

The Applicability of Robustness Measures to Water Resources Decision Making

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Abstract: The choice among alternative water supply sources is generally based on the fundamental objective of cost minimisation. There is, however, a need to consider sustainability, the environment and social implications in regional water resources planning, in addition to economics. In order to achieve this, multi-criteria decision analysis (MCDA) techniques can be used. There are a large number of MCDA methods, however, none of them can be considered as appropriate for all decision-making situations. Selection of the most suitable method can therefore be difficult. Various sources of uncertainty exist in the application of MCDA methods including the definition of criteria weights and the assignment of criteria performance values. Robustness / sensitivity analysis can be used to analyse the effects of these uncertainties and in this paper, two existing methods are applied to two water resources case studies. The results indicate that consideration of the various sources of uncertainty should be an integral part of the decision-making process. However, the existing methods could be improved by enabling concurrent alteration of the subjective input values and the method should also be applicable to a range of MCDA techniques.

Keywords: *Multi-criteria decision making; Robustness; Uncertainty; Water resources.*

1. INTRODUCTION

The allocation of water is an imperative decision-making problem, which is often characterised by a large number of alternatives, uncertain consequences, complex interactions, participation of multiple stakeholders, conflicting interests and competing objectives. In the past, solutions to water resource allocation problems have been based on the fundamental objective of maximising the ratio of benefits to costs using the benefit cost analysis (BCA) approach. Multi-criteria decision analysis (MCDA) can be used as an alternative methodology, as it allows the combination of quantitative and qualitative criteria, measured in different units, thereby enabling social and environmental factors to be taken into consideration. The benefit also lies in the insight to the problem the methodology provides, rather than the specific numbers it produces. However, the use of MCDA also presents a number of challenges. One difficulty associated with the use of MCDA is due to the large number of methods that exist for analysing MCDA problems. The choice is predominantly based on familiarity with the method, as no standard selection criteria have been developed.

The subjective nature of the input data required to perform the analysis presents another challenge. Generally, a value must be assigned to each

alternative, indicating its performance in relation to each decision criterion. This value is not fixed or known exactly and is often affected by the following three phenomena: imprecision, indetermination and uncertainty. Most MCDA methods also require some measure of relative importance to be attached to the criteria by using weights, which enables different views and their impact on the ranking of alternatives to be expressed explicitly. Research has shown that the weights assigned by stakeholders may not be reliable or consistent (Larichev, 1992).

The above factors have been found to influence the resultant ranking of alternatives and therefore should be taken into consideration as part of the decision-making process. However, despite their importance, these factors have been largely ignored in studies in which MCDA has been applied to water resources problems (see for example Gershon et al., 1982; Jaber et al., 2001; Kheireldin et al., 2001). Although different MCDA methods have been used in all of the above studies, there is little to suggest which approach is most appropriate, and consequently, it is difficult to determine the best course of action when the rankings of alternatives produced by the various methods differ. In addition, investigations into the impact that uncertainties in the criteria weights and performance values have on the rankings has been limited. In cases where

sensitivity analyses was conducted, it was generally only in the form of altering the criteria weights. Sensitivity of the results to the MCDA method used, and the performance values assigned to the criteria, were commonly ignored.

The objective of this paper is to investigate whether the use of existing robustness / sensitivity analysis methods can assist with overcoming some of the limitations of MCDA outlined above. Robustness is connected to the fact that, as discussed above, MCDA methods often contain parameters whose values have to be chosen (more or less arbitrarily) by the decision maker (DM). An alternative is considered robust if the solutions obtained for different plausible values of its parameters do not contradict each other and the ranking remains stable i.e. a robust solution performs reasonably well under a range of possible conditions. If the rankings of a number of solutions are robust to the values of the criteria weights, performance values assigned to the criteria and the MCDA method used, the level of confidence in the selected option is increased significantly. Conversely, if the rankings of alternatives are sensitive to the factors mentioned above, it is difficult to say that one solution is superior to another, and other factors might need to be considered to make a final decision.

2. METHODS

In order to investigate whether the use of robustness / sensitivity analysis methods can assist with the MCDA process, the robustness measure proposed by Guillen et al. (1998) and the sensitivity analysis approach proposed by Triantaphyllou and Sanchez (1997) are applied to two case studies for which a number of MCDA methods have been used previously (Raju et al., 2000 and Duckstein et al., 1994). No prior applications of these robustness / sensitivity analysis methods to case studies have been found in the literature reviewed to date. For both case studies, the Guillen et al. (1998) robustness measure, is used to investigate the robustness of the rankings obtained to the criteria weights and the sensitivity analysis approach suggested by Triantaphyllou and Sanchez (1997) is employed to explore the sensitivity of the rankings obtained to both the criteria weights and the performance values assigned to the criteria.

In order to apply the robustness / sensitivity analysis methods to the two case studies, the Weighted Sum Method (WSM) approach is used, which is a simple and often used MCDA method. An appraisal score is calculated for each alternative by multiplying each performance value ($x_{m,n}$) by its appropriate weight (w_m),

followed by summing the weighted scores for all criteria as follows:

$$\sum_{m=1}^M w_m x_{m,n} \quad (1)$$

Quantitative ranking algorithms, such as WSM, require criterion measures to be standardised into commensurable units. There are many types of standardisation formulae, but there is no obvious reason for selecting one method over another. A commonly used method, which is used in case study 2, is contained in Table 1.

Table 1. Standardisation formulae

Higher value is better	Lower value is better
$S_{mn} = \frac{x_{mn}}{x_n \max}$	$S_{mn} = \frac{x_n \min}{x_{mn}}$

Source: Hajkowicz et al. (2000)

2.1. Robustness Measure

The robustness measure proposed by Guillen et al. (1998) allows the DM to “determine the robustness of the preference between two alternatives”. It is defined as the proportion by which the DM must modify the weights to change the preferences between two alternatives. The robustness measure can be calculated for each pair of alternatives using the following equation:

$$r(a_1, a_2) = \frac{w_1 \times (x_{1,1} - x_{1,2}) + \dots + w_m \times (x_{1,n} - x_{2,n})}{w_1 \times (x_{1,1} - x_{1,2}) + \dots + w_m \times (x_{1,n} - x_{2,n})} \quad (2)$$

where $r(a_1, a_2)$ is the robustness between alternative 1 and 2, w_m is the weight applied to criteria n and $x_{1,n}$ is the performance measure of criteria n of alternative 1. $r(a_1, a_2)$ takes its value in the interval $(-1,1)$. a_1 dominates a_2 on all criteria when $r(a_1, a_2) = 1$.

The weights required to reverse a ranking can be calculated using the following equation(s):

If $x_{1,1} > x_{1,2}$ then:

$$w_1^* = \begin{cases} (w_1 - w_1 \times r(a_1, a_2)) & \text{otherwise} \\ (w_1 + w_1 \times r(a_1, a_2)) & \end{cases} \quad (3)$$

2.2. Sensitivity Analysis

Separate methods are proposed by Triantaphyllou and Sanchez (1997) for three MCDA methods (weighted sum model (WSM), weighted product model (WPM) and analytic hierarchy process (AHP)). The method described in this paper is based upon the WSM.

The minimum quantity that a criterion weight needs to be changed by to reverse the ranking can be calculated for each pair of alternatives for each criterion by:

$$\delta_{1,1,2} = \frac{(P_2 - P_1)}{(x_{2,1} - x_{1,1})} \quad (4)$$

where P_1 is the final value of alternative 1.

The following condition must be satisfied for the new weight to be feasible:

$$\delta_{1,1,2} \leq w_1 \quad (5)$$

Sometimes there may not be a feasible value as it may be impossible to reverse the existing ranking by making changes to the current weight. In such instances, the modified weight of the first criterion is calculated by:

$$w_1^* = w_1 - \delta_{1,1,2} \quad (6)$$

The percentage change in the weights is given by:

$$\%w = \frac{w_1^*}{w_1} \times 100 \quad (7)$$

The threshold value, R (in %), by which the performance measure of alternative a_n in terms of criterion c_m , denoted as P_i , needs to be modified so that the ranking of the alternative a_n and a_p will be reversed, is as follows:

$$R_{n,p} = \frac{(P_n - P_p)}{w_m} \times \frac{100}{x_{m,n}} \quad (8)$$

Furthermore, the following condition should also be satisfied for the threshold value to be feasible:

$$R_{n,p} \leq 100 \quad (9)$$

3. CASE STUDY 1 RESULTS

In this case study, Raju et al. (2000) used five MCDA techniques to evaluate seven alternative irrigation scenarios with 10 criteria. The opinions of DMs were reflected by criteria weights. The final rankings obtained by Raju et al. (2000) are given in Table 2. In addition, the results acquired by applying the WSM (Eq. 1) are displayed. This analysis was undertaken for the purposes of performing the robustness / sensitivity analysis using the techniques described in section 2. Application of the standardisation method was not required as all performance values were already presented in the same units. The results in Table 2 show that each MCDA method selected alternative 1 (A_1) as the preferred strategy.

