A scaling approach to capture sub-time-step rainfall variability in rainfall-runoff and erosion modelling

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Abstract: Scaling in space and time is a fundamental problem in hydrological and erosion modelling. Owing mainly to the lack of adequate data to support modelling at the process timescales, modelling of small time scale processes using coarse time scale data is often undertaken. Generally process descriptions are not modified but rather effective parameter values are used. A similar approach is taken spatially. It is generally recognized that this approach to scaling in process-based modelling can be problematic when non-linear interactions occur. This paper presents a method for modelling surface runoff and erosion processes that accounts for sub-timestep variability in rainfall while retaining a daily timestep and utilizing daily rainfall totals. The method uses the cumulative distribution function (CDF) of rainfall intensities to represent the effect of temporal variability of rainfall at a time-scale of minutes. While any CDF can be used, the rainfall intensity distribution model chosen here is the lognormal distribution. The distribution parameters are determined from daily rainfall totals. The rainfall CDF is modified by the interception, infiltration, and saturation excess processes to derive CDFs of throughfall, infiltration and surface runoff. These are then applied to the erosion algorithm to determine the erosion CDF. The resulting CDFs are integrated to predict respective daily loads. While a specific model is used here, it is worth noting that the approach used for rainfall scaling here is general and could be applied to many (but not all) rainfall-runoff and erosion models.

Keywords: *Temporal Scaling; Rainfall-Runoff; Erosion Modelling; Distribution Function*

1. INTRODUCTION

Scaling is a fundamental problem in hydrologic and erosion modelling. Variability, nonlinearity and the interacting nature of processes over various scales significantly influences the mechanics of surface runoff generation, erosion/deposition, and sediment production at all scales. In particular, the temporal dynamics of precipitation strongly influences the runoff and resulting soil erosion, due in particular to the nonlinear nature of infiltration, soil detachment and transport processes. The current generation process-based models describe detailed understanding of the processes at fine spatial and temporal scales but require extensive data. The scales of readily available data, the scales of processes being represented and the scales of the model application are wide and varied and in many occasions, mismatched (e.g. Beven, 1993; Blöschl and Sivapalan, 1995; Grayson *et al.*, 1993). In recent years, scaling-up of process-based models has received increasing attention to address large-scale issues and to make use of commonly available data.

Surface erosion is mainly initiated by the impact energy of falling raindrops on the soil surface and surface runoff transports sediment entrained into the flow (Hairsine and Rose, 1991; Rose et al., 1994). These processes occur instantly at the time scales of a minute or two. Therefore, both surface runoff and erosion are modelled better at fine time scales than at daily scales (e.g. Kandel et al., 2002; Kandel et al., *in press*). Past studies indicate that peak rates of rainfall for rainfall-runoff modelling, and peak rates of both rainfall and runoff for erosion modelling are the most important hydrologic variables (e.g. Kandel et al. *in press*; Yu et al., 1998). These studies suggest that the process-based hydrologic and erosion models ideally require fine time scale input. Yet, the option available to modellers often is to use only daily scale observations. As a consequence, modelling of the fine time scale processes using coarser time scale data has become the norm in the hydrologic and erosion modelling community. The time-averaging involved causes a considerable loss of detail in the temporal distribution of rainfall and runoff, resulting in significant attenuation of the peak rates that are critical to the processes involved.

The time scale difference between the physical processes and the models significantly affects the results of process parameterisation and simulation (e.g. Cameron et al., 2001; Mertens et al., 2002), often leading to a reduction in model performance (e.g. Kandel et al., *in review*). Rainfall scaling has significant scope for modelling of these processes at various time scales since it tends to preserve prediction quality as time scales become coarser (Kandel et al., *in press*). In this paper we develop an alternative approach by scaling rainfall that captures the fine time scale processes while retaining the coarser modelling time steps.

2. TEMPORAL SCALING APPROACHES

Four potential solutions to the problem of representing fine (minutes) time scales using daily information in surface runoff and erosion models are described below. These are (1) the use of effective parameters, (2) the use of effective rates, (3) disaggregation of daily rainfall data, and (4) a distribution function approach.

2.1 Effective parameter approach

In this approach, parameters are identified at the modelling time step irrespective of the time scales of processes included in the model. The resulting parameters are used to simulate the processes at modelling scales. This is mostly the case with surface runoff and erosion modelling studies, in which models are run at coarse time scales (e.g. daily). These temporally up-scaled parameters are assumed to effectively account for the fine time scale processes. To some extent, this approach is successful. However, in addition to the parameters losing any physical significance, this approach leads to a reduction in model performance compared with fine timescale computations due to the time-averaging effect. This is particularly problematic for erosion simulations (Kandel et al., *in review, in press*).

2.2 Effective rate approach

Rainfall and runoff rates are the most important hydrological variables in process-based soil erosion modelling that are attenuated when timescales move from finer to coarser. A common approach is to assume uniform conditions within a spatial/temporal unit and represent these rates by single effective values, purely as a compromise between the peak instantaneous rates that cause most of the erosion and the average rate at the coarse time scale.

Effective rates can be estimated from instantaneous rates (e.g. Rose and Yu, 1998) or from daily data (e.g. Kandel et al., *in press*). Owing to limited availability of data with instantaneous rates, determination of effective rates from the daily data has a practical advantage in that it can be applied when only daily information is available. Kandel et al. (*in press*) applied this approach in modelling surface runoff and erosion at the plot scale by calculating effective rates of rainfall and runoff using relationships empirically derived via model calibration, and the model was run at daily time steps with fine time scale process parameters. They found that this approach is a considerable improvement over the effective parameter approach; however, it increases the number of model parameters substantially.

2.3 Disaggregation approach

Ideally high temporal resolution (minutes) pluviograph data would be used to model erosion. Where only daily data are available, it is possible to disaggregate daily rainfall totals to an appropriate process time scale, using stochastic disaggregation approaches and then run a model at fine time steps.

Various studies show that model performance is improved when disaggregation approaches are used to enable simulation of the physical processes at shorter time scales (e.g. Kandel et al., *in press*; Socolofsky et al., 2001; Mertens et al., 2002). Kandel et al. (*in press*) showed that this approach is comparable to simulations using pluviograph data and that it produces better results than the effective parameter or effective rate approaches mentioned here for erosion plots from Nepal. This may tempt many researchers to increase the temporal resolution of the models until it is appropriate to explicitly represent the short time-step processes but this can lead to potentially high computational demands and there are conceptually and practically more appealing approaches.

2.4 Distribution approach

Rather than using knowledge of the statistical distribution of rainfall during a day to disaggregate daily totals, it is often possible to use the distribution information more directly. This is where a distribution approach differs from disaggregation. It is an improvement over the disaggregation in terms of saving a great deal of computational demand. It can be considered as a compromise between fine and coarse time scale simulations in which models are modified to use a distribution directly enabling simulation at coarser time-steps. In this approach, a distribution function representing the instantaneous rates of rainfall is used to capture the effect of sub-daily temporal variability of rainfall at a daily time-step. Therefore, the essence of distribution approach is computational efficiency. Distribution function approaches have been utilized to represent spatial variability of soil moisture in particular (e.g. Beven and Kirkby, 1979; Sivapalan and Woods, 1995; Wood et al., 1992, Wooldridge et al., 2002; Zhau et al., 1992) and use of distributions has been suggested by Beven (1995) as a more general scaling tool.

3. MODELLING RAINFALL-RUNOFF AND EROSION BY DISTRIBUTION APPROACH

The rainfall-runoff and erosion model used here is a hybrid that simulates interception and uses the infiltration model from GUEST after modification by Kandel et al. (2002) to calculate Hortonian runoff. It also incorporates a bucket-type storage concept for saturation excess runoff and uses a one-parameter erosion algorithm for erosion prediction. It is described by Kandel et al. (*in press*, *in review*) and here only the distribution version is described.

3.1 Rainfall distribution

In this study, a cumulative distribution function (CDF) is used to represent the sub-daily temporal variability of daily rainfalls. Although there are many rainfall distribution models, none appears clearly superior. While any CDF can be used, the distribution chosen here is the two-parameter lognormal distribution. The parameters are predicted empirically from daily rainfall totals using relationships developed from pluviograph data. It is worth noting that the approach used for rainfall scaling here is general and could be applied to many (but not all) rainfall-runoff models.

Assuming a lognormal distribution of rainfall in time, the rainfall intensity, $p_j(\rho)$ in mm/h corresponding to the cumulative probability (ρ) within the wet part of a day can be expressed as:

$$p_j(\rho) = e^{\mu + \sigma z(\rho)} \tag{1}$$

where μ and σ are distribution parameters, and $z(\rho)$ is the normal standard variate.

