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Switching Surfaces for Optimal Natural Resource Extraction under Uncertainty

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Abstract: The flexibility to revise managerial and/or operational decisions over time in response to uncertain market conditions can significantly increase the value of a project. In order to maximise the project value, the operational decisions need to be made sequentially, in an optimal manner, in response to the evolution of uncertainties. Although dynamic strategies brings substantial improvements of the project, its complexity from stochastic control algorithm makes modern real option theory rarely adopted by industry. Thus it calls for a methodology to display graphically the results obtained by real options analysis. An intuitive display of the information about the boundaries between the regions of different optimal decisions (called switching boundaries) would greatly assist industry with optimal sequential decision-making under uncertainty.

This paper presents a methodology to construct switching boundaries/surfaces for optimal natural resource extraction under uncertainty, based on the regression Monte Carlo approach. We extend previous research by (1) incorporating recently proposed advanced techniques (such as adaptive local basis and memory reduction methods) that allow considerable improvement of the accuracy of the switching boundaries; and (2) constructing and analysing the higher-dimensional switching boundaries.

We illustrate how to construct and use switching boundaries using a classical model of a copper mine with flexibility to delay, temporarily close, reopen or completely abandon the mineral extraction in response to the stochastic behaviour of the copper price. For such a model, the switching boundaries are the critical copper prices that trigger a change of operating regime. For this example, the switching boundaries are two-dimensional copper price surfaces that depend on the remaining reserve and the remaining time horizon. We display and analyse these surfaces using both 3D graphs and dynamic 2D graphs.

The paper demonstrates several benefits of the switching boundaries. They can be used by mining companies:

1. as a simple and intuitive decision support tool for identification of optimal operational strategies and for optimal management of resources projects;
2. to gain insight into optimal strategies under different market conditions and project settings;
3. to benefit financially from dynamic strategies.

Keywords: *Real options, stochastic optimal control, least-squares Monte Carlo, memory reduction method, stochastic switching*

1 INTRODUCTION

The flexibility to revise managerial and/or operational decisions over time in response to uncertain market conditions can significantly increase the value of a project. In the minerals industry, it is well known since the seminal paper of Brennan and Schwartz (1985) [henceforth B&S] that the value of flexibility to delay, temporarily close, reopen and completely abandon mineral extraction project in response to commodity price fluctuations can be pronounced. These types of flexibility remain important for optimal management of modern natural resource investments and continue to draw research interest (see, e.g., Trigeorgis (1996), Slade (2001), Dimitrakopoulos and Abdel Sabour (2007), Carmona and Ludkovski (2008), Tsekrekos et al. (2012), Bao et al. (2013), Mortazavi-Naeini et al. (2014), Tarnopolskaya et al. (2015)). In order to maximise the project value, the operational decisions need to be made sequentially in an optimal manner, in response to the evolution of uncertainties. The complexity of the real options thoery and stochastic dynamic programming algorithms is among the reasons for real options methods being rarely used by industry. An intuitive display of the information about the boundaries between the regimes of different optimal decisions (called switching boundaries) would greatly assist industry with optimal sequential decision-making under uncertainty, and could also potentially accelerate the adoption of real options analysis by industry. The switching boundaries, also known as critical (or threshold) curves, or dispersal curves, have been studied in a number of applications in both deterministic and stochastic optimal control problems (see, e.g., Carmona and Ludkovski (2008), Tarnopolskaya and Fulton (2010a), Tarnopolskaya and Fulton (2010b), Mortazavi-Naeini et al. (2014), Tarnopolskaya et al. (2015)).

