A Proto-type Dynamic Model of the Australian Sheep Flock

A.J. Clark[a] and B. Forester[a].

[a] Bureau of Rural Sciences, Agriculture Fisheries and Forestry—Australia, Barton, GPO Box 858 Canberra A.C.T, 2601. anthony.clark@brs.gov.au

Abstract: The sheep meat industry experiences production variability associated with climatic inputs, policy signals and flock management. It is apparent that driven by the buoyant markets and a run of good seasons coming into the new century, the industry was extracting increasing numbers of potential breeding females from the flock to market as sheep meat. This may have long-term implications for flock dynamics, particularly the ability of the flock to recover from a drought, exploit market opportunities or manage disease. The challenge was to develop a simple, inexpensive yet robust model as a way of evaluating, and possibly predicting long term flock decline.

This prototype is a deterministic model using a simple projection matrix (6-age, 2-sex structure). The vital rates of the flock were estimated using a Delphi technique with 5 key producers in the industry. This model was able to hind cast the flock decline evident in national level Australian Bureau of Statistics records. Sensitivity and Eigen-analysis of the flock growth rate pointed to the survival and reproductive output of two and four year old merino ewes as being important determinants of the decline. This simple approach was also extended to include year-to-year production variability and the inter-regional migration of sheep.

Collectively these prototype models demonstrate that it is possible to undertake simple projections of the national flock and ask 'what-if' questions to explore a range of plausible outcomes. The approach requires further work to explore its application, including: the collection of production data suitable for demographic modeling of the flock at a regional to national scale; construction of economic and environmental sub-models to enhance the applications flexibility; and development of an alternative matrix structure, like weight or breed matrices to analyze evolution in industry practice and technology. The way to further extend this approach is to populate the model through a modest survey undertaken concurrently with projects that benchmark biological productivity on groups of farms and across regions. This may lead to a suite of models that are suitable for enhance productivity benchmarking and undertaking program evaluation.

Keywords: *Sheep flock; Matrix modeling; Population dynamics.*

1. INTRODUCTION

A first draft dynamic model of the Australian sheep flock was developed using matrix population models. The primary aim was to test an approach that could be used by the industry to ask 'what if' questions relating to the fertility, survival (slaughter rates) and management of sheep, and the likely consequences for the flock as a whole. The broad goal was to investigate an approach that could be used to evaluate program development in the industry. A prototype model was built to:

- test an approach which could be used to address these 'biological' and 'economic' decision making problems at the regional and national level;
- explore the trade offs between model simplicity and robustness;
- use existing data and or undertake a convenient data collection; and
- scope out directions for further model development.

2. MODEL DESCRIPTION

2.1. Matrix Structure

The model is a transitional matrix of vital rate probabilities (*An*) solved for vectors of sheep numbers (*n*) using linear equations (1) over an annual birth pulse (*t*):

$$n\left(t+1\right)=\boldsymbol{An}\left(t\right) \tag{1}$$

The matrix is structured as a generic sheep lifecycle (Figure 1) where there are 6 age classes, the first class being infertile individuals.

The lifecycle is partitioned into ewes (E1-E6) and wethers (W1-W6), which remain infertile. 'Pe' is the probability of survival for ewes, 'We' is the probability of survival for wethers and 'F' is the probability for fertility. The role of rams in the flock is included in the estimate of fertility, but the abundance and use of rams as a source of genetic improvement was not included in this matrix. The partition of fertility between ewes and wethers was assumed to be a ratio of 50:50. As shown later, the model can be either deterministic with a fixed transitional matrix over time, or stochastic with varying vital rates.

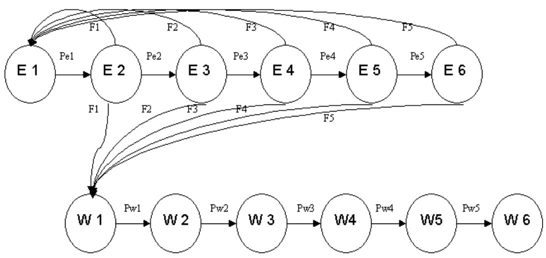

Figure 1. A simple age structured sheep life cycle.

The vital rates are an integrated function of flock management (including agronomic practices and decisions under price variability), genetic resources and environmental variability. As discussed in section 4.2 it is possible to build detailed sub-models to estimate the vital rates, such as partitioning survivorship between background mortality, slaughter, live sales and agistment.

2.2. Life Table Construction

The vital rates are derived from a life table (Table 1), where the survivorship and fecundity functions are the median estimates from by 5 industry experts. These were made using an iterative a Delphi technique (Morgan and Henrion 1990). This was a convenient method for life table construction for the purpose of prototype model development. The estimates were made for a calendar year ending 31 December, consistent with a birth pulse model (Caughley 1977, Caswell 2001).

Age Class	M(x)	eL(x)	wL(x)
1	0	0.72	0.5
2	0.8	0.72	0.6
3	0.8	0.7	0.77
4	0.78	0.7	0.77
5	0.76	0.7	0.77
6	0.75	0	0

Table 1. Life table Fecundity M(x), ewe survivorship eL(x) and wether survivorship wL(x).

2.3 Model analysis

The value of linear matrix population models is not in accurately forecasting abundance in the short term, but in conveniently projecting likely long term changes in the growth rate (fitness) of a population (λ), and examining the influence of different parts of the matrix on λ (Caswell 2001). This is achieved through calculating of the Eigenvector (characteristic λ values) of the projection matrix. The sensitivity of the Eigenvector to changes in the projection matrix, as well as the elasticity of different groups survival and fertility on λ can also be determined.

3. MODEL APPLICATIONS

3.1. Sheep Flock Dynamics

National flock decline

The national flock was modeled deterministically; with the starting condition (n) set as sheep numbers from the 1989 Australian Bureau of Statistics (ABS) Agricultural Census. Starting conditions (n) were perturbed by randomly selecting 100 values above and below the census count.

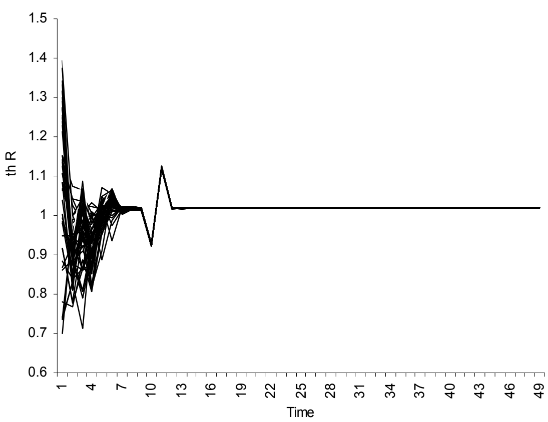

Figure 2. Perturbation of the initial starting conditions.

The results show a strong tendency for the intrinsic growth rate to converge after 7-10 time steps (Figure 2). This strongly ergodic behavior suggests that an analysis of the projection matrix can proceed with a degree of confidence in the results (Caswell op cit.).

This model was used to project flock abundance and growth rate for the period 1990-2000, giving a dominant Eigienvalue λ=0.92, a maximum generation time of 3.6 years, and an average lifetime reproductive value per individual ewe of 3.6 lambs. When compared to ABS data, the model appears to capture the 10 year decline in the flock, but not the year-to-year variation, as shown in Figure 3.

Figure 3 also shows the lift in flock abundance over a decade when testing some course scenarios: a 5 per cent increase in overall flock fertility and increasing the fertility of ewes in age classes 2 and 3 by 20 per cent.

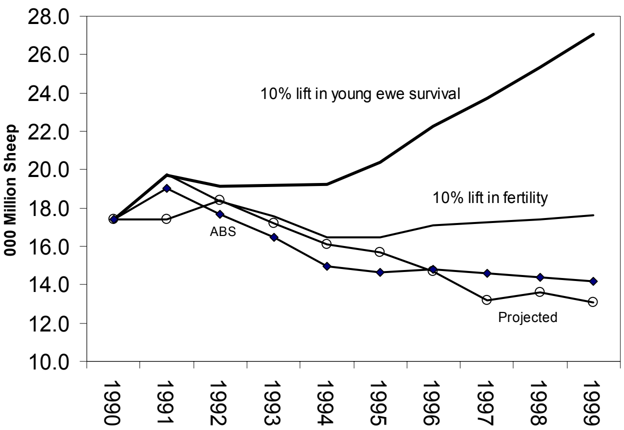

Figure 3. Model projection using median vital rates, and results of fertility and survivorship increases.

The large difference between the projections suggest that changes in parts of the flock matrix, in this case the fertility of 2-3 year old ewes, can have higher leverage in determining abundance and growth rate.

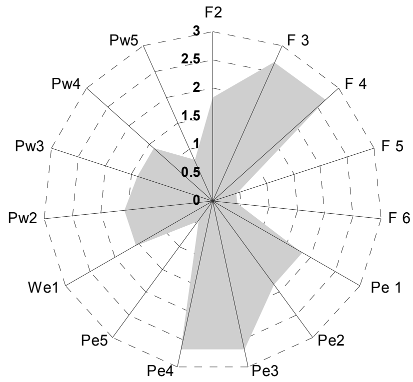

Figure 4. Sensitivity of the 6 age, two-sex national projection matrix

A full sensitivity analysis of the projection matrix highlights that λ is most sensitive to changes in the survival and fertility of ewes in age 3-4, followed by ewes at age 2 (Figure 4). This not only suggests that this age class is important, but the management practices that influence fertility (joining decisions and health) are of equal importance to survivorship (driven by sales). λ is comparatively less sensitive to Wether survival.