The daily rainfall reproduced by the model ($\widetilde{P}$) is obtained by integrating rainfall distribution over wet period of the day as given below:

$$\widetilde{P} = 24\, WF \int_0^1 p_j(\rho)\, d\rho \tag{2}$$

The mean of the modelled incremental intensities ($\overline{P_j}$) during the wet period of the day and the observed 24-hour mean intensity ($\overline{P}$) are estimated as shown in equations (3) and (4) respectively.

$$\overline{P_j} = \frac{\widetilde{P}}{24 \times WF} \tag{3}$$

$$\overline{P} = \frac{P}{24 \times WF} \tag{4}$$

where WF is wet fraction of a rainy day and P is observed daily rainfall in mm.

Both distribution parameters (μ and σ) are strongly correlated with the mean intensity over the wet period, $\overline{P}$ (see Figure 8; Kandel et al., *in press*). It is found that a simple log-linear regression is able to adequately describe the relationship of σ with $\overline{P}$ as shown in equation (5).

$$\sigma = k_1 \ln\left(\overline{P}\right) - k_2 \tag{5}$$

μ can then be calculated analytically as:

$$\mu = \ln\left(\overline{P}\right) - \frac{\sigma^2}{2} \tag{6}$$

The constants k_1 and k_2 in equation (5) have been determined using two-minute tipping bucket data (1997 - 98) from Jhikhu Khola catchment in Nepal and six-minute pluviograph data (1990 – 96) from Gunnedah, NSW in Australia. They are respectively 0.55 and 0.87 for the Nepalese sites, and 0.24 and – 0.65 for the Australian sites. These are the generalised parameters, which actually drive the distribution model.

3.2 Surface runoff and erosion distribution

The rainfall distribution is transformed into surface runoff and erosion distributions by applying the rainfall CDF given in equation (1) to runoff and erosion algorithm. The rainfall CDF is modified by the interception, infiltration, and saturation excess processes to derive CDFs of throughfall, infiltration and surface runoff. The CDFs of throughfall and surface runoff are then applied to the erosion algorithm to determine the erosion CDF. The resulting CDFs are integrated over the wet period to predict daily totals. The model runs with daily rainfall values as input at daily time steps but captures the processes that occur at small time scales of the order of minutes. Algorithms for surface runoff and erosion distribution model are described below.

The throughfall distribution, $p_t(\rho)$ can be estimated as:

$$p_t(\rho)=\left(\frac{P_t}{P}\right)p_j(\rho) \tag{7}$$

where P_t is the daily throughfall in mm that can be calculated as:

$$P_t = Max\begin{cases} 0 \\ P-\frac{C_{\max}-C_0}{\Delta t}-E_{ac} \end{cases} \tag{8}$$

where C_0 is the initial canopy storage (mm) for the day, C_{max} is the canopy interception capacity (mm), Δt is the time-step (i.e. daily), and E_{ac} is the actual evapo-transpiration from vegetation and canopy (mm) within the day.

The infiltration distribution, $i(\rho)$ is:

$$i(\rho) = I_p\left(1-e^{-\frac{p_t(\rho)}{I_p}}\right) \tag{9}$$

where I_p is the spatially averaged potential infiltration capacity in mm/h, time dynamics of which is linked with temporal variation of soil moisture (Kandel et al., 2002). The daily infiltration total (I) in mm is obtained by integrating the infiltration distribution over the wet period of a day as given below.

$$I = 24WF\int_0^1 i(\rho)d\rho \tag{10}$$

The Hortonian runoff distribution, $r_{IE}(\rho)$ is the difference between the throughfall and infiltration distributions.