This paper continues the study of the switching boundaries in natural resource investment problems by Tarnopolskaya et al. (2015) and extends this previous work to the situation when the switching boundaries depend on the remaining reserves (which is typically the case for natural resource management problems with longer time horizons). For such case, the switching boundaries become multi-dimensional surfaces. The construction of switching surfaces in this paper is based on the regression Monte Carlo approach. Recently proposed advanced techniques, such as adaptive local basis approach (Bouchard and Warin (2012)) and memory reduction method (Aïd et al. (2014)), have been implemented to improve the accuracy and efficiency of the algorithm. We benchmark the algorithm against the classical results by B&S for the infinite horizon, finite reserve copper mine valuation problem. We also construct the switching boundaries and analyse their structure using both 3D graphs and dynamic 2D graphs.

The paper is organized as follows. The problem formulation is discussed in Section 2. The least squares Monte Carlo (LSMC) method and algorithm for construction of the switching surfaces are described in Section 3. The numerical results, visualization of the switching surface and discussion are described in Section 4. Finally, the conclusions are given in Section 5.

2 PROBLEM FORMULATION

We study the optimal extraction of an exhaustible resource by the company which owns a mining license. The company possesses the operational flexibility to delay, temporarily stop, restart or completely abandon the extraction operations in response to uncertain commodity price S. Such problem has been studied on an infinite time horizon by B&S who formulated this problem as a quasi-variational inequality. Realistic mine planning problems usually have a finite time horizon T, as mining licenses have finite lifetime. In this section, we formulate the problem as a discrete time, finite horizon stochastic switching problem.

We assume that management has the option to change operating regimes at pre-specified discrete decision times $t_n = n\Delta t, n = 0, 1, ..., N-1$, where $\Delta t = T/N$.

The commodity price at t_n is denoted S_n. As in B&S, we assume that the commodity price follows a one-dimensional geometric Brownian motion. In a discretised form, it is given by

$$S_{n+1} = S_n e^{\left(r-\delta-\frac{\sigma^2}{2}\right)\Delta t+\sigma(W_{n+1}-W_n)}, \qquad W_{n+1} - W_n \sim \mathcal{N}(0, \Delta t) \quad \text{i.i.d.} \tag{1}$$

where r is the risk-free rate, δ is the instantaneous convenience yield of the commodity, σ is the volatility of the spot price S.

We define the set of operating regimes as $\mathbb{Z} = \{o, c, a\}$, where o, c and a denote an open (operating), a temporarily closed and an abandoned mine respectively. The change of the operating regime at time t_n from the regime $i \in \{o, c\}$ to $j \in \mathbb{Z}$ incurs the switching cost $k(t_n, i, j) = k(0, i, j)e^{\pi t_n}$, where π is the inflation rate. $k(t_n, i, i) = 0$ if no switching occurs.

The discretised remaining reserve is given by $Q_{n+1} = Q_n - q\Delta t \mathbb{1}\{i_n = o\}$, where q is a constant extraction rate. The maximum number of possible reserve states is $J = Q_0/(q\Delta t)$. At time t_n, the feasible reserve levels are $\mathcal{Q}_n = \{(J-k)q\Delta t, k = 0, 1, ..., \min[n, J]\}$. At each decision time, only feasible reserve levels are considered.

The cash flow $\Pi_i(t_n, S_n)$ in the operating regime $i \in \mathbb{Z}$ between two decision times $[t_n, t_{n+1}), n = 0, ..., N-1$ is given by

- For an open mine: $\Pi_o(t_n, S_n) = q(S_n - A_{t_n}) - Tax(S_n)$, where $A_{t_n} = A_0 e^{\pi t_n}$ is the operating cost, $Tax(S_n) = p_1 q S_n + p_2 q(S_n(1-p_1) - A_n)$ is the total income tax and royalties, with p_1 and p_2 denoting the royalty rate and the income tax respectively.
- For a closed mine: $\Pi_c(t_n, S_n) = -M_n$, where M_0 is the initial maintenance cost, and $M_n = M_0 e^{\pi t_n}$ is the after-tax maintenance cost.
- $\Pi_a(t_n, S_n) = 0$;

