Modelling Volatility Asymmetry of Business Cycles in the U.S.

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EXTENDED ABSTRACT

Most studies on the asymmetric and non-linear properties of US business cycles exclude the dimension of asymmetric conditional volatility. Engle (1982) proposes an autoregressive conditional heteroskedasticity (ARCH) model to capture the time-varying volatility of inflation rates in the United Kingdom. Weiss (1984) finds evidence of ARCH in the US industrial production. The ARCH model is then extended to generalized ARCH (GARCH) models by Bollerslev (1986) and exponential GARCH models by Nelson (1991). Recently Stock and Watson (2002) find that a substantial reduction in the variability of the US output growth since the early 1980s can be explained by a reduction in the variance of macroeconomic shocks. However, a few researchers have attempted to formally model asymmetries in the conditional variance of business-cycle variables (see Brunner 1992, Hamori 2000, and Ho and Tsui 2001, 2003 and 2004). All these studies are confined to univariate GARCH analysis. One major drawback of the univariate GARCH framework is that it does not capture the co-movement of business-cycle variables, nor analyze the empirical evidence of asymmetric volatilities in the context of multivariate GARCH approach.

In this paper, we use the multivariate GARCH framework to investigate the evidence of asymmetric volatility and time-varying conditional correlations between sectors of the monthly US industrial production (IIP) indices in 1961-1997. We propose three new multivariate asymmetric GARCH models, which are developed based on a synthesis and improvement of the methodologies of Ding et al (1993), Sentana (1995) and Tse and Tsui (2002), including the VC-Quadratic GARCH (VC-QGARCH) model, VC-Leveraged GARCH (VC-LGARCH) model, and VC-Threshold GARCH (VC-TGