Max-Min Ant System Applied to Water Distribution System Optimisation

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Abstract: Water distribution systems (WDSs) are costly infrastructure in terms of materials, construction, maintenance and energy requirements. Much attention has been given to the application of optimisation methods to minimise the costs associated with such infrastructure. Historically, traditional optimisation techniques have been used, such as linear and non-linear programming, but within the past decade the focus has shifted to the use of Evolutionary Algorithms, for example Genetic Algorithms, Simulated Annealing and more recently Ant Colony Optimisation (ACO). Advancements on the basic formulation of ACO have been developed, these advancements differ from one another in their utilisation of information learned about the search-space to manage the trade-off between exploitation and exploration in the algorithms searching behaviour. Exploration is the algorithms ability to search broadly through the problems search space and exploitation is the algorithms ability to search locally around good solutions that have been previously found. One such advanced ACO algorithm, which is presented within this paper, is the Max-Min Ant System (MMAS). This algorithm encourages local searching around the best solution found in each iteration while implementing methods to slow convergence and facilitate exploration. The performance of MMAS is compared to that of the most basic ACO formulation Ant System (AS) for two commonly used WDS case studies. The sophistication of MMAS is shown to be effective as it outperforms AS for both case studies and performs competitively in comparison to other algorithms in the literature.

Keywords: *Ant Colony Optimisation; Water Distribution Systems; Exploitation; Exploration*

1. INTRODUCTION

Due to the high costs associated with the construction of water distribution systems (WDSs) much research over the last 25 years has been dedicated to the development of techniques to minimise the capital costs associated with such infrastructure. This process has been given the title of "optimisation" or "optimal design" of WDSs.

Within the last decade, many researchers have shifted the focus of WDS optimisation from traditional optimisation techniques based on linear and non-linear programming to the implementation of Evolutionary Algorithms (EAs) namely; genetic algorithms (GAs) (Dandy *et al.* 1996, Savic & Walters 1997, Lippai *et al.* 1999, Wu *et al* 2001), simulated annealing (Cunha & Sousa 1999) and ant colony optimisation (ACO) (Maier *et al.* 2003). Noted advantages that exist with the use of EAs for application to WDSs are; (i) only commercial pipe diameters are considered (e.g. they treat the optimisation problem as discrete), (ii) they deal only with objective function information and avoid complications associated with determining derivatives or other auxiliary information, (iii) they are global optimisation procedures, and (iv) as they deal with a population of solutions numerous optimal or near-optimal solutions can be determined.

Due to the iterative nature of the solution generation of EAs, they can be intuitively seen as algorithms that incrementally search through the solution-space using knowledge about solutions that have already been found to further guide the search. The searching behaviour of EAs can be characterised by two main features (Colorni *et al.* 1996), (i) *exploration*, which is the ability of the algorithm to search broadly through the solution-space and (ii) *exploitation*, which is the ability of the algorithm to search more thoroughly in the local neighbourhood where good solutions have previously been found. By definition, these attributes are in conflict with one another.

ACO is an EA based on the foraging behaviour of ants (Dorigo *et al.* 1996). It has seen a wide and successful application to many different optimisation problems (see Dorigo *et al.* (1999) for an overview) and recently it has been seen to perform very competitively for WDS optimisation (Maier *et al.* 2003).

Advancements have been developed on the initial and most simple formulation of ACO, Ant System (AS) (Dorigo *et al.* 1996), to improve the operation of the decision policy and the manner in which the policy incorporates new information, to help in exploring the search space. These developments have primarily been aimed at managing the trade-off between the two conflicting search attributes of exploration and exploitation.

Many notable advances on the simple AS have been developed (Dorigo *et al.* 1999), however, only one of these is considered in this paper; the Max-Min Ant System (MMAS) (Stützle & Hoos 2000).

The objective of this paper is to assess the efficacy of the additional mechanisms incorporated in the Max-Min Ant System, compared to the more basic Ant System, for WDS optimisation. To undertake this, a comparison between the performance of AS and MMAS for two case studies has been presented. These algorithms have also been compared to the best performing algorithms previously presented in the literature for the two case studies considered.

2. THE WATER DISTRIBUTION SYSTEM OPTIMISATION PROBLEM

The optimisation of WDSs is loosely defined as the selection of the lowest cost combination of appropriate component sizes and component settings such that the criteria of demands and other design constraints are satisfied. In practice the optimisation of WDSs can take many forms as WDSs are comprised of a number of different components and have many different performance criteria.

