A probabilistic approach to climate shift detection based on the Maronna-Yohai bivariate test

J.H. Ricketts ^a

^a Victoria Institute of Strategic Economic Studies, Victoria University, 300 Flinders St, Melbourne, Victoria, Australia Email: james.ricketts@live.vu.edu.au

Abstract: This paper presents an extension of a method from previously published work showing that discontinuous shifts, or step changes, are present in recent climate.

The global mean temperature (GMT) time series is a composite variable. It contains natural variability centered about Earth's biospheric heat. Historically, this has varied due to volcanic, geographic, solar and orbital effects. At the decadal scale, it has varied due to a complex set of internal variations, of which the Pacific decadal oscillation (PDO) and the Atlantic multi-decadal oscillation (AMO) are possibly the most influential. Anthropogenically-induced global warming, due to changes in land use, ozone depleting chemicals, and increasingly, greenhouse gases (GHG), leaves a complex signature in this time series. However, due to the need to convey information about climate change impacts to a wide and complex readership, the annual GMT series is regarded as the primary measure in monitoring the climate and its rate of change, and for comparisons with projections. Embedded in many projections, is the assumption that the rate of change should be largely free of discontinuities.

There is now an existing body of work based on a homogeneity test, the Maronna-Yohai bivariate test. This test was originally devised to assess single inhomogeneities in a serially independent time series. This paper describes an extension of this test to detect multiple shifts in the mean of a time series subject to external forcing. This has been achieved by the application of a set of decision rules designed to produce a robust set of results from noisy time series. It has been applied to estimated annual surface temperature at global, hemispheric, tropical, extra-tropical and polar scales

The procedure is presented as an objective test for determining the most likely time of change. The test's basis, equations and decision rules are described in some detail, and preliminary results are then presented, demonstrating that (a) climate regime shifts occurring at sub-global scale leave traces in the global record, and (b) recent shifts are at least as coherent as similar shifts found earlier in the instrumental record.

Keywords: Maronna bivariate, global temperatures, shift, step change, regime change

1. INTRODUCTION

The proposition that climate changes in a non-linear manner has been put forward by a number of researchers over the years but has not really gained favour for two reasons: one is that the methods for carrying out the analysis lack robustness and the other is that climate change approximates to a linear process on both observations and models. Theoretically however, the process of how the climate changes, is still an open question (Kirtman et al., 2013). Jones has used the bivariate test extensively over the years to show that climate in south-eastern Australia and in many other regions, changes by a series of steps (Jones, 2010; Jones, 2012; Jones et al., 2013), and hypothesized that they "may occur in the transition between decadal scale modes of climate variability". He used a bivariate test (Maronna and Yohai, 1978) to detect both univariate (temperature) and multivariate (temperature/rainfall co-variation) shifts.

This test has been widely used to assess step changes and inhomogeneities in climate and related variables, but is suited to assessing only one change in a time series. One work-around has been to divide a time series into segments, but this technique has a number of weaknesses. For example, the experimenter needs to make decisions about which segments to analyse in order, and whether a finding is likely to be true or false. The analysis can also only be semi-automated, so is not suitable for analysing large numbers of time series. For this reason, a rule-based probabilistic application of the bivariate test has been developed and is presented here.

Global mean annual temperature (GMT) is a composite estimate of regional temperature variations across the earth. Each ocean basin has its own systems of currents and gyres, which influence its long term patterns of temperature. These temperatures are strongly influenced by a heat memory, whereas over land they are not. The composite nature of GMT means that regional shifts in temperature would be expected to leave traces larger scale averages. GMT is also influenced by potentially confounding influences, such as autocorrelation, extrinsic events, and intrinsic, sub-decadal and multi-decadal regional processes.

This paper is structured as follows: the basic equations are given then the algorithmic components of the test are detailed in Section 2. A comparison of global, and zonal land, ocean and combine land/ocean temperature records is in Section 3. A brief discussion and conclusion comprises Section 4.

