Modelling Multivariate Asymmetric Financial Volatility

Felix Chan[a], **Michael McAleer**[b]
[a]*Department of Economics, University of Western Australia (Felix.Chan@uwa.edu.au)*
[b]*Department of Economics, University of Western Australia*

Abstract: The univariate Generalised Autoregressive Conditional Heterscedasticity (GARCH) model has successfully captured the symmetric conditional volatility in a wide range of time series financial returns. Although multivariate effects across assets can be captured through modelling the conditional correlations, the univariate GARCH model has two important restrictions in that it: (1) does not accommodate the asymmetric effects of positive and negative shocks; and (2) assumes independence between conditional volatilities across different assets and/or markets. In order to capture such asymmetric effects, Glosten et al. (1993) proposed a univariate asymmetric GARCH (or GJR) model. However, the univariate GJR model also assumes independence between conditional volatilities across different assets and/or markets. Several multivariate GARCH models have been proposed to capture such interdependencies, but none has been designed to capture asymmetry, apart from the constant correlation multivariate asymmetric GARCH (CC-MGJR) model of Hoti et al. (2002). The CC-MGJR model captures asymmetric effects and permits interdependencies between conditional volatilities across different assets and/or markets. In addition, the structural and statistical properties of the CC-MGJR model have been established, and the sufficient conditions for consistency and asymptotic normality can be verified in practice. The aim of this paper is to model the multivariate asymmetric conditional volatility of three different stock indexes, namely S&P 500, Nikkei and Hang Sang, using the CC-MGARCH model of Bollerslev (1990), the vector ARMA-GARCH model of Ling and McAleer (2003), and the CC-MGJR model of Hoti et al. (2002). Extensive empirical results support the presence of asymmetric effects across the stock indexes, as well as interdependencies in conditional volatilities across different markets.

Keywords: Multivariate GARCH, asymmetry, multivariate volatility models, conditional correlation, interdependence, structural properties, statistical properties.

1. INTRODUCTION

Engle's (1982) Autoregressive Conditional Heteroscedasticity (ARCH) and Bollerslev's (1986) Generalised ARCH (GARCH) models have been used extensively in the finance and financial econometrics literature to capture the dynamics of conditional volatility in financial returns (see Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Li, Ling and McAleer (2002) for details). It is well known that the GARCH model can accommodate several unique features that occur frequently in financial returns data, such as the ability to capture thick tails and volatility clustering. However, GARCH is also well known for two major deficiencies. Both ARCH and GARCH assume symmetric impacts of the unconditional shocks, so that a positive shock has the same impact on volatility as a negative shock. This restriction contradicts one of the stylised facts of financial returns, in which negative shocks tend to have larger impacts on volatility than do positive shocks. The (G)ARCH model is also a univariate model and does not permit interdependencies across different assets and/or markets. Thus, GARCH does not test for a relationship between the volatilities of different assets and/or markets, which is of primary importance in areas such as optimal portfolio management.

Previous research has tried to resolve these deficiencies with some success. Glosten et al. (1992) (GJR) proposed a univariate asymmetric GARCH model to accommodate the asymmetric impacts of unconditional shocks on volatility. Bollerslev (1990) proposed a constant correlation multivariate GARCH (CC-MGARCH) model, but without establishing its structural or statistical properties. Ling and McAleer (2003) proposed a vector ARMA-GARCH model which has the CC-MGARCH as a special case. In addition, Ling and McAleer (2003) established the structural and statistical properties of the model, including the necessary and sufficient conditions for stationarity and ergodicity, as well as sufficient conditions for the existence of moments, and sufficient conditions for consistency and asymptotic normality of the Quasi Maximum Likelihood Estimator (QMLE) for the vector ARMA-GARCH model (see Li, Ling and McAleer (2002) for a comprehensive survey of recent theoretical developments of univariate GARCH models). Moreover, the vector ARMA-GARCH model allows interdependencies of volatilities across different assets and/or markets. As QMLE is consistent and asymptotically normal, valid inferences can be obtained to determine the existence of cross-asset, cross-market and cross-country effects.