The Guillen robustness measures for the highest ranked alternative (A_1) paired with the three next best alternatives (A_2 , A_3 and A_4) are summarised in Table 3. It can be seen that a 23% change in each of the criteria weights is required for A_2 to rank equally with A_1 . A change in the weights

will have no impact on the ranking of A_3 with regard to A_1 , as a robustness measure of 1.0 was obtained. A_4 will not be ranked equally with A_1 unless a 78% change in each of the criteria weights occurs.

Table 2. Ranking patterns obtained by various MCDA techniques for case study 1

Method	Alternatives						
	1	2	3	4	5	6	7
PROMETHEE-2	1	2	3	4	6	5	7
EXPROM-2	1	2	3	4	6	5	7
ELECTRE III	1	2	2	3	3	3	4
ELECTRE IV	1	3	4	2	4	3	5
CP (p = 1)	1	2	3	4	6	5	7
CP (p = 2)	1	2	3	4	5	6	7
CP (p = ∞)	1	3	2	4	5	6	7
WSM*	1	2	3	4	5	6	7

* Additional results obtained as part of this study

Table 3. Guillen et al. (1998) robustness values for the 4 highest ranked alternatives, case study 1

	Robustness measure
r (1, 2)	0.23
r (1, 3)	1.00
r (1, 4)	0.78

The results of applying the Triantaphyllou and Sanchez sensitivity analysis method to this case study are summarised in Table 4 for the highest ranked alternative (A_1) paired with the three next best alternatives (A_2 , A_3 and A_4).

Table 4. Percent change in criteria weights and performance values required to reverse the ranking obtained using the Triantaphyllou and Sanchez (1997) method for the 4 highest ranked alternatives, case study 1

Pair of A's	% Change in Criteria Weights
$A_1 - A_2$	-97% (W_1), 97% (W_5), 156% (W_{10}), NF for remainder of weights
$A_1 - A_3$	NF for all weights
$A_1 - A_4$	-700% (W_5), NF for remainder of weights
Pair of A's	% Change in Performance Values
$A_1 - A_2$	5% (C_3), 9% (C_9), 10% (C_{10}), 16% (C_4)
$A_1 - A_3$	23% (C_3), 42% (C_9), 48% (C_{10}), 79% (C_4)
$A_1 - A_4$	33% (C_3), 42% (C_9), 48% (C_{10}), NF (C_4)

Note: A = alternative, W = weight, C = criteria, NF = not feasible; results rounded to a whole number.

The analysis of criteria weights shows that the ranking of A_1 and A_2 will be equal if there is either a 97% reduction of the weight assigned to criterion 1 (W_1), a 97% increase of the weight of criterion 5 (W_5) or a 156% increase of the weight of criterion 10 (W_{10}). Changing the weights of the criteria will not alter the ranking of alternative 3 with respect to alternative 1.

A relatively small change in the performance values, compared to the criteria weights, of either of criterion 3, 9, 10 or 4 will result in the ranking of A_1 and A_2 being equal i.e. if the performance measure of criterion 3 for A_1 is reduced by 5%, the ranking of A_1 and A_2 will be equal. A significantly larger change in the performance values of criteria 3, 9, 10 and 4 will result in A_1 being equally ranked with A_3 and A_4 respectively.

4. CASE STUDY 2 RESULTS

Four MCDA methods were applied to a groundwater management problem investigated by Duckstein et al. (1994). 13 alternatives were assessed using three criteria. The results obtained for each technique considered by Duckstein et al. (1994) are presented in Table 5. In addition, results acquired by applying the WSM (Eq. 1) and the standardisation method in Table 1, are displayed.