We denote by $V(t_n, S_n, Q_n, i)$ the value of the mine at t_n when the commodity price is S_n, with Q_n reserves and status i and assume that $V(T, S_N, Q_N, i) = 0$ (that is, the value is 0 after the expiration of mining licence). The goal of the mining company is to maximize the value $V(t_0, S_0, Q_0, i)$ though the optimal strategy. Thus, the Bellman value function for this problem is given by

$$V(t_n, S_n, Q_n, i_{n^-}) = \max_{j \in \mathbb{Z}} \left\{ \Pi_j(t_n, S_n)\Delta t - k(t_n, i_{n^-}, j) + \mathbb{E}\left[e^{-\tilde{r}\Delta t} V(t_{n+1}, S_{n+1}, Q_{n+1}, j) \middle| S_n, Q_n \right] \right\},$$

where $\tilde{r} = r + \lambda$, i_{n^-} denotes the operating regime right before the decision time t_n, and λ is the property tax rate, proportional to the project value.

3 NUMERICAL METHOD

3.1 Regression Monte Carlo Approach

Regression Monte Carlo approach is a versatile simulation-based technique based on the approximation of the continuation function (the last term in Bellman equation at the end of Section 2) via basis functions. One of the earlier regression Monte Carlo methods, known as least-squares Monte Carlo (LSMC), was proposed by Longstaff and Schwartz (2001) for valuation of American option, and has become popular for stochastic control problems in minerals industries due to its flexibility and ease of implementation (see, e.g., Dimitrakopoulos and Abdel Sabour (2007), Tsekrekos et al. (2012),Bao et al. (2013), Tarnopolskaya et al. (2015)) .

The regression Monte Carlo method combines two parts:

1. In the forward loop, the algorithm generates M Monte Carlo paths of the commodity prices $\{S_n^m : m = 1, 2, ..., M; n = 0, 1, ..., N\}$ with the same initial price $S_0^m = S_0$ forward in time.
2. In the backward loop, the continuation functions $\Phi(t_n, S_n, Q_n, i)$ are computed by regressing the discounted realized cashflows along the simulated paths onto a set of selected basis functions $\{\phi_l(S_n^m)\}$ for each operating regime $i \in \{o, c\}$ and each feasible reserve level $Q_n \in \mathcal{Q}_n$. At time t_n, for each simulated path S_n^m, an optimal switch from the current operating regime i to another operating regime j happens if and only if

$$\Pi_i(t_n, S_n^m)\Delta t + \Phi(t_n, S_n^m, Q_n, i) < \max_{j \neq i} \left[\Pi_j(t_n, S_n^m)\Delta t - k(t_n, i, j) + \Phi(t_n, S_n^m, Q_n, j) \right].$$

Despite its advantages, LSMC has two significant drawbacks. Specifically, (1) the choice of basis function is problematic; and (2) increasing the size of the basis may cause overfitting problem, thus affecting the accuracy. In this paper, we use two recent advanced techniques in order to overcome the convergence difficulties and to improve the accuracy of the numerical algorithm: (1) we adopt a local basis function method (Bouchard and Warin (2012)) which overcomes an issue with appropriate selection of basis function in LSMC; (2) we implement a memory reduction method to reduce memory consumption. Both methods are described in more details in Langrené et al. (2015). The improved accuracy achieved as a result of these modifications is especially important for approximating the switching surfaces.

3.2 Constructing the Switching Surfaces

The structure of the Bellman value function for optimal resource extraction problem (Section 2) suggests that at each decision time the optimal operating regime depends on the operating regime at the previous decision time, the current commodity price, the remaining inventory and the time to the end of the horizon. We then define the switching sets $\Lambda^{i,j}(t_n), n = 1, ..., N-1,$, from one operating regime $i \in \{o, c\}$ to another $j \in \mathbb{Z}$, as the locus of all combinations of simulated commodity price and feasible reserve level $(S_n^m, Q_n); m = 1, ..., M, Q_n \in \mathcal{Q}_n$, where it is optimal to switch from the regime i to j at time t_n (note that this definition includes non-switching set i to i). The regression Monte Carlo algorithm described in the previous section can be used to establish a mapping between the simulated state variables and the optimal operating regimes, and therefore to produce the switching sets.

There are generally three switching sets at each decision time from the operating mode $i = o$: $\Lambda^{o,c}(t_n,)$, $\Lambda^{o,a}(t_n)$, $\Lambda^{o,o}(t_n)$, and three switching sets from the operating mode $i = c$: $\Lambda^{c,a}(t_n)$, $\Lambda^{c,o}(t_n)$ $\Lambda^{c,c}(t_n)$. Our numerical results suggest that the switching sets form connected components in (S, Q) plane and do not overlap. It is then straightforward to construct the boundaries between the switching sets. For a given decision time t_n, the switching boundaries between the operating modes i and j are the critical commodity prices $S^*_{i,j}(Q_n)$ that trigger a regime switch from i to j when the commodity price crosses in the direction from the optimal region i to j.

In this paper, in order to increase the number of points in each switching set, instead of generating the switching sets from the optimal strategies on the realised paths (as in Tarnopolskaya et al. (2015)), we generate 'hypothetical' switching sets based on optimal strategies obtained by using the continuation functions computed at each time step t_n during the backward induction process for every feasible reserve level and every operating mode at the previous time step. We can see that for large switching sets, the switching boundaries are sufficiently smooth (see the two figures on the left-hand side of Fig.1).

By superimposing the switching sets $\Lambda^{o,c}(t_n,)$, $\Lambda^{o,a}(t_n)$, $\Lambda^{o,o}(t_n)$ and $\Lambda^{c,a}(t_n)$, $\Lambda^{c,o}(t_n)$ $\Lambda^{c,c}(t_n)$, a complete set of decision regions can be produced. This is illustrated in the figure on the right-hand side of Fig.1. New regions of optimal decisions produced in this way are: hysteresis band, abandon from close and abandon from open.

The switching surface $\mathcal{S}^*_{i,j}(t_n, Q_n)$ combines all the switching boundaries $S^*_{i,j}(Q_n)$ for all decision times.

4 Numerical Results and Discussion

In this section we: (1) validate our numerical implementation by benchmarking to the solution of the classical infinite horizon finite inventory natural resource investment problem by B&S; and (2) visualize the switching

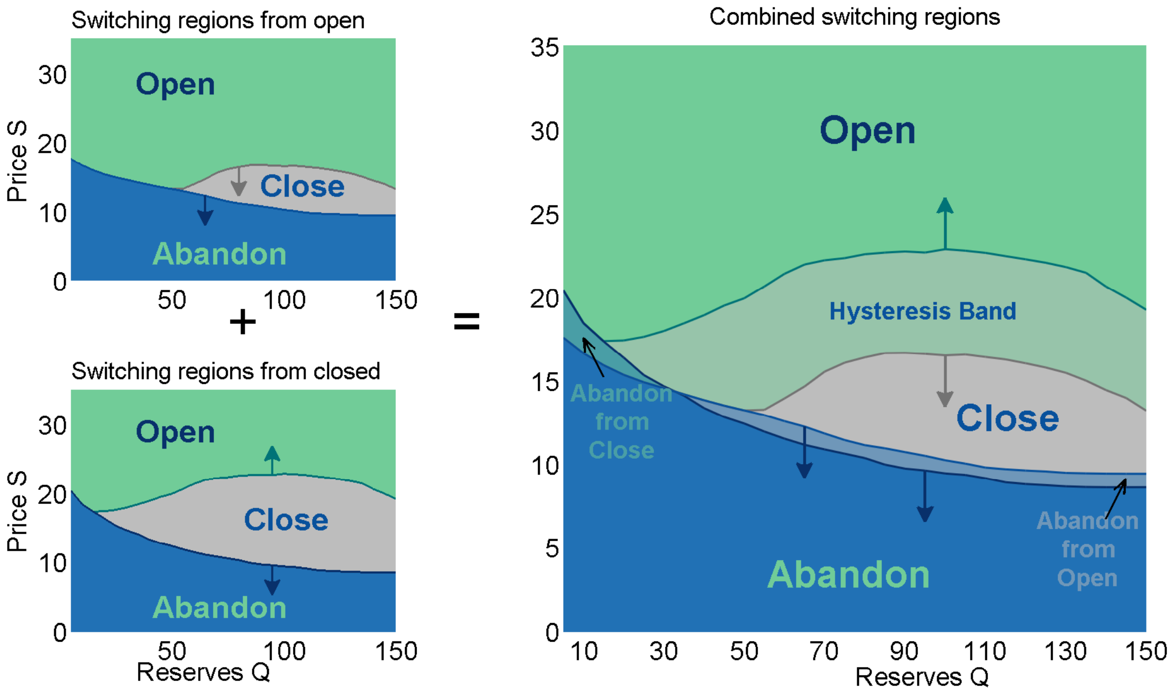

Figure 1. Constructing the switching regions for a given decision time ($t = 45$); 10^6 Monte Carlo simulations.

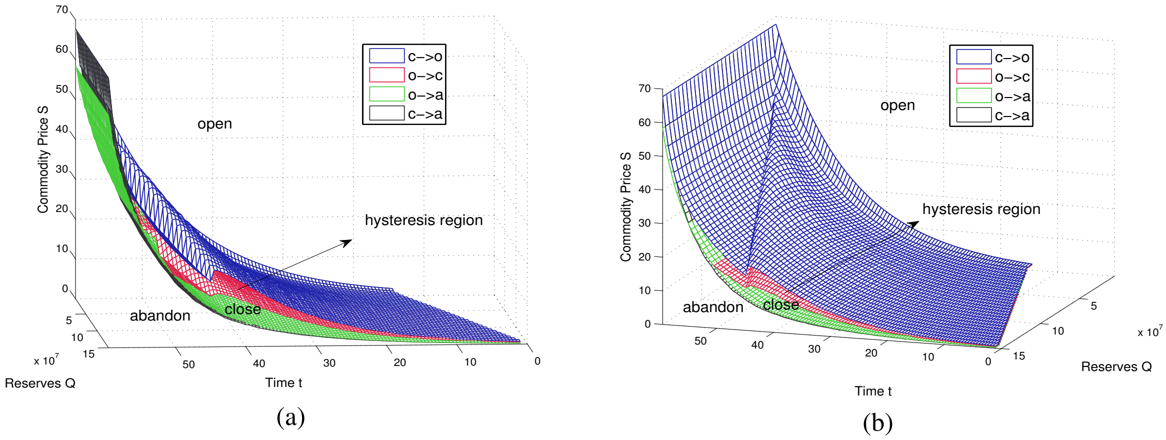

Figure 2. Switching surfaces for $T = 60$, two decisions per year, 10^6 Monte Carlo paths.

surfaces and discuss their structure.

4.1 Benchmarking to Infinite Horizon Problem

B&S studied the continuous, infinite horizon, finite resource copper mine valuation problem. In order to test our method, we approximate the solution of this infinite horizon continuous problem by that of a finite horizon discrete time stochastic control problem described in Section 2. We use the parameters of this model as in B&S: $q = 10^6$ pounds/year, $Q_0 = 150 \times 10^6$ pounds, $A_0 = \$0.5$/pound, $k(0, o, c) = k(0, c, o) = \$200\,000$, $k(0, o, a) = k(0, c, a) = \$0$, $M_0 = \$500\,000$/year, $\sigma = 0.08$, $\delta = 0.01$, $r = 0.1$, $p_1 = 0.02$, $p_2 = 0.5$, $\pi = 0.08$. A good agreement with B&S is achieved for a 60-year time horizon with 10 decisions per year ($N = 600$) and with $L = 20$ local basis functions described in Section 3.1 (see Table 1). We can see that the agreement with B&S is better than in Tarnopolskaya et al. (2015), where a shorter horizon and smaller number of annual decisions were used.