Traditionally in the literature the optimisation of WDSs has dealt with a relatively simple and idealised problem. The decision variables have primarily been the pipe diameters within the system, where more specifically, the decision options have typically been the selection of (i) a diameter for a new pipe, or (ii) a diameter for a duplicate pipe. The design constraints on the system have normally been the requirement of minimum allowable pressures at each of the nodes. In addition to the design constraints the hydraulic equations governing fluid flow through a network (e.g. nodal continuity and conservation of energy around a closed loop) must also be satisfied.

3. ANT COLONY OPTIMISATION

3.1. General Analogy

ACO, developed by Dorigo *et al.* (1991) (cited in Dorigo *et al.* 1996) is a discrete combinatorial optimisation algorithm based upon the foraging behaviour of ants. Over a period of time a colony of ants are able to determine the shortest path from their home to a food source. The ‘swarm intelligence’ of the ant colony is achieved via an indirect form of communication that involves the ants following and depositing a chemical substance, called pheromone, on the paths that they travel. Over time, shorter (or more desirable) paths are reinforced with greater amounts of pheromone, as they require less time to be traversed, thus becoming the dominant paths for the colony.

To apply ACO to a combinatorial optimisation problem, it is important to outline some basic concepts. Within ACO, the optimisation problem is represented as a graph consisting of n decision points where each decision point is connected to its adjacent decision point via a set of edges[1]. For example, θ_i is the set of edges available from decision point i. A solution, termed a path in ACO (symbolised by S), is comprised of a selection of an edge at each decision point. Therefore a path (i.e. solution) can be seen as a vector of the selected edges, that is

$$S = (selection_i \mid selection_i \in \theta_i, \forall\, i = 1,\ldots,n) \quad (1)$$

The ACO algorithm operates by iteratively generating a population of solutions where each solution is representative of the path that a single ant has travelled. An ant generates a solution by selecting an edge at each decision point based upon a decision policy. Once each ant has generated a solution, an amount of pheromone proportional to the quality of the solution is deposited upon all the edges on the path. In this way, better solution components (i.e. edges) are reinforced with greater amounts of pheromone.

As stated, at each decision point, an ant selects an edge governed by a non-deterministic decision policy. This policy considers a trade-off between the pheromone intensity on a particular edge and the desirability of that edge with respect to its individual influence on the objective function. The desirability has different definitions for different problems. For example, if the objective is to minimise cost, the desirability of an edge maybe set equal to the inverse of the cost

[1] The definition of the graph in this case slightly differs from that represented in other papers (e.g. Dorigo et al. 1999) to allow for a more intuitive application to WDS optimisation.

associated with that edge (e.g. cheaper edges are more desirable). Taking these two properties of an edge into account, ACO algorithms effectively utilise heuristic information that has been learned (represented as pheromone intensity) in addition to incorporating a bias towards edges that are of a greater desirability.

Incorporated within this process is a mechanism to model the pheromone evaporation. Pheromone evaporation is analogous to a gradual loss of memory and is important as it allows for ACO algorithms to forget poor information that was learned early on in the search and focus on using the better information that has been gained at later stages of the search.

The mathematical formulations of the ACO algorithms presented in this paper, namely AS and MMAS, are given in the following sections.

3.2. Ant System

Ant System (Dorigo *et al.* 1996) is the original and most simplistic ACO algorithm. As such, it has been the most influential in the development of more advanced ACO algorithms (Dorigo *et al.* 1999). The decision policy used within AS is as follows: the probability that edge (i,j) will be selected at decision point i is given by (Dorigo *et al.* 1996)

$$p_{i,j}(t) = \frac{\left[\tau_{i,j}(t)\right]^{\alpha}\left[\eta_{i,j}\right]^{\beta}}{\sum_{l\in\theta_i}\left[\tau_{i,l}(t)\right]^{\alpha}\left[\eta_{i,l}\right]^{\beta}} \tag{2}$$

where $p_{i,j}(t)$ is the probability that edge (i,j) is chosen in iteration t, $\tau_{i,j}(t)$ is the concentration of pheromone associated with edge (i,j) in iteration t, $\eta_{i,j}$ is the desirability of edge (i,j) and α and β are the parameters controlling relative importance of the pheromone intensity and desirability, respectively, for each ants' decision. If $\alpha >> \beta$ then the algorithm will make decisions based mainly on the learned information, as represented by the pheromone and if $\beta >> \alpha$ the algorithm will act as a greedy heuristic selecting mainly the shortest or cheapest edges, disregarding the impact of these decisions on the final solution quality.

At the end of an iteration (i.e. each ant has generated a solution) the pheromone on each edge is updated. The pheromone updating equation for AS is given by (Dorigo *et al.* 1996)

$$\tau_{i,j}(t+1) = \rho\tau_{i,j}(t) + \Delta\tau_{i,j}(t) \tag{3}$$

where ρ is the coefficient representing pheromone persistence (note: $0 \leq \rho \leq 1$) and $\Delta\tau_{i,j}(t)$ is the pheromone addition for edge (i,j). The pheromone persistence factor is the mechanism by which the pheromone trails are decayed, enabling the colony to 'forget' poor edges and increasing the probability of selecting good edges. For $\rho \rightarrow 1$ only small amounts of pheromone are decayed between iterations and the convergence rate is slower, whereas for $\rho \rightarrow 0$ more pheromone is decayed resulting in faster convergence. $\Delta\tau_{i,j}(t)$ is a function of the solutions found at iteration t and is given by (Dorigo *et al.* 1996)

$$\Delta\tau_{i,j}(t) = \sum_{k=1}^{m}\Delta\tau_{i,j}^{k}(t) \tag{4}$$

where m is the number of ants and $\Delta\tau_{i,j}^{k}(t)$ is the pheromone addition laid on edge (i,j) by the k^{th} ant at the end of iteration t. This is given by (Dorigo *et al.* 1996)

$$\Delta\tau_{i,j}^{k}(t) = \begin{cases} \dfrac{Q}{f(S_k(t))} & \text{if } (i,j)\in S_k(t) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where Q is the pheromone addition factor (a constant) and $S_k(t)$ is the set of edges selected by ant k in iteration t and $f(\cdot)$ is the objective function. From (5) it is clear that better solutions (e.g. solutions with lower $f(\cdot)$ values) are rewarded with greater pheromone additions.

3.3. Max-Min Ant System

Premature convergence to sub-optimal solutions is an issue that can be experienced by all EAs, especially those that have a greater emphasis on exploitation. To overcome this issue the Max-Min Ant System (MMAS) was developed by Stützle and Hoos (2000). The basis of MMAS is to provide *dynamically evolving bounds* on the pheromone trail intensities such that the pheromone intensity on all paths is always within a specified limit of the path with the greatest pheromone intensity. As a result all paths will always have a non-trivial probability of being selected and thus wider exploration of the search space is encouraged.

MMAS uses upper and lower bounds to ensure pheromone intensities lie within a given range, that is $\tau_{min}(t) \leq \tau_{i,j}(t) \leq \tau_{max}(t)$. The upper bound $\tau_{max}(t)$ is given by[2] (Stützle & Hoos 2000)

$$\tau_{\max}(t) = \frac{1}{1-\rho}\frac{Q}{f\left(S^{gb}(t-1)\right)} \tag{6}$$

and the lower bound $\tau_{min}(t)$ is given by (Stützle & Hoos 2000)

[2] Stützle and Hoos (2000) omit Q from their formulation, but for continuity sake with AS, it is included in this study.

$$\tau_{min}(t) = \frac{\tau_{max}(t)(1-\sqrt[n]{p_{best}})}{(NO_{avg}-1)\sqrt[n]{p_{best}}} \quad (7)$$

where p_{best} is the probability that the current global-best path, $S^{gb}(t)$, will be selected given that all non-global best edges have a pheromone level of $\tau_{min}(t)$ and all global-best edges have a pheromone level of $\tau_{max}(t)$, n is the number of decision points and NO_{avg} is the average number of edges at each decision point.

Theoretical justifications of the bounds are given in Stützle and Hoos (2000). An analysis of (7) shows that lower values of p_{best} indicate tighter pheromone bounds, that is $\tau_{min}(t) \rightarrow \tau_{max}(t)$ as $p_{best} \rightarrow 0$.

As the bounds serve to encourage exploration, to provide an emphasis on exploitation, MMAS updates only the iteration best ant's path at the end of an iteration to ensure that good information is being retained and reinforced. Consequently the updating scheme is given as in (3) where $\Delta\tau_{i,j}(t)$ is given by[2] (Stützle & Hoos 2000)

$$\Delta\tau_{i,j}(t) = \begin{cases} \dfrac{Q}{f(S^{ib}(t))} & if\ (i,j) \in S^{ib}(t) \\ 0 & otherwise \end{cases} \quad (8)$$

where $S^{ib}(t)$ is the iteration best path found in iteration t. The pheromone paths are initialised to an arbitrarily high value such that in iteration 2 the paths are set to $\tau_{max}(t)$. MMAS, as formulated in Stützle and Hoos (2000), also incorporates additional mechanisms that are not included here.

3.4. Application of Ant Colony Optimisation to Water Distribution System Optimisation

Transformation of problem

ACO, as for all EAs, is unable to deal directly with constrained optimisation problems as it cannot adhere to constraints that separate feasible regions of the search space from infeasible regions. The standard technique to convert constrained problems to unconstrained problems is to use a penalty function. To guide the search away from the infeasible region and towards the feasible region, the penalty function increases the cost of infeasible solutions such that they are considered to be poor solutions. The unconstrained optimisation problem takes the form

$$\min \quad NC(\Omega) = C(\Omega) + PC(\Omega) \quad (9)$$

where $NC(\Omega)$ is the network cost for design Ω, $C(\Omega)$ is the material and installation cost of Ω and $PC(\Omega)$ is the penalty cost incurred by Ω. Within this study, $PC(\Omega)$ was taken to be proportional to the maximum nodal pressure deficit induced by Ω as in Maier *et al.* (2003).

Modification of ACO elements

Visibility is a measure of the desirability of an option with respect to its influence on the objective function value. As the objective is to minimise cost, cheaper options are more desirable. Therefore the visibility of an option is taken as the inverse of the cost of implementing that option (Maier *et al.* 2003). In other words

$$\eta_{i,j} = \frac{1}{c_{i,j}} \quad (10)$$

where $c_{i,j}$ is the unit cost of implementing diameter j at pipe i. As cheaper diameter options are more desirable, a higher bias in the probability of selection for cheaper diameters results. For options with zero cost, a virtual-zero-cost was selected such that it was in proportion to the other costs.

A summary of the conversion of the general ACO problem formulation to the WDS optimisation is given in Table 1.

Table 1. Conversion from the general ACO problem formulation to the WDSP

General ACO problem formulation		WDSP equivalent	
Element	Symbol	Element	Symbol
Path	S	Design	Ω
Edge	(i,j)	Diameter option	$dia_{i,j}$
Set of edges available from decision point i	θ_i	Set of diameter options available for pipe i	$\begin{pmatrix} dia_{i,1}, . \\ ..,dia_{i,NO_i} \end{pmatrix}$
Objective function	$f(S)$	Network cost	$NC(\Omega)$

4. CASE STUDIES

The simulations were performed on two different case studies, the New York Tunnels Problem (NYTP) and the Hanoi Problem (HP). The AS program was coded in FORTRAN 90 with EPANET2 as the hydraulic solver. The MMAS program, coded in FORTRAN 77, used WADISO as the hydraulic solver with adjustments to the head-loss coefficients such that they were equivalent to EPANET2. Simulations were typically performed on a dual processor 1 GHz Pentium LINUX system. For each simulation, the runtime was long enough to allow the ACO algorithms to converge to a solution.

4.1. Parameter Settings

Based on a preliminary sensitivity analysis the parameters were set as follows for both AS and MMAS within both of the case studies; $\alpha = 1.0$, $\beta = 0.5$, $\rho = 0.98$. The other parameters τ_0 and Q (for AS only), p_{best} (for MMAS only) and m (for both) were found to be case study dependent and were consequently calibrated independently.

4.2. Case Study 1: The New York Tunnels Problem

The WDS for the NYTP is a gravity fed system from a single reservoir and consists of 20 nodes connected via 21 tunnels. There is a single demand case for the problem (see Dandy *et al.* (1996) for network details). For each of the tunnels there is the option to leave the tunnel (e.g. a 'do nothing' option) or the option to provide a duplicate tunnel with one of fifteen different diameter sizes.

As there is a 'do nothing' option, a virtual-zero-cost of \$110 per metre was used in this study. This is approximately 1/3 of the cost of the cheapest duplicate option. This case study has a search space of approximately 1.934×10^{25} possible designs. For AS, $\tau_0 = 140$ and $m = 90$. For MMAS, $m = 84$ and $p_{best} = 0.01$. Q was set to 2.94×10^8 for both algorithms.

The known-optimum solution is \$38.638 million found first by ACOA (a version of ACO with a similar updating scheme to MMAS but without the pheromone bounds) in Maier *et al.* (2003) with a minimum search-time of 7,014 evaluations. It is important to note that other authors (Savic & Walters 1997; Lippai *et al.* 1999; Wu *et al.* 2001) have proposed cheaper solutions to the NYTP, however these solutions were assessed as being infeasible by EPANET2 (Maier *et al.* 2003), which was the benchmark hydraulic analysis tool used in this research.

Table 2 shows a comparison of the two ACO algorithms with current best performing algorithms from the literature; an improved GA (GA) (Dandy *et al.* 1996), and ACOA (Maier, *et al.* 2003).