$$r_{IE}(\rho) = p_t(\rho) - i(\rho) \tag{11}$$

The daily total of infiltration excess runoff, R_{IE} in mm is

$$R_{IE} = 24WF\int_0^1 r_{IE}(\rho)d\rho \tag{12}$$

The soil storage is updated for the day by adding the infiltration. It is noted that the canopy part of evapo-transpiration from above ground surface is already accounted for in the throughfall model (i.e. equation 8). Other losses, namely evapo-transpiration from ground surface and deep seepage are calculated and removed, and any excess water becomes saturation excess runoff (R_{SE}). This can be estimated as:

$$R_{SE} = Max\begin{cases} 0 \\ \frac{S_0-S_{\max}}{\Delta t}+I-E_{as}-K_z \end{cases} \tag{13}$$

where S_0 is initial soil storage (mm) for the day, S_{max} is soil storage capacity (mm), E_{as} is actual evapo-transpiration from soil surface (mm) within the day and K_z is sub-surface vertical hydraulic conductivity for the day. It is assumed that the occurrence of saturation excess runoff is independent of $i(\rho)$. This implies that the saturation excess runoff distribution, $r_{SE}(\rho)$ can be modelled as proportional to the infiltration distribution as given in equation (14).

$$r_{SE}(\rho) = \left(\frac{R_{SE}}{I}\right)i(\rho) \tag{14}$$

The distribution of surface runoff, $r(\rho)$ includes both the infiltration excess and saturation excess distributions as shown in equation (15).

$$r(\rho)=r_{IE}(\rho)+r_{SE}(\rho) \tag{15}$$

This is integrated to daily runoff total, R in mm as shown in equation (16).

$$R = 24WF\int_0^1 r(\rho)d\rho \tag{16}$$

The soil erosion distribution, $d_{er}(\rho)$ is obtained by incorporating throughfall and runoff distributions into the erosion algorithm as shown in equation (17).

$$d_{er}(\rho)=\frac{K_{er}\,p_t(\rho)\,r(\rho)\,S_f\,SDR}{3600000} \tag{17}$$

where K_{er}, S_f and SDR are respectively spatially averaged soil erodibility in kg s/m^4, slope factor, and sediment delivery ratio. The daily soil erosion total, D_{er} in g/m^2 is obtained by integrating the erosion distribution over the wet period of a rainy day as given below.

$$D_{er} = 24WF\int_{0}^{1} d_{er}(\rho)d\rho \qquad (18)$$

The integration of the distribution functions in equations (2), (10), (12), (16) and (18) are performed numerically. This is done by dividing the cumulative distribution function (cdf) into a number of uniform increments in the range $0 < \rho < 1$ for each rainy day. It adds computational demand compared to the single daily run models but is very computation-efficient compared to the sub-hourly models aiming to capture the fine time-scale processes. A preliminary study indicates that the distribution function model, numerically integrating these equations with a cdf division of 20 uniform increments for a day is able to produce similar simulated results to a two-minute time-step model (i.e. 1440/2 = 720 runs for a day). This indicates a computational saving of 36 times (i.e. 720/20).

4. DISCUSSION AND CONCLUSION

Scaling in space and time is a fundamental problem in hydrologic and erosion modelling. A common approach is to assume that the effects of small time and space scale processes can be adequately modelled using coarser scale data with effective parameters that are derived by calibration. While this approach has been successful in rainfall-runoff modelling, it can cause significant reductions in model performance when used for erosion modelling. This is because erosion depends mainly on the peak rainfall and runoff rates at sub-hourly time scales, rather than daily average rates used in daily time-step modelling.

In light of the scaling issues discussed above, this paper briefly discusses four methods of overcoming these problems and then presents a method for accounting for fine time-scale variations in rainfall intensity when modelling surface runoff and erosion processes at coarser timesteps. The method uses a cumulative distribution function (CDF) of rainfall intensities to represent the effect of sub-daily temporal variability of daily rainfall. The rainfall distribution model used here is lognormal but the scaling approach is general and any CDF can be used, and the distribution parameters are determined from daily rainfalls. The rainfall CDF is modified by the hydrologic and erosion processes to derive CDFs of surface runoff and erosion, which are then integrated over the wet period in a day to get daily totals.

The model is meant to run at a daily time-step but is able to represent the small time scale processes that occur at the temporal resolutions of the order of minutes. This supports continuous simulation keeping track of storage characteristics and saves a great deal of computational demand compared to the sub-hourly or finer time-step models. It is hoped that the approach will be, particularly useful to improve the predictive efficiency of the daily surface runoff and erosion models in making use of daily time scale hydro-meteorological information available around the world. The method is simple and could be applied to many (but not all) rainfall-runoff and erosion models, particularly to those aiming to capture the sub-daily time scale fluxes with no explicit sub-daily temporal pattern. The work is continuing on testing the approach at various sites in Nepal and Australia.

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