Table 5. Ranking of the 13 alternatives obtained using different MCDA methods, case study 2

Alt.	CP			ELECTRE III	MUF	UTA	WSM*
	$P=1$	$P=2$	$P=\infty$				
1	8	2	1	6	7	8	12
2	10	10	8	2	10	10	11
3	11	11	11	3	11	11	10
4	12	12	12	5	12	12	7
5	13	13	13	7	13	13	9
6	9	5	4	5	8	9	13
7	7	6	2	4	6	6	8
8	6	7	6	3	2	2	5
9	4	8	9	1	1	1	1
10	3	3	5	5	5	7	6
11	1	1	2	1	3	4	4
12	2	4	6	2	4	3	3
13	5	9	9	6	9	5	2

* Additional results obtained as part of this study

The rankings produced by the ELECTRE III, MUF and UTA methods show that there are four alternatives (A_8, A_9, A_{11}, A_{12}) that are selected as “good alternatives”. CP yields slightly different results.

The Guillen robustness measures for the highest ranked alternative (A_9) paired with the three next best alternatives (A_{13}, A_{12} and A_{11}) are summarised in Table 6. These results indicate that A_9 is least robust with respect to the second and third ranked alternatives (A_{12} and A_{13}), requiring only a 4% or 16% change, respectively, in each of the criteria weights for the ranking to be equalled.

Table 6. Guillen robustness values for the four highest ranked alternatives, case study 2

	Robustness measure
$r(9, 13)$	0.04
$r(9, 12)$	0.16
$r(9, 11)$	0.25

The results of the sensitivity analysis proposed by Triantaphyllou and Sanchez (1997) for criteria weights and performance measures of the highest ranked alternative paired with the subsequent three most highly ranked alternatives are summarised in Table 7.

Table 7. Percent change in criteria weights and performance values required to reverse the ranking obtained using the Triantaphyllou and Sanchez (1997) method for the 4 highest ranked alternatives, case study 2

Pair of A's	% Change in Criteria Weights
$A_9 - A_{13}$	NF (W_1), 8% (W_2), -8% (W_3)
$A_9 - A_{12}$	-415% (W_1), 28% (W_2), -42% (W_3)
$A_9 - A_{11}$	-311% (W_1), 41% (W_2), -87% (W_3)
Pair of A's	% Change in Performance Values
$A_9 - A_{13}$	46% (C_1), 7% (C_2), NF (C_3)
$A_9 - A_{12}$	NF (C_1), 24% (C_2), NF (C_3)
$A_9 - A_{11}$	NF (C_1), 36% (C_2), NF (C_3)

Note: A = alternative, W = weight, C = criteria, NF = not feasible; results rounded to a whole number.

The analysis of the criteria weights shows that A_9 and A_{13} will be ranked equally if there is either an 8% reduction in criterion weight 2 or an 8% increase in criterion weight 3. Significantly larger changes in the criteria weights are required for the ranking of A_{12} or A_{11} to equal that of A_9 .

A small change in the performance value of criterion 2 is required for A_9 and A_{13} to be ranked

equally, while larger changes in the performance values of criterion 2 are necessary for the rankings of A_{12} and A_{11} to equal that of A_9 .

5. DISCUSSION

Due to space limitations, the results presented in sections 3 and 4 and discussed below concentrate on how changes in criteria weights and performance values will affect the highest ranking alternatives. The same analysis can be conducted for the lower ranked alternatives.

The results of case study 1 (Table 2) show that the ranking of alternatives obtained using a number of MCDA methods are in good agreement. However, application of these methods provides no information with regard to how close the various alternatives are to each other, nor how much any uncertainties in the assumptions made in relation to criteria weights and performance values impact on the rankings.

Investigation of the effect that changes in criteria weights would have on the ranking between alternatives in case study 1, by applying the methods proposed by Guillen et al. (1998) and Triantaphyllou and Sanchez (1997), found that of the seven alternatives, alternative 2 will equal the highest ranked alternative with the smallest changes in criteria weights. The analysis showed that a large (i.e. > 95%) change in a single criterion weight is required to equal the ranking of alternatives 1 and 2, in comparison to a 23% change to all the criteria weights. Both of the methods also established that no changes in the weights would produce any alteration to the ranking of alternatives 1 and 3. From this information it can be concluded that minor changes to either all of the weights or individual weights will not have an impact on the final ranking obtained from the WSM in case study 1.