2. BIVARIATE TEST

The bivariate test appears an efficient equivalent to a sliding pair wise regression which finds the time point t_n which maximizes the absolute difference in estimated responses of a pair of linear regressions lines.

Given
$$y_t = \begin{cases} a_1 + b_1 x_t, t \le i \\ a_2 + b_2 x_t, t > i \end{cases}$$
 then $t_n = t : \max_{t \in 1..n} \left(\operatorname{abs} \left((a_2 + b_2 x_t) - (a_1 + b_1 x_t) \right) \right)$ (1)

The equations used are those published in (Bücher and Dessens, 1991).

Step 1. Standardize series.

$$\overline{X}' = \frac{\sum_{j=1}^{n} x_j'}{n}, \ \overline{Y}' = \frac{\sum_{j=1}^{n} y_j'}{n}, \ S_x' = \left(\frac{\sum_{j=1}^{n} (x_j' - \overline{X})^2}{n}\right)^{1/2}, \ S_y' = \left(\frac{\sum_{j=1}^{n} (y_j' - \overline{Y})^2}{n}\right)^{1/2},$$
$$x_j = \frac{(x_j' - \overline{X})}{s_x'}, \ y_j = \frac{(y_j' - \overline{Y})}{s_y'} \text{ for all } j \le n.$$
(2)

Step 2. Compute test statistics.

$$S_{xy} = \sum_{j=1}^{n} x_j y_j \tag{3}$$

$$X_i = \frac{\sum_{j=1}^i x_j}{i}, \ Y_i = \frac{\sum_{j=1}^i y_j}{i} \text{ for all } i < n$$

$$\tag{4}$$

$$F_i = n - \frac{X_i^2 ni}{(n-i)} \text{ for all } i < n$$
(5)

$$D_i = \frac{\left(S_{xy}X_i - nY_i\right)n}{(n-i)F_i} \text{ for all } i < n$$
(6)

$$T_{i} = \frac{[i(n-i)D_{i}^{2}F_{i}]}{(n^{2}-S_{xy}^{2})}, \text{ for all } i < n \text{ and } T_{i0} = \max(T_{i})$$
(7)

Let i_0^* be the value of *i* for which $T_i = T_{i0}$, the time after which a change occurred. Its successor is the first time of the new regime. D_i^* is defined as the maximum likelihood estimator of a shift at i_0^* . T_{i0}^* is the test statistic that tested against some constant, discriminates with a specified probability, a null hypothesis H₀ of no shift against H₁ that a shift exists (Maronna and Yohai, 1978). A mean shift can be computed as $\Delta \bar{y} = D_{i0}^* S_y$. For the null trend case, analyzed in Maronna and Yohai (1978), critical values of T_i are given for probabilities of (0.25, 0.1, 0.05, and 0.01) for the null hypothesis of no change, given time series lengths *n* of 10, 15, 20, 30, 40 and 70. Potter (1981) provides these for *n*=100.

Based on the published values, I have fitted an empirically derived interpolating function, using a genetic programming framework, Eureqa (Schmidt and Lipson, 2013), with its default error metric of mean absolute deviation, to generalize these results. Given data and a user defined objective function, Eureqa produces sets of equations ranked by complexity/accuracy. The objective function was $T^{critical} = f(Pr, n)$. The top ranked equation was selected and encoded as below (Equation 8).

Let *n* be the time series length and *P* the probability of interest,

Let
$$p_1 = n - 0.045266/(nP(1 - 8.4878))$$
, and $p_2 = 4.2994 - 3.6824P$
 $T_i^{critical} = \begin{cases} p_2, p_1 \le 0\\ p_2 - 0.40572 \cdot ln(P) \cdot ln(p_1), p_1 > 0 \end{cases}$
(8)

2.1. Probabalistic version

The bivariate test above has been coded in the Python language and incorporated in a computing framework, the probabilistic bivariate test (PBV). Since a single variate is being tested, we use a flat random time series as the second variate, following Vives and Jones (2005). The important outputs of the test in a time series of length N are, (1) The T_i statistic, which is defined for times i < N, (2) the T_{i0} value, (3) i_0 , the time associated with T_{i0} , and (4) shift at that time. The probability $Pr(T_{i0})$ can be computed by a bisection method from the function $T_i^{critical}$.

