hypothesis:

$$H_0 : \quad \text{Model (2)} \quad \text{is correct} \quad \text{vs}$$

$$H_1 : \quad \text{Model (2)} \quad \text{is not correct}$$

The portmanteau statistic is defined as

$$Q_M = n\hat{\rho}^{'}(\, I_M - D^{'}\Omega^{-1}D\,)^{-1}\hat{\rho}.$$

By Theorem 3.1, under H_0, Q_M is asymptotically follows a χ^2 distribution with M degrees of freedom. Q_M can be used to test the hypothesis H_0 against H_1. When H_0 is accepted, we can claim that model (2) is adequate for the data $\{Y_t\}_{t=1}^{n}$.

3.2 LINITING DISTRIBUTION OF THE SQUARED RESIDUAL ACFS

We define the squared residual autocovariance (ACV) as

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=1}^{n-k}\big(\,\hat{\varepsilon}_t^{\,2} - 1\,\big)\,\big(\,\hat{\varepsilon}_{t+k}^{\,2} - 1\,\big)\,. \tag{14}$$

We assume $E\varepsilon_t^2 = \sigma_\varepsilon^2 = 1$. When $\sigma_\varepsilon^2 \neq 1$, σ_ε^2 can be estimated by $\hat{\sigma}_\varepsilon^2$ defined in (10). When $k = 0$, we can show that

$$\hat{\gamma}_0 = \frac{1}{n}\sum_{t=1}^{n}\big(\hat{\varepsilon}_t^2 - 1\big)^2 \xrightarrow{p} E\varepsilon_t^4 - 1 \equiv \sigma_4, \tag{15}$$

as $n \longrightarrow \infty$. The corresponding squared residual ACF is defined as

$$\tilde{r}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k}\big(\hat{\varepsilon}_t^2 - 1\big)\big(\hat{\varepsilon}_{t+k}^2 - 1\big)}{\sum_{t=1}^{n}\big(\hat{\varepsilon}_t^2 - 1\big)^2}. \tag{16}$$

Denote $\widetilde{R} = (\tilde{r}_1, \cdots, \tilde{r}_M)'$. The aim of this section is to derive the limiting distribution of the vector $\sqrt{n}\widetilde{R}$. Now, we first consider $\hat{\gamma}_k$. Since $\varepsilon_t^2(\beta_0) = \varepsilon_t^2$, by Taylor's expansion, we have

$$\begin{aligned}
&\frac{1}{\sqrt{n}}\,\sum_{t=1}^{n-k}\big(\,\hat{\varepsilon}_t^{\,2} - 1\,\big)\,\big(\,\hat{\varepsilon}_{t+k}^{\,2} - 1\,\big) \\
&\quad = \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k}\big(\,\varepsilon_t^{\,2}(\hat{\beta}) - 1\,\big)\,\big(\,\varepsilon_{t+k}^{\,2}(\hat{\beta}) - 1\,\big) \\
&\quad = \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k}\big(\,\varepsilon_t^{\,2} - 1\,\big)\,\big(\,\varepsilon_{t+k}^{\,2} - 1\,\big) \\
&\qquad + \frac{1}{\sqrt{n}}\sum_{t=1}^{n-k}\big(\,\hat{\beta} - \beta_0\,\big)^{'}\frac{\partial\Big[\,\big(\,\varepsilon_t^{\,2}(\beta) - 1\,\big)\,\big(\,\varepsilon_{t+k}^{\,2}(\beta) - 1\,\big)\,\Big]}{\partial\beta}\Big|_{\beta=\beta_0} \\
&\qquad + \frac{1}{2\sqrt{n}}\sum_{t=1}^{n-k}\big(\,\hat{\beta} - \beta_0\,\big)^{'}\frac{\partial^2\Big[\,\big(\,\varepsilon_t^{\,2}(\beta) - 1\,\big)\,\big(\,\varepsilon_{t+k}^{\,2}(\beta) - 1\,\big)\,\Big]}{\partial\beta\,\partial\beta^{'}}\Big|_{\beta=\beta^*}\big(\,\hat{\beta} - \beta_0\,\big) \\
&\quad \equiv B_1 + B_2 + B_3\,,
\end{aligned} \tag{17}$$

where $\beta^* = \beta_0 + \rho\,(\,\hat{\beta} - \beta_0\,),$ with $|\,\rho\,| < 1$. Then, we give two basic lemmas and their proofs are given in the Appendix.

Lemma 3.3 *Suppose* Y_t, $\widetilde{W}_{t-1}$, *and* $\widetilde{\varepsilon}_{t-1}$ *are defined as in (12). Let* $\xi_t = \varepsilon_t(\varepsilon^2_{t+k} - 1)$ *and* $\xi_{t+k} = \varepsilon_{t+k}(\varepsilon^2_t - 1)$. *If* $\sigma_4 < \infty$, *then it follows that*

(i) $\frac{1}{n\sqrt{n}}\sum_{t=1}^{n-k}\left[\,Y_{t+k-h-1}\,\xi_{t+k}\,\right] = o_p(1),$

(ii) $\frac{1}{n\sqrt{n}}\sum_{t=1}^{n-k}\left[\,Y_{t-h-1}\,\xi_t\,\right] = o_p(1),$

(iii) $\frac{1}{n}\sum_{t=1}^{n-k}\left[\,\theta^{-1}(L)\,\widetilde{W}_{t+k-1}\,\right]\xi_{t+k} = o_p(1),$

(iv) $\frac{1}{n}\sum_{t=1}^{n-k}\left[\,\theta^{-1}(L)\,\widetilde{W}_{t-1}\,\right]\xi_t = o_p(1),$

(v) $\frac{1}{n}\sum_{t=1}^{n-k}\left[\,\theta^{-1}(L)\,\widetilde{\varepsilon}_{t+k-1}\,\right]\xi_{t+k} = o_p(1),$

(vi) $\frac{1}{n}\sum_{t=1}^{n-k}\left[\,\theta^{-1}(L)\,\widetilde{\varepsilon}_{t-1}\,\right]\xi_t = o_p(1).$

Lemma 3.4 *Under the assumption of model (1),*

(i) $B_2 = o_p(1),$

(ii) $B_3 = o_p(1).$

Thus, we have the following theorem.

Theorem 3.2 *If the assumption of model (1) or (2) holds and* $\sigma_4 < \infty$, *then*

$$\sqrt{n}\widetilde{R} = \sqrt{n}(\tilde{r}_1, \cdots, \tilde{r}_M)' \xrightarrow{D} N(0, I_M).$$

We consider the following hypothesis:

$$H_0: \quad \text{Model (2)} \quad \text{is correct} \quad \text{vs}$$

$$H_1: \quad \text{Model (2)} \quad \text{is not correct}$$

The new portmantaue statistic is defined as

$$\widetilde{Q}_M = n\widetilde{R}'\widetilde{R}.$$

By Theorem 3.2, under H_0, $\widetilde{Q}_M$ is asymptotically follows a χ^2 distribution with M degrees of freedom. $\widetilde{Q}_M$ can be used to test the hypothesis H_0 against H_1. When H_0 is accepted, we can claim that model (2) is adequate for the data $\{Y_t\}_{t=1}^n$.

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