3.2. Stochastic Model

The assumption that the vital rates are constant through time was relaxed by internally generating variability in the projection matrix, where:

$$n\left(t+1\right)=\boldsymbol{A}_N\boldsymbol{n}\left(t\right) \quad (2)$$

The co-variant (N) determines the vital rates in year (t+1). N is generated by randomly selecting values from a binomial distribution for fertility and a lognormal distribution for survival. Using Monte Carlo methods, the model is perturbed to generate a vector of λ.

Vital rates estimated for one property, a sheep-meat and wool production system near Mount Gambier in South Australia were used to populate a regional projection matrix where n was estimated using ABS data.

The results in Figure 5, showing the 100 independent trials of a Monte Carlo simulation, demonstrate that it is possible for individual flocks to attain different, in some cases positive, growth rates against a background of overall regional decline.

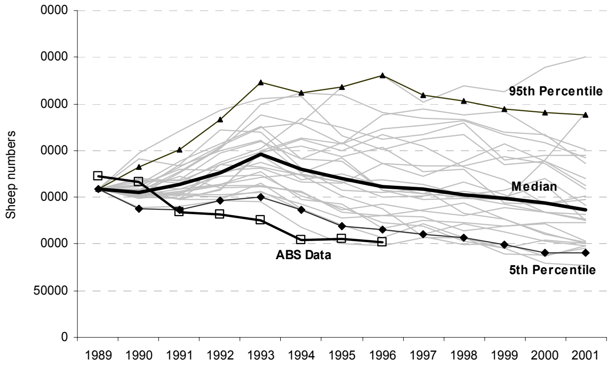

Figure 5. Stochastic simulation for one property near Mount Gambier in South Australia.

In Figure 6 the stochastic model was run using the median life table for the national flock and 1989 starting conditions, with increases in the overall flocks fertility by 10 and 20 percent. The differences in growth rate suggest that an increase of this order of magnitude will be required to increase abundance (increase λ above 1) . Clearly the biological feasibility of attaining this type of fertility increase needs to be investigated further.

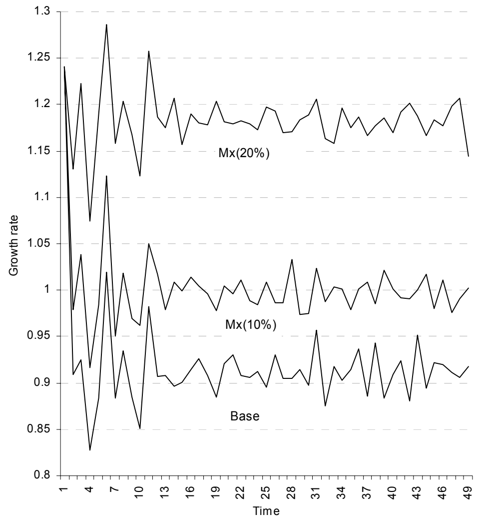

Figure 6. Stochastic growth rate of the national flock after increases in fertility (Mx).

3.3. Migration

Migration

A simple migration model was developed by linking two age-structured matrices, representing regions, with a common covariate, the net migration rate of individuals (Caswell op cit.). The first matrix was set up to represent a finishing system *to be* established near Oberon in NSW (based on *target* fertility and survivorship estimates from *one* producer) that sources lambs from the second region, a breeding system near Cowra-Boorowa.

Figure 7 shows the results with a net migration rate of zero lambs. The finishing system that is planned to be established had much lower long term abundance than ABS records—with low levels of lamb survival because of high rates of meat production had a population of near zero after two years. The breeding system, managed for higher levels of ewe survivorship and fertility, had a positive projected λ.

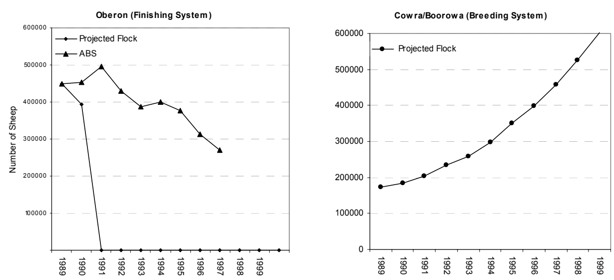

Figure 7. Migration model with a zero net migration of lambs

Figure 8 shows the results of introducing a 30 per cent net migration of male and female lambs. This resulted in the population stabilizing at around 2000 sheep in the Oberon region (again much lower abundance than the regional data), while the breeding systems population changed considerably, with a stable λ.