The outer loop is iterated 100 times, and from this a small set of step-series is returned (most commonly just one). I define the majority count as the *degree of consensus*. If more than one step-series is returned then these can be further analysed, however in this paper the consensus list is returned as the result of the analysis.

Outer loop. A single iteration of the test consists of a screening pass then convergent pass (see below). It returns a list of break-years that segment a time series such that each shift is statistically significant at a selectable level (default is Pr=0.01) based on $T_i^{critical}$. Whilst most often, all 100 iterations return identical lists, analysis of some time series will several variations – mostly as individual shift dates move by a year or so. Occasional datasets are encountered where distinct subsets of dates are detected. The most common – the modal – list is returned as the result of the analysis. At all times within this loop, all breaks are determined by applying a *resampling test* (below).

Resampling test. The bivariate test is repeated 100 times using different random sequences and the i_0 values and means of the associated T_{i0} , and shifts are collated. On the *screening pass* only modal values are examined. On the *convergent pass* the modal and the second modal values (if present) are returned. The first modal value (i.e. most frequent) is returned as i_0 for those runs. The mean of T_{i0} , $\overline{T_{i0}}$, and shift hose values associated with i_0 . A segment contains a breakpoint in position i_0 if $\overline{T_{i0}} \ge T_i^{critical}$.

Binary segmentation. This is a recursive segmentation technique. The entire time series is analysed for a single break-point using the resampling test. If $\overline{T_{i0}} \ge T_i^{critical}$, then the segment up to and including i_0 is analysed for an earlier break, and the segment after i_0 is analysed for a later break. This process is repeated for the sub-segments so formed until no significant breaks are found. The result is a series of break-points which are then refined on the convergent pass. It should be noted that as break-points found on this pass are returned on the basis of a recursive process, end point effects may perturb the results.

Screening pass. This pass produces a provisional list of break-points using binary segmentation, which serves as a starting point for the convergent pass (below).

Convergent pass. The list of *n* break-points breaks the original time series into s=n+1 segments. The algorithm then works its way from earliest to latest segments combining consecutive segments into one, and then searching within that segment using binary segmentation to produce new estimates of included shift dates. These estimates are processed by the decision rules below. There are two special cases, segments 1 and *s* which are analysed individually at either end of this process to cover the impact of end point adjustments. This pass iterates until a break-point list is found for the second time, and this is returned as the result.

Decision rules

- A tunable prohibition period defaulting to seven years is applied after a break-point before another point will be accepted, to minimize false positives associated with sub-decadal variation and redness.
- If the modal year is within the minimum defined prohibition interval of the last discovered then the two are compared. Firstly a resample test is conducted by extending the segment backwards to the start date of the segment within which the last discovered one was defined. If it is a valid break this then replaces the previous break otherwise the previous break is retained, and as small "safety margin" is added for one iteration to the low bound of the first segment next time round.
- If the modal value is >= 90% or the modal value is >50% and the second modal value > 20% then the modal year is accepted, else it is dropped.
- If a segment contains a single break-point that break-point is retained.
- If a segment no longer contains a break-point then the segment and the next are merged and treated as a single segment on the next iteration.
- If the candidate list contains more than one point then the earliest two are retained and the rest discarded. The two points are then trialed using a resampling test to determine if the interval up to the later of the two still contains a break, and if this is still present, it is retained. If not then the second candidate is similarly tested.

2.2. Sensitivity of the bivariate test



Figure 1. Error in absolute years for the identification of a shift point located randomly from years 25 to 75.