Interestingly, these models have resolved one deficiency or the other of the GARCH model, but not both. For this reason, Hoti et al. (2002) proposed a constant correlation multivariate GJR (CC-MGJR) model. This model differentiates the asymmetric impacts of positive shocks and negative shocks, and also allows interdependencies between different assets and/or markets in the conditional mean, as well as in the conditional volatilities. As in Ling and McAleer (2003), Hoti et al. (2002) also established the necessary and sufficient conditions for stationarity and ergodicity, as well as sufficient conditions for the existence of moments, and sufficient conditions for consistency and asymptotic normality of the QMLE for the CC-MGJR model. Thus, valid inferences can be conducted to determine the presence of asymmetric and/or interdependent effects.

The purpose of this paper is to evaluate three multivariate GARCH models, namely CC-MGARCH, vector ARMA-GARCH and CC-MGJR, using three data sets, namely the returns of Standard and Poor's 500 Composite Index, Nikkei and Hang Sang. The empirical results show that there are interdependencies across different markets, and the presence of asymmetric impacts from unconditional shocks is also detected. Conditional correlation matrices are estimated for the three models, and their stability is investigated through the use of rolling windows. The impacts of extreme observations and outliers on the conditional correlation matrices are also analysed.

The plan of the paper is as follows. Section 2 reviews the three multivariate models and describes the data used in the paper. Section 3 contains the empirical results. Concluding remarks are given in Section 4.

2. MODELS AND DATA

Consider the following multivariate GARCH model:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_{0t},$$

$$\varepsilon_{0t} = D_{0t}\eta_{0t} \tag{1}$$

$$Var(\varepsilon_{0t} \mid F_{t-1}) = D_{0t}\Gamma_0 D_{0t}$$

where F_t is the past information available up to time *t*, $D_{0t} = diag(h_{0it}^{1/2})$, $i = 1,...,m$, and $\Gamma_0 = [\rho_{0ij}]$, $i, j = 1,...,m$.

As $\Gamma_0 = E(\eta_{0t}\eta_{0t}')$, the correlation matrix of the unconditional shocks, ε_{0t} is, by definition, the same as the correlation matrix of the conditional shocks, η_{0t}. Bollerslev (1990) proposed the above framework with

$$h_{0it} = \omega_{0i} + \sum_{l=1}^{r}\alpha_{0i}\varepsilon_{0it-l}^2 + \sum_{l=1}^{s}\beta_{0i}h_{0it-l} \tag{2}$$

where $i = 1,..,m$, and *m* is the total number of assets or markets. Equation (2) is essentially a standard GARCH(*r,s*) model for asset *i*, in which $\sum_{i=1}^{r}\alpha_{0i}$ denotes short run persistence (or ARCH effects) and $\sum_{i=1}^{r}\alpha_{0i} + \sum_{i=1}^{s}\beta_{0i}$ denotes long run persistence (in which $\sum_{i=1}^{s}\beta_{0i}$ are the GARCH effects). Although the conditional correlation is modelled, and hence can be estimated in practice, the CC-MGARCH model does not allow any interdependencies of volatilities across different assets and/or markets, and does not accommodate asymmetric behaviour.

In order to allow for interdependencies of volatilities across different assets and/or markets, Ling and McAleer (2003) proposed the following vector ARMA-GARCH model:

$$\Phi_0(L)(Y_t - \mu_0) = \Psi_0(L)\varepsilon_{0t} \tag{3}$$

$$\varepsilon_{0t} = D_{0t}\eta_{0t}$$

$$H_{0t} = W_0 + \sum_{l=1}^{r}A_{0l}\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{s}B_{0l}H_{0t-l} \tag{4}$$

where $D_{0t} = diag(h_{0it}^{1/2})$, A_{0l} and B_{0l} are $m \times m$ matrices with typical elements α_{0ij} and β_{0ij}, respectively, for $i, j = 1,...,m$, $\Phi_0(L) = I_m - \Phi_{01}L - ... - \Phi_{0p}L^p$ and $\Psi_0(L) = I_m - \Psi_{01}L - ... - \Psi_{0q}L^q$ are polynomials in *L*, and $\vec{\varepsilon}_{0t} = (\varepsilon_{01t}^2,..., \varepsilon_{0mt}^2)'$.

It is clear that when A_{0l} and B_{0l} are diagonal matrices, equation (4) reduces to equation (2), so that the vector ARMA-GARCH model has CC-MGARCH as a special case. Ling and McAleer (2003) established the structural and statistical properties of the model, including the necessary and sufficient conditions for stationarity and ergodicity, sufficient conditions for the existence of moments, and sufficient conditions for consistency and asymptotic normality of the QMLE for the model.

Hoti et al. (2002) extended the vector ARMA-GARCH model of Ling and McAleer (2003) to accommodate the asymmetric impacts of the unconditional shocks on the conditional variances. They proposed the CC-MGJR model as follows:

$$\Phi_0(L)(Y_t - \mu_0) = \Psi_0(L)\varepsilon_{0t} \tag{5}$$

$$\varepsilon_{0t} = D_{0t}\eta_{0t}$$

$$H_{0t} = W_0 + \sum_{l=1}^{r}A_{0l}\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{r}C_{0l}I(\eta_{0t-l})\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{s}B_{0l}H_{0t-l} \tag{6}$$

where C_{0l} is an $m \times m$ matrix with typical element γ_{0ij}, and $I(\eta_{0it})$ is an indicator function, given as:

$$I(\eta_{0it}) = \begin{cases} 1, & \varepsilon_{0it} \le 0 \\ 0, & \varepsilon_{0it} > 0. \end{cases} \tag{7}$$

It is clear that if $m = 1$, equation (6) reduces to the asymmetric GARCH, or GJR, model of Glosten et al. (1992). If $C_{0l} = 0$, equations (5) and (6) collapse to the vector ARMA-GARCH model of Ling and McAleer

(2003). Hoti et al. (2002) established the structural and statistical properties of the CC-MGJR model. As in to Ling and McAleer (2003), this includes the necessary and sufficient conditions for stationarity and ergodicity, sufficient conditions for the existence of moments, and sufficient conditions of consistency and asymptotic normality of the QMLE for CC-MGJR. Hoti et al. (2002) also provided a concise summary and comparison of various multivariate GARCH models, including Engle and Kroner's (1995) Vech (or VAR) model, Bollerslev, Engle and Wooldridge's (1988) Diagonal model, Engle and Kroner's (1995) BEKK model, and the Dynamic Conditional Correlation (DCC) model of Engle (2002), which is equivalent to the Varying Correlation Multivariate GARCH (VC-MGARCH) model of Tse and Tsui (2002). The structural and statistical theory for the DCC and VC-MGARCH models have not yet been established, which means there is no foundation for statistical inference.

Multivariate GARCH models are typically estimated by MLE, and are defined as follows:

$$\hat{\theta} = \max_{\theta} -\frac{1}{2}\sum_{t=1}^{T}\left(\log\left|H_t\right| + \left|\varepsilon_t' H_t^{-1}\varepsilon_t\right|\right) \qquad (8)$$

where $|A|$ denotes the determinant of matrix A. When η_{0t} does not follow a joint normal distribution, equation (8) is defined as the Quasi-MLE (QMLE). The properties of the QMLE for the vector ARMA-GARCH and CC-MGJR models can be found in Ling and McAleer (2003) and Hoti et al. (2002), respectively.

The primary purpose of this paper is to investigate the interdependent effects of volatilities across different markets. As discussed in Section 2, the three multivariate models, namely CC-MGARCH, vector ARMA-GARCH and CC-MGJR, will be estimated using three daily data sets, namely Standard and Poor's 500 Composite (S&P), Nikkei (NK) and Hang Sang (HS). The data were obtained through DataStream DataBase Services, with the sample from 1/1/1986 to 11/4/2000, giving 3726 observations.

Of primary interest are the returns from each of these series, which are calculated as $r_t = (y_t - y_{t-1})/y_{t-1}$, where y_t denotes the stock price at time *t*, and r_t denotes the corresponding return at time *t*. In the next section, the CC-MGARCH, vector ARMA-GARCH and CC-MGJR models will be estimated using the returns from S&P, Nikkei and Hang Sang. Tests of cross-country and asymmetric effects will be conducted, and the stability of the conditional correlation matrices will be examined through the use of rolling windows.

3. EMPIRICAL RESULTS

Tables 1 to 3 contain the estimates for CC-MGARCH, vector ARMA-GARCH and CC-MGJR, respectively. The three entries corresponding to each parameter are the respective estimate, asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio. In addition, the conditional correlation matrices of CC-MGARCH, vector ARMA-GARCH and CC-MGJR are given in Table 4. Finally, Table 5 contains a summary regarding the cross-markets effects on volatilities implied by these models. All the models in this paper are estimated by EViews 4 with $p = 1, q = 0$ and $r = s = 1$.

Hoti et al. (2002) showed that the QMLE for CC-MGJR is asymptotically normal, which facilitates statistical inference. Similar results hold for the vector ARMA-GARCH and CC-MGARCH models, as they are special cases of CC-MGJR (see Ling and McAleer (2003) and Hoti et al. (2002) for further details).

The significance of the γ_i estimates for $i = S, N, H$, where $S =$ S&P, $N =$ Nikkei and $H =$ Hang Sang, in CC-MGJR in Table 3 suggests the presence of asymmetric impacts from the unconditional shocks on the volatilities in all three markets. Therefore, the CC-MGJR model is preferred to the vector ARMA-GARCH and CC-MGARCH models. Furthermore, the results in Table 3 also suggest the presence of cross-markets effects. In fact, the results show that the conditional volatility of S&P is affected by its previous short and long run shocks (namely, $\alpha_S + \frac{1}{2}\gamma_S$ and $\alpha_S + \frac{1}{2}\gamma_S + \beta_S$, respectively), as well as the previous long run shocks from Nikkei (namely, $\alpha_N + \frac{1}{2}\gamma_N + \beta_N$). Interestingly, the conditional volatility of Nikkei returns is also significantly affected by its own previous short and long run shocks, and previous long run shocks from S&P, as well as the short and long shocks from Hang Sang. In the case of Hang Sang, the conditional volatility is affected by its previous short and long run shocks, and previous long run shocks from Nikkei. The conditional volatilities of Hang Sang and S&P returns do not seem to affect each other, even though they are both affected by Nikkei.

It is worth noting that the vector ARMA-GARCH model revealed the same cross-country effects as did CC-MGJR. Moreover, the estimates of the corresponding parameters between the two models do not differ substantially, but their absolute magnitudes are generally lower in CC-MGJR, with no change in sign. In addition, the Bollerslev-Wooldridge (1992) robust t-ratios are lower than the asymptotic t-ratios for all the estimates in the three models.

As in Hoti et al. (2002), the conditional correlation matrices of the three models do not seem to differ substantially. It is worth noting that CC-MGJR produces the lowest correlations compared with the other two models. Interestingly, the correlations between the conditional shocks are all positive in the three cases. S&P and Hang Sang, (SP, HS), has the highest correlation of 0.315, and the correlation between S&P and Nikkei, (SP, NK), is second with 0.259, followed closely by the correlation of 0.255 between Hang Sang and Nikkei, (HS, NK). Although the conditional correlations differ slightly from the other two matrices, the rankings remain the same.