Table 1. Comparison with benchmark results

Initial Price	B & S	($millions)	Our results	($millions)
($)	Open	Closed	Open	Closed
0.4	4.15	4.35*	4.13	4.33*
0.5	7.95	8.11	7.93	8.08
0.6	12.52	12.49	12.52	12.49
0.7	17.56	17.38	17.56	17.39
0.8	22.88**	22.68	22.88**	22.68
0.9	28.38**	28.18	28.40**	28.20
1	34.01**	33.81	34.03**	33.83

Note: * Optimal to close, ** optimal to open.

4.2 Switching surfaces

For the exhaustible resource extraction problem described in Section 2, the switching boundaries are generally functions of both the time t_n and the remaining reserve Q_n and represent two-dimensional copper price surfaces. Fig. 2 shows four switching surfaces ("$c \to o$","$o \to c$","$o \to a$", "$c \to a$") for the mine valuation problem with 60-year time horizon ($T = 60$) and 2 decision times per year ($\Delta t = 1/2$) from two different angles.

An interesting feature in Fig. 2 is a sharp decrease w.r.t. Q of the switching surfaces "$o \to c$" (red) and "$c \to o$" (blue) along the line from $(t = 45, Q = Q_0)$ to $(t = T, Q = 0)$. After the sharp decrease, the switching surfaces become independent of the remaining reserve. The explanation of this effect is as follows: at a time when the remaining reserves is sufficient for continuous operation until the the end of time horizon, the reserve is no longer a constraint for the operating decisions and therefore the switching surfaces become