Estimates of the year of shift given by the bivariate test could be impacted by amongst other things, a coincident trend, the timing of the shift within the year and the extent of the shift. To gain an idea of the sensitivity of the estimate of the year to these factors, 4,500 trials of the bivariate test were conducted using random sequences to represent stationary climates. These were perturbed by adding various uniform trends, and various step changes commencing after randomly selected points. The step value at the selected time was adjusted to represent the effect of a shift commencing on any of the months. A representative equation was sought linking the absolute error of time (ε) to the month (M), shift (S), and trend (T), again using Eureqa similarly to the determination of Equation 8, with an objective function of $\varepsilon = f(M, S, T)$. The best ranked function, $(\varepsilon = 59.35 \cdot e^{-12.57 \cdot S} + 0.0878 \cdot M)$ shows that once a shift exceeds 1/3 standard deviations, error is low

and dominated by the uncertainty of the time of shift within a year (See Figure 1). Trends were not influential of the timing once a definite shift was present.

3. COMPARING GLOBAL AND ZONAL LAND, OCEAN AND COMBINED LAND/OCEAN

Annual files in ASCII format covering surface type of Land, Ocean, and combined Land and Ocean were downloaded on 29 May 2015 from National Climatic Data Centre (NCDC) via <u>ftp://ftp.ncdc.noaa.gov/pub/data/mlost/operational/products/</u>.

The zonal averages were over latitude ranges of 90°S-90°N (Global), 90°S-0°S (Southern hemisphere), 0°N-90°N (Northern hemisphere), 90°S-20°S, 60°S-30°S, 60°S-60°N, 30°S-0°N, 0°N-3°0N, 20°S-2°0N, 20°N-90°N, and 60°N-90°N. Data in the files labelled as 90°S-60°S for all three subsets was clearly corrupted and was not used. In total there were 33 datasets.



Figure 2. Mean annual temperatures for the globe and two hemispheres plotted with resultant trends. Significance levels of >0.05, 0.05, 0.01 and 0.001 indicated with 'n/s', *, **, and *** respectively. 'S' or 'T' indicate which of shift or trend change are more significant. 'R' indicates that by ANCOVA the two piece regression is preferred indicating combined shift and trend change. Magenta is Land, Blue is Ocean, Green is Land and Ocean.

The probabilistic bivariate test was run for all 33 datasets. Of these 100% shift-year set consensus was achieved for all but seven analyses. Of these seven only one, (ocean $60^{\circ}N-90^{\circ}N$) had more than one alternate.

Due to its assumptions the bivariate test tends to detect shifts that more conventional approaches may reject. To further assess the shifts every one was tested within the bounds of its neighbouring breaks by ANCOVA (scripted in R) to determine four diagnostics; (a) significance of preference of a two piece regression over a single piece regression, (b) significance of change of trends, (c) significance of shift and, (d) significance of joint independent probabilities of trend change and shift. Criteria (a) and (d) differ in their assumption of independence of a change of trend and of shift. In particular, if there is a significant shift coupled with a change of trend, such that the second line of a paired regression intersects the line of the single regression after the shift date (a "post shift convergence"), then it is possible that criterion (c) will be met and (a) will not.

Table 1 summarizes the outcomes. Shifted years delineated by the probabilistic bivariate test test (the year after the year of i_{0} are listed by decade for each dataset and marked up according to the criteria which they met at Pr<0.05. For the most part breaks either remain significant by some criterion or occasionally are correlated with significant breaks in other series. No breaks were found the 1940s and only one in the 1890s (and that did not meet any of the above criteria). Figure 2 illustrates the global and hemispheric results with the most significant criterion and its significance. Notably, at these scales shifts dominate pre 1970. Post 1970, even where trend changes are seen in the late 1970s and mid 1980s they interact with shifts. The phenomenon of regional shifts showing as traces in more global records is demonstrated by the difference between the shifts in the Southern hemisphere and globally of 1979. In Table 1,a shift over land in 1956 seems to relate to the Southern tropics and otherwise only shows in overlapping zones.