Recently, Engle (2002) and Tse and Tsui (2002) proposed the multivariate GARCH model with time-varying conditional correlations, such that Γ_0 is no longer a constant matrix but follows a GARCH-type process. However, no structural or statistical properties of these models have yet been developed. The primary difficulty lies in the fact that Γ_0 is the conditional correlation matrix of η_{0t}, which is assumed to be a vector of independently and identically distributed random variables. If Γ_0 is assumed to be time varying with an autoregressive structure, the iid assumption of η_{0t} would be violated, so that any existing proofs of consistency and asymptotic normality would not be valid for these models.

In order to examine the time-varying nature of the conditional correlations, as well as investigating the effects of extreme observations and outliers on the conditional correlations of CC-MGARCH, vector ARMA-GARCH and CC-MGJR, all three models also are estimated using rolling windows. The dynamic paths of the condition correlations of each model should provide insights to two important issues, namely: (i) variations in the conditional correlations fluctuate over time; and (ii) the impacts of aberrant observations on the conditional correlations. In order to strike a balance between efficiency in estimation and a sensible number of rolling windows, the rolling windows size is selected to be 3000 for all three data sets.

Figures 1-3 contain the dynamic paths of the conditional correlation matrices for CC-MGARCH, vector ARMA-GARCH and CC-MGJR, respectively. The conditional correlations of (SP, NK) and (NK, HS) seem to have upward trends for all three models. In addition, the conditional correlation of (NK, HS) seems to be rising the most rapidly. Interestingly, although the conditional correlation of (SP, NK) seems to be slightly higher than (NK, HS) when using the whole sample for all three models, the rolling windows show that there are, in fact, regime changes in all three cases. The conditional correlation of (SP, NK) seems to be much higher than for (NK, HS) in the early rolling samples, and remains at a similar level throughout the entire sample, but the conditional correlation of (NK, HS) rises rapidly, and eventually exceeds the conditional correlation of (SP, NK).

An interesting question arises when the conditional correlation of (NK, HS) becomes consistently higher than for (SP, NK) for each model. For CC-MGJR and vector ARMA-GARCH, the conditional correlation of (SP, NK) becomes consistently lower than for (NK, HS) in rolling samples 173 and 183, respectively. However, the conditional correlation of (SP, NK) does not become consistently lower than for (NK, HS) until much later, specifically in rolling sample 338. This may be due to the misspecification of the conditional volatilities, and would be an interesting area for future research.

Another interesting observation is the dramatic decline in the conditional correlation of (SP, NK) when the outlier in observation 466 is removed from the rolling sample. The conditional correlation of (SP, NK) in CC-MGARCH decreases from 0.254 to 0.239 in five consecutive rolling samples, then stabilises at around 0.238 for the remaining rolling samples. A similar explanation holds for the vector ARMA-GARCH model, where the conditional correlation of (SP, NK) decreases from 0.247 to 0.235 in six consecutive rolling samples, then stabilises at around 0.234. Interestingly, the conditional correlation of (SP, NK) does not decrease as dramatically in CC-MGJR, as it decreases from 0.243 to 0.229 in nine consecutive rolling samples, and remains at around 0.229. It would appear, therefore, that accommodating asymmetric behaviour leads to more robust inferences.

4. CONCLUDING REMARKS

This paper analysed the cross-markets and asymmetric effects of conditional volatilities in the returns of the S&P 500 Composite Index, Nikkei and Hang Sang using three different multivariate GARCH models, namely the CC-MGARCH model of Bollerslev (1990), the vector ARMA-GARCH model of Ling and McAleer (2003), and the asymmetric CC-MGJR model of Hoti et al. (2002). The empirical results showed that there are interdependencies of volatilities between S&P and Nikkei, and between Hang Sang and Nikkei, but no interdependencies in the conditional volatilities between S&P and Hang Sang. The time-varying nature of conditional correlations was examined through the use of rolling windows. These empirical results showed that the conditional correlations do not vary substantially for (SP, NK) and (SP, HS), but exhibit a slight upward trend in the correlations of (NK, HS). Models that allow time-varying conditional correlations may provide greater information about the underlying structures of the processes, even though the structural and statistical theory for these models have yet to be established for purposes of valid inference.

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6. REFERENCES

Bollerslev, T., (1986) Generalised autoregressive conditional heteroskedasticity, *Journal of Econometrics,* 31, 307-327.

Bollerslev, T., (1990) Modelling the coherence in short-run nominal exchange rate: A multivariate generalized ARCH approach, *Review of Economics and Statistics*, 72, 498-505.

Bollerslev, T., R. Y. Chou and K. F. Kroner, (1992) ARCH modelling in finance: a review of the theory and empirical evidence, *Journal of Econometrics,* 52, 5-59.

Bollerslev, T., R. F. Engle and D. B. Nelson, (1994) ARCH models, in R. F. Engle and D. L. McFadden (eds.), *Handbook of Econometrics*, 4 (North-Holland, Amsterdam) 2961-3038.

Bollerslev, T., R.F. Engle and J.M. Wooldridge, (1988) A capital asset pricing model with time varying covariance, *Journal of Political Economy*, 96, 116-131.

Bollerslev, T. and J.M. Wooldridge, (1992) Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, *Econometric Reviews*, 11, 143-173.

Engle, R. F., (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, *Econometrica,* 50, 987-1007.

Engle, R.F., (2002) Dynamic conditional correlation: A new simple class of multivariate GARCH models, *Journal of Business and Economic Statistics*, 20, 339-350.

Engle, R.F. and K.F. Kroner, (1995) Multivariate simultaneous generalized ARCH, *Econometric Theory*, 11, 122-150.

Glosten, L., R. Jagannathan and D. Runkle, (1992) On the relation between the expected value and volatility of nominal excess return on stocks, *Journal of Finance,* 46, 1779-1801.

Hoti, S., F. Chan and M. McAleer, (2002) Structure and asymptotic theory for multivariate asymmetric volatility: Empirical evidence for country risk ratings. Paper presented to the Australasian Meeting of the Econometric Society, Brisbane, July 2002.

Li, W. K., S. Ling and M. McAleer, (2002) Recent theoretical results for time series models with GARCH errors, *Journal of Economic Surveys*, 16, 245-269. Reprinted in M. McAleer and L. Oxley (eds.) *Contributions to Financial Econometrics: Theoretical and Practical Issues*, Blackwell, Oxford, 2002, pp. 9-33.

Ling, S. and M. McAleer, (2003) Asymptotic theory for a vector ARMA-GARCH model, *Econometric Theory*, 19, 278-308.

Tse, Y.K. and A.K.C. Tsui, (2002) A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations, *Journal of Business and Economic Statistics*, 20, 351-362.

Table 1. CC-MGARCH Estimates

Returns	ω_i	α_i	β_i
S&P 500	1.25E-06	0.077	0.914
	7.536	34.436	228.828
	3.123	2.356	32.540
Nikkei	2.88E-06	0.140	0.861
	7.813	36.025	173.436
	3.177	3.405	27.153
Hang Sang	7.29E-06	0.150	0.838
	14.653	24.675	126.609
	2.970	4.656	39.008

Notes: 1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The index i denotes $i = S, N, H$.