From Table 2, it is seen that AS does not find the known-optimum, and its lowest solution deviates 1.5% from the known-optimum. MMAS was able to find the known-optimum for four out of five runs with a mean best-cost deviating only 0.2% from the known-optimum. Even though ACOA searches more efficiently it is known that it was not able to find the known-optimum as frequently as MMAS. MMAS is more efficient than AS and GA.

Table 2. Comparison of algorithmic performance for the New York Tunnels Problem. Performance statistics are ordered as follows; minimum, [mean] and {maximum}.

Algorithm	Best-cost (\$M), *(% deviation from known optimum)*	Search-time (evaluation number)
AS	39.221, *(1.5)*	26,329
	[39.784, *(3.0)*]	[37,001]
	{40.318, *(4.3)*}	{43,971}
MMAS	38.638, *(0.0)*	23,542
	[38.7, *(0.2)*]	[24,978]
	{38.949, *(0.8)*}	{27,285}
GA[a]	38.796, *(0.4)*	96,750
ACOA[b]	38.638, *(0.0)*	7,014

NOTES: [a]Dandy *et al.* (1996), [b] Maier *et al.* (2003). AS and MMAS results are based on 5 runs.

4.3. Case Study 2: The Hanoi Problem

The Hanoi Problem (HP) has been considered by numerous authors in its discrete problem formulation (Savic & Walters 1997; Cunha & Sousa 1999; Wu *et al.* 2001). Unlike the NYTP, it is a new design as there are no existing pipes in the system. The network consists of 34 pipes and 32 nodes organised in three loops. The system is gravity fed by a single reservoir and has only a single demand case (see Wu *et al.* (2001) for network details). For each link there are six different new pipe options where a minimum diameter constraint is enforced.

This case study has a problem size of approximately 2.87×10^{26} possible designs. For AS, $\tau_0 = 26$ and $m = 80$. For MMAS, $m = 83$ and $p_{best} = 0.9$. Q was set to 1.1×10^7 for both algorithms.

The known-optimum solution in the literature is \$6.182 million found by the fast messy genetic algorithm (fmGA1) in 113 626 evaluations (Wu *et al.* 2001). Again it is important to note that other authors found solutions cheaper than this (Savic & Walters 1997; Cunha & Sousa 1999; Wu *et al.* 2001), but these were determined as infeasible by EPANET2.

Table 3 shows a comparison of the two ACO algorithms with two other algorithms; GA-No.2, a version of the standard GA (Savic & Walters 1997) and fmGA1 (Wu *et al.* 2001).

No feasible solutions were found by AS in any run for the HP. As the lowest cost solution for the HP contains many of the larger size diameters it can be deduced that the problem has a small feasible region, thus explaining AS's poor performance. MMAS found a minimum best cost that deviated 3.7% and a mean best cost that deviated 8.1% from the known-optimum. This performance is worse than that of GA-No.2 and fmGA1, however it is a great improvement on AS, which was unable to even find the feasible region.

Table 3. Comparison of algorithmic performance for the Hanoi Problem. Performance statistics are ordered as follows; minimum, [mean] and {maximum}.

Algorithm	Best-cost ($M) *(% deviation from known optimum)*	Search-time (evaluation number)
AS	[c]NFS	-
MMAS	6.412,*(3.7)*	25,092
	[6.685, *(8.1)*]	[31,595]
	{6.905, *(11.7)*}	{38,693}
GA-No.2[a]	6.195, *(0.2)*	~10^6
fmGA1[b]	6.182, *(0.0)*	113,626

NOTES: [a] Savic & Walters (1997), [b] Wu *et al.* (2001). [c] NFS means no feasible solutions were found. AS and MMAS results were based on 5 runs.

5. CONCLUSIONS

Within this paper, the advanced ACO algorithm, MMAS, is compared to the simplistic ACO algorithm, AS, and other best performing algorithms from the literature for two WDS case studies. For both case studies MMAS is shown to outperform AS.

Within the first case study, the New York Tunnels Problem, MMAS found the known-optimum 80% of the time and at a faster rate than AS, which did not find it once. This degree of robust performance for the NYTP has previously been unseen in the literature.

For the second case study, the Hanoi Problem, AS was unable to find any feasible solutions. MMAS was unable to find known-optimum in the literature, however, its performance was a vast improvement on that of AS. Even though the GAs performed better on the HP, the ACO algorithms were found to be more computationally efficient.

The additional mechanisms incorporated in MMAS to manage the exploit-explore relationship have been seen to be effective in improving performance compared with AS. MMAS has also been seen to perform competitively with respect to other algorithms in the literature.

As MMAS is only one of many advanced ACO algorithms, future work should focus on the testing of the other algorithms to determine the algorithmic characteristics that are most suited to WDS optimisation.

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