At least one shift was highly significant (Pr<0.001) by criteria (a) and (c) in all data sets. In 15 data sets there was at least one shift date which was not significant (Pr>0.05) by criterion (a). However, of these by criterion (d), six were significant at 0.05 level and two only at the 0.10 level. In four cases, whilst neither the change of trend nor the shift were especially significant, the two-piece regression was preferred (Pr<0.01) by criterion (a).

The two most wide-spread shifts are those associated with 1977/79, the so-called Pacific reorganization, and 1997 which just precedes a record El Niño in 1998; both in the latter half of the record. The so-called hiatus period (commencing between 1995and 1997 and continuing until now) is of interest due to much recent

debate. Corresponding years were found in 28 data sets. Of these, six were not significant by criterion (a) and all of these show post-shift convergence. The five in which it was not found were *Land* and *Land and Ocean* 60°N-90°N, and the *Southern Hemisphere Land* sets, 90S-20°S, 90S-0°S, 60°S-30°S, and *Tropical Land* 20°S-20°N. However there is a tendency for a shift to occur round 2001 in the *Southern Hemisphere Land*.

Table 1. Shift dates, the first shifted year, determined by PBV are shown by decade. Exponents on the years
indicate if the point remained significant under an ANCOVA and why. a-two segment preferred, b-trend
change, c-step, d-only joint independent probability of step and trend.

Zone	Cover	1800s	1900s	1910s	1920s	1930s	1950s	1960s	1970s	1980s	1990s	21C
90°S-90°N (Glob)	Land				25 [°]				80 ^{ab}		97 ^c	
90°S-90°N (Glob)	Ocean	90 ^{ac}			30 ^{ac}				77	87	97 ^c	
90°S-90°N (Glob)	Globe				30 ^{abc}				79 ^{abc}		97 ^{ac}	
00°N-90°N (N Hem)	Land			2	1 ^c				80 ^b		97 [°]	
00°N-90°N (N Hem)	Ocean		03 ^{abc}		26 ^{abc}					87 ^a	97 [°]	
00°N-90°N (N Hem)	Globe				25 ^{abc}					87 ^{ac}	97 ^{ac}	
90°S-00°N (S Hem)	Land					37 ^c	57 [°]		79 ^{ac}			02
90°S-00°N (S Hem)	Ocean	90 ^{ac}				37 ^c		69 [°]	79 ^{ac}		97 ^{ac}	
90°S-00°N (S Hem)	Globe	90 ^{ac}				37 ^c		69 [°]	79 ^{ac}		97 ^c	
00°N-30°N (N Tropics)	Land		03 ^{ac}		26 ^{ac}				79 ^a		98 ^{ac}	
00°N-30°N (N Tropics)	Ocean		07 ^{ac}		26 ^{ac}					87 ^{ac}		
00°N-30°N (N Tropics)	Globe		03 ^{ac}		26 ^{ac}				79 ^a		97	
20°S-20°N (Tropics)	Land		04 ^{ac}		26 ^{ac}		57		79 ^d		97	
20°S-20°N (Tropics)	Ocean				26 ^{abc}				79 ^a		97	
20°S-20°N (Tropics)	Globe					36 ^c			79 ^{ac}		97	
30°S-00°N (S Tropics)	Land					37 ^{ac}	57 ^{ac}		78 ^{ac}			01 ^d
30°S-00°N (S Tropics)	Ocean					37 ^{ac}			78 ^{ac}		97 ^c	
30°S-00°N (S Tropics)	Globe					37 ^{abc}			78 ^{ac}		97	
30°N-60°N (N Mid Lat)	Land	94		21 ^{ac}					80 ^{ac}		97 ^{ac}	
30°N-60°N (N Mid Lat)	Ocean		02 ^{ac}	14 ^{ac}	30 ^a			64 ^{ac}		89 ^{ac}	98 ^{ac}	
30°N-60°N (N Mid Lat)	Globe			2:	1 ^{ac}					88 ^{ac}	97 ^{ac}	
60°S-30°S (S Mid Lat)	Land					38			76 ^{ac}			03 ^a
60°S-30°S (S Mid Lat)	Ocean	87 ^{ac}				37 ^{ac}		70 ^a	76 ^a		96 ^{ac}	
60°S-30°S (S Mid Lat)	Globe	87 ^{ac}				37 ^{ac}		68 ^a	76 ^{ab}		96 ^{ac}	
60°N-90°N (N Polar)	Land			20 ^{ac}						88 ^{abc}		
60°N-90°N (N Polar)	Ocean				26 ^{ac}						2000	abc
60°N-90°N (N Polar)	Globe			20 ^{ac}						88 ^{ac}		02
60°S-60°N (Non polar)	Land			21		38			78 ^{abc}		97 ^{ac}	
60°S-60°N (Non polar)	Ocean	90 ^{ac}			30 ^{ac}				76	87 ^c	97 ^{ac}	
60°S-60°N (Non polar)	Globe		03 ^{ac}	14 ^{ac}	25 ^d	37 ^{ac}			78 ^{abc}		97 ^{ac}	
20°N-90°N (N Ex Trop)	Land				1 ^c					87 ^{ac}	98 ^{ac}	
20°N-90°N (N Ex Trop)	Ocean		02 ^{ac}	15 ^{abc}	30 ^{abc}			64 ^{ac}		88 ^{ac}	97 ^{ac}	
20°N-90°N (N Ex Trop)	Globe				24 ^{ac}					87 ^{ac}	98 ^{ac}	
90°S-20°S (S Ex Trop)	Land				26 ^{ac}		57 ^{ac}		76 ^{ac}			02
90°S-20°S (S Ex Trop)	Ocean	87 ^{ac}				37 ^{ac}		69 ^a	76 ^a		96 ^{ac}	
90°S-20°S (S Ex Trop)	Globe	87 ^{ac}				37 ^{ac}		69 ^a	76 ^{ac}		97 ^{ac}	