Results of the Triantaphyllou and Sanchez (1997) method for assessing the effect of changes in the performance values of decision criteria for case study 1 show that changes to the performance values of the same four criteria (3, 4, 9 and 10) by small amounts (Table 4), compared to the criteria weights, will result in equal rankings between alternative 1 and alternatives 2, 3 and 4, respectively. This analysis informs the DM that of the 10 criteria used in the assessment, the performance values of these four are the most critical to the overall ranking, therefore, the accuracy of these values, in particular, should be ensured. It is interesting to note that criterion 3 is the highest weighted criteria (0.2), followed by criteria 9 and 10 (0.125).

The results of the analysis undertaken for case study 1 has established that the rankings are most

sensitive to the performance values of particular criteria, rather than the weights assigned to the criteria. However, the impact of combined changes in performance values and criteria weights on the ranking of the alternatives is still unknown.

The results of case study 2 (Table 5) are puzzling to the DM, as the rankings obtained using a number of MCDA methods are not in agreement. In addition, application of these methods provides no information with regard to how close various alternatives are to each other, why the rankings differ, nor how much various uncertainties influence the rankings.

The outcomes of applying the methods proposed by Guillen et al. (1998) and Triantaphyllou and Sanchez (1997) to case study 2 using the WSM indicate that of the 13 alternatives, alternative 13 will equal the highest ranked alternative (A_9) with the smallest change in criteria weights. In comparison with case study 1, a minor (i.e. <10%) change in a single criterion weight or a 4% change in all of the criteria weights will enable this to occur. The results of both methods, therefore, concur that small changes in criteria weights may have an impact on the ranking of the alternatives.

Changes to the performance values of criterion 2 in case study 2 will result in equal rankings between alternative 9 and alternatives 13, 12 and 11, respectively. The application of the Triantaphyllou and Sanchez (1997) method for performance values informs the DM that of the three criteria used, the performance value of criterion 2 is the most critical to the ranking of the alternatives. It is again noted, as with case study 1, that the criterion that has the most influence on alterations in the ranking (i.e. C_2) is the highest weighted criterion.

From the analysis undertaken it is evident that either alterations to the performance values or changes to the criteria weights will have an impact on the ranking of the alternatives in case study 2. This is in contrast to case study 1, where only amendments to the criteria weights were deemed to be significant.

In general, based on the results obtained in both case studies, the Guillen et al. (1998) and Triantaphyllou and Sanchez (1997) methods are in good agreement with regard to which rankings will be affected with a change in criteria weights. The application of the Triantaphyllou and Sanchez (1997) method to case study 1 confirms that the uncertainty of performance values can have an impact on the ranking of alternatives and should be taken into consideration in all robustness / sensitivity analysis.

The Guillen et al. (1998) and Triantaphyllou and Sanchez (1997) methods provided additional information with regard to whether changes in criteria weights and / or performance values will affect the ranking between two alternatives in the two case studies. Identification of the most critical criteria weights and / or performance values from the results of the Triantaphyllou and Sanchez (1997) method is also beneficial. Despite the insight provided by the two methods, a number of shortcomings remain.

The foremost limitation of both methods, which is common with other existing methods, is that they only consider one data input at a time, with the remaining data inputs being fixed. A methodology which is capable of assessing the effect of conjunctive alterations in input parameter values is therefore required to be developed as part of future research.

The existing methodologies are also inadequate in the sense that they are only applicable to certain MCDA methods. The potential of developing a robustness measure that is applicable to a range of MCDA methods also needs to be investigated.

Both case studies illustrated the varying effect utilisation of different MCDA methods can have on the ranking, with agreement of the rankings in case study 1 compared to disagreement in the rankings in case study 2. The application of the two robustness / sensitivity analysis methods to case study 2 only provided additional information on the stability of the rankings produced by the WSM and did not impart any enlightenment on the differing rankings obtained by the four MCDA methods applied by Duckstein et al. (1994). This is a problem that should be addressed by further research.

6. CONCLUSIONS

Analyses of water resource allocation problems, involving tradeoffs among multiple criteria, can be undertaken using MCDA. However, the arbitrariness of the selection of the MCDA method, in addition to the subjectivity of the assignment of weights to the criteria and the uncertainty in the determination of the performance values of the criteria, creates significant doubt in the rankings obtained.

The application of the two existing robustness / sensitivity analysis methods to two water resources case studies has illustrated the insight that the existing methods can provide into the impact that changes in methods, criteria weights and performance values have on the ranking of alternatives. Further research is required, however, in order to develop a robustness measure that allows the concurrent alteration of

input values and is applicable to a range of MCDA methods.

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