Table 2. Vector ARMA-GARCH Estimates

Returns	ω	α_S	β_S	α_N	β_N	α_H	β_H
S&P 500	1.64E-06	0.062	0.927	0.005	-0.007	0.004	-0.004
	9.970	22.189	242.639	4.161	-5.382	6.227	-6.568
	3.203	2.539	41.343	1.758	-2.209	1.436	-1.438
Nikkei	2.52E-06	0.025	-0.026	0.089	0.900	0.014	-0.010
	9.174	5.484	-7.647	15.586	161.004	9.610	-8.384
	4.526	1.501	-2.442	6.253	61.487	2.025	-2.412
Hang Sang	8.51E-06	0.068	-0.056	0.022	-0.023	0.116	0.856
	11.958	13.207	-4.883	4.707	-5.209	17.066	125.353
	3.528	0.934	-1.134	1.600	-2.138	5.738	43.054

Notes: 1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (4) associated with S&P, Nikkei and Hang Sang Returns are denoted by subscripts S, N and H, respectively.

Table 3. CC-MGJR Estimates

Returns	ω	α_S	γ_S	β_S	α_N	γ_N	β_N	α_H	γ_H	β_H
S&P 500	2.19E-06	0.019	0.090	0.919	0.005		-0.007	0.002		-0.002
	13.167	2.874	12.226	205.506	3.684		-4.938	4.281		-4.744
	3.736	1.943	2.876	55.831	1.682		-2.286	1.069		-1.082
Nikkei	2.89E-06	0.027		-0.026	0.024	0.132	0.898	0.011		-0.008
	9.369	5.841		-6.948	4.792	12.352	157.551	6.722		-7.106
	5.870	1.663		-2.768	2.238	5.741	72.871	2.019		-2.535
Hang Sang	9.09E-06	0.054		-0.031	0.022		-0.021	0.052	0.141	0.842
	11.849	9.149		-2.410	4.546		-4.392	6.322	10.839	115.188
	3.695	0.758		-0.608	1.641		-2.032	3.154	4.242	43.060

Notes 1. The three entries for each parameter are their respective estimate, the asymptotic t-ratio and the Bollerslev-Wooldridge (1992) robust t-ratio.
2. The parameters in equation (6) associated with S&P, Nikkei and Hang Sang Returns are denoted by subscripts S, N and H, respectively.

Table 4. Conditional Correlations for CC-MGARCH, Vector ARMA-GARCH and CC-MGJR

Returns	S&P 500	Nikkei	Hang Sang
S&P 500	1.000	0.272 (0.263) [0.259]	0.316 (0.319) [0.315]
Nikkei		1.000	0.256 (0.262) [0.255]
Hang Sang			1.000

Note: The three conditional correlation entries correspond to CC-MGARCH, (Vector ARMA-GARCH) and [CC-MGJR], respectively.

Table 5. Summary of Cross-Markets Effect

Relationships	Interpretation
$N \leftrightarrow S$	Interdependent effects between Nikkei and S&P 500
$H \leftrightarrow N$	Interdependent effects between Hang Sang and Nikkei
$H \cap S = \phi$	Independent effects between Hang Sang and S&P 500

Figure 1. Dynamic Paths of Conditional Correlations for CC-MGARCH

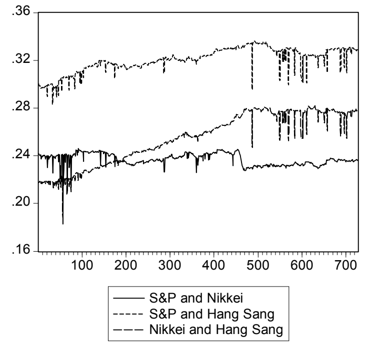

Figure 2. Dynamic Paths of Conditional Correlations for Vector ARMA-GARCH

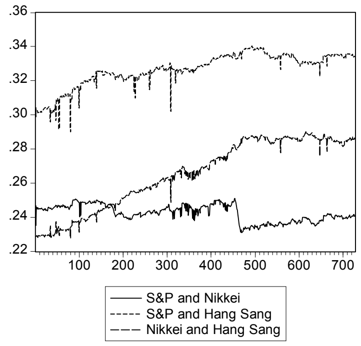

Figure 3. Dynamic of Conditional Correlations for CC-MGJR