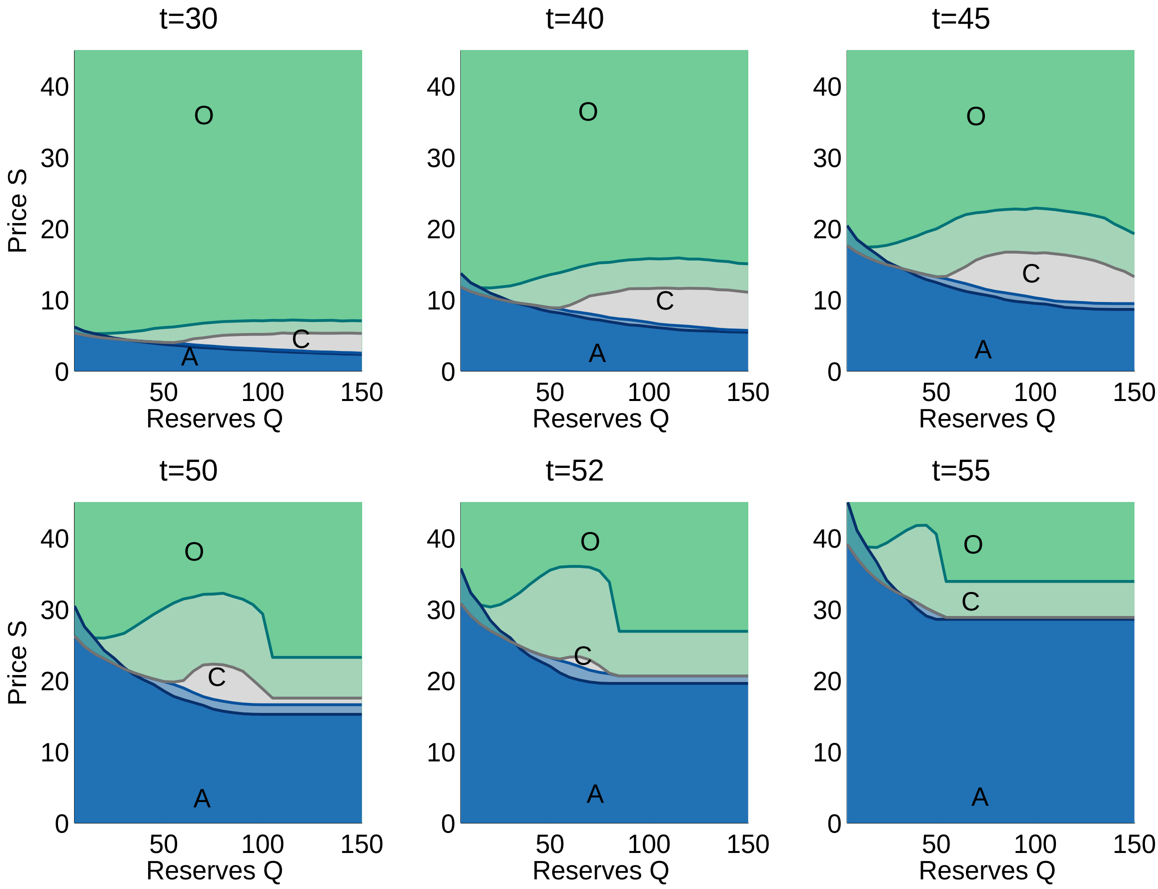

Figure 3. Switching boundaries as functions of remaining reserves Q at different decision times t, computed from one million Monte Carlo paths. (More detailed information of the coloured regions can be seen in Fig.1.)

independent of Q. A decrease in the switching surfaces values at these times indicates that it is beneficial to keep the mine open at lower commodity prices when plenty of reserves are left.

In Fig. 2, a hysteresis region between the switching surfaces "$o \to c$" and "$c \to o$" can be observed, which is due to the switching cost (see discussions in Carmona and Ludkovski (2008), Tarnopolskaya et al. (2015)). To further investigate the structure of the switching surfaces, we plot the two-dimensional cross-sections at different decision times during the planning horizon in Fig. 3. We notice the following features:

1. All the switching boundaries move upwards as time increases.

2. After $t = 45$, horizontal segments of the switching boundaries appear, starting at the remaining reserve level which is sufficient for continuous operation for the remaining time horizon (see discussion above).

3. The concave down sections of the switching boundaries "$o \to c$" (red) and "$c \to o$" (blue) suggest that the optimal strategy for higher reserve levels is to wait for prices to go up. However, as the remaining reserves decreases, it becomes beneficial to open the mine at lower commodity prices.

4. Towards the end of time horizon, the switching boundary "$o \to c$" (red) disappears, which means it is not optimal to close an open mine near the end of time horizon. In such situations, an open mine should be either kept open or be abandoned. This result is consistent with that in Tarnopolskaya et al. (2015).

5. A sharp increase in the "$c \to a$" switching boundaries near $Q = 0$ in the last three subfigures of Fig. 2 indicates that it becomes beneficial to abandon the mine when the reserve is sparse and the price is not sufficiently high. Similarly, a sharp increase in the "$c \to o$" near $Q = 0$ in the last three subfigures of Fig. 2 indicates that much higher commodity prices are required for reopening the mine to become optimal when the reserve is sparse and the end of the time horizon approaches.

Remark 1. *Our numerical analysis suggests that the Monte Carlo sample size required to produce smooth switching surfaces is generally larger than that required to produce sufficiently accurate valuation results.*

Remark 2. *The switching surfaces are computed for a given initial commodity price S_0. However, numerical results suggest that the switching surfaces do not change with the change in the initial commodity price.*

5 CONCLUSION

An improved methodology for construction of the switching surfaces dependent on both the remaining reserve level and time is presented, based on the advanced regression Monte Carlo method. Optimal switching surfaces can be used by mining companies as an efficient and intuitive decision support tool for optimal resource extraction under uncertainty. The switching surfaces provide insight into the optimal strategies under different conditions and can be used by companies to benefit financially from dynamic strategies.

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