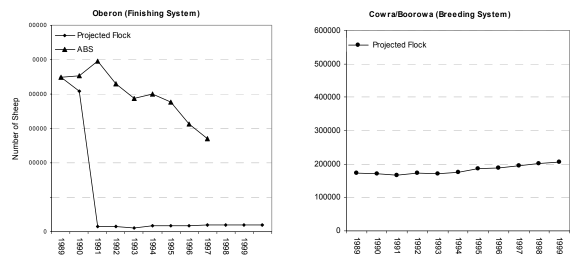

Figure 8. Migration model with a 30 percent net migration of lambs.

4. DISCUSSION

4.1 Model assumptions

The prototype models are, deliberately, simple and convenient projections of the sheep flock. They show that it is possible to undertake simple projections of the national flock and ask 'what-if' questions to explore a range of plausible outcomes. This simple approach was chosen to address the stated purpose of the exercise, which was to build models that were capable of providing some answers to the problem of national flock decline without the need for expensive computation and data collection.

While the matrix population models provided convenience and simplicity in this application, they also have a number of tradeoffs. For example they assume that it is biologically plausible to describe regional and national scale flock by annual age in order to simulate its dynamics. This makes the model relatively coarse when compared to detailed biological simulations like BREW (White 1983) or GRAZPLAN (Freer 1997) that are capable of simulating seasonal changes in sheep abundance associated with pasture availability. It is equally not possible to project many specific factors that are of interest to farm production using this matrix, like weight gain, fat score and carcass weight.

Any scenario tested in the model, such as the impact of a drought or the potential impact of a strategy to change fertility or survival, will involve approximating the likely change in the vital rates. The biological plausibility of this approximation needs to be considered carefully independent of the model. The prototype flock model assumes that the vital rates, or the structure of the flock itself will not change or evolve dramatically over time or within each age class, and that the inherent biological and economic variability can be estimated by a statistical method.

Importantly, the prototype model is not a seasonal forecasting tool and should not be used to make estimates of what will happen in the season ahead. The value of the approach is in the simple 'what if' analysis of flock growth rate to change over longer time frames and is largely suitable for a heuristic (learning) function in a planning exercise.

4.2 Future work

While there are important assumptions that will limit the application of the first draft model, there is potential to further explore (and improve) the application of matrix population models in sheep industry analysis. The prototype models demonstrate that the approach has broader potential application than the scenarios analyzed by the first draft dynamic flock model, and a number of opportunities exist to extend the work further.

For example regional and national prototype models show that there may be potential to include both price and climate variability. This could be achieved by building statistical sub-models to determine the vital rates, like partitioning survivorship into its components of sales, slaughter and background mortality, and determining these by simulation. Equally, fertility has been simulated by the use of stochastic models that include the effect of climate variability (White op cit. and Freer op cit.) and as a Markov series which allow segregation of genetics (Carrick 1998). These might be used to assess the stability of different production systems under these risks.

Migration models could be extended to assess the full set of changes to the flock through the establishment sheep finishing systems in one region (increase in net migration), seasonal management of drought through agistment and trade, and the impact of restricting stock movements when managing diseases like Ovine Johnes.

A major area of future work is the possibility of structuring the prototype model according to sheep development stage, genetics and production system. Coupled with improvements to the stochastic determination of the vital rates there may be potential to build a simple yet robust regional benchmarking tool. There are also opportunities to undertake analyses of flock evolution and gene flow by structuring a matrix according to the genetics of the flock.

By structuring the model according to factors like weight and genetic components that also integrate climatic and price signals in a stochastic process, it may be possible to build a forecasting tool that operates at seasonal timescales. However, experience in application in human demography and population genetics highlight that forecasting models require significant increase in detail, with often-limited increase in precision (Lee and Tuljapurkar 1994).

Additionally, proven macroeconomic approaches to forecast seasonal prices and production are already in operation through our colleagues in the Australian Bureau of Agricultural and Resource Economics (ABARE) and Meat and Livestock Australia (MLA). These models are a complex set of simultaneous non-linear equations, where collecting the data through the ABS and survey infrastructure, estimating the parameters and solving the equations is far from a trivial task. This level of analysis goes well beyond the capacity of matrix population models.

Returning to first principles, the major factor in future model development is matching the goals of the modeling exercise with data collection and the approach (Gaunt 1997). Given goals of either farm benchmarking in an educational context or exploring a range of possible outcomes given proposed changes in a research and extension program, this simple matrix approach is best extended by associating its development with the collection of biological and production data. While existing data sets from LAMBPLAN and OVIS have some potential, they are only representative of the breeding nuclei of the flock. Clearly, the way to further extend this approach is to populate the model through a modest survey undertaken concurrently with projects that benchmark biological and productivity on groups of farms and across regions.

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