4. DISCUSSION AND CONCLUSIONS

The step changes identified correspond well to those previously reported, for example page 27 of Jones et al. (2013); see also references therein.

A step change in 1997 is global in extent. A step change circa 1979, associated with a trend change, is present globally but absent in the Northern mid, and high, latitudes. Shifts in the global ocean in 1977 and 1987, which were not classed as significant by ANCOVA, were consistent with shifts elsewhere that were. Other shifts are more variously regional, some present only over land or over ocean. Thus, regional shifts leave traces in the mean global record, and these are detectable by the PBV test. The test not only locates times of shifts in the mean, but when its assumptions hold it provides estimates of shift and likelihood. To

date when analysed this way, the significance of shifts in current climate records is mostly greater than significance of change of trend.

Uncertainties present in the determined shift-dates spring from (a) uncertainty of the time in the year of onset, since breaks starting in later months will contribute less to the average, (b) variability in shift sizes, (c) non-stationarity of internal, variability including trend changes, and (d) extrinsic events and transient events such as El Niño. The probabilistic nature of the test helps to reduce the impacts of uncertainties due to (a) and (b). Concerning (c), non-stationarity, notably trend and/or trend change, can affect the significance of shift-points within the bivariate test, mostly by inflating the significance of shift. False positives can be detected in post-processing, whereas false negatives are difficult, hence false positives are deemed preferable. Even in the presence of substantial trend it is capable of accurately locating shifts of less than 0.4 of the standard deviation of synthetic data. Concerning (d) the consideration of the second modal value and the prohibition period limits the impact of periods of redness, and transience in the data.

This work has treated the probabilistic bivariate test as an exploratory tool and used secondary analysis to quantify the results. This can utilise many other approaches, either using frequentist statistics, or even by taking a Bayesian style approach and comparing its findings to climatic events and other findings. Even though the ANCOVA analysis does not show 1997 to be significant in the tropics for example, it *was* found by the probabilistic bivariate test and is coincident with strongly significant shifts elsewhere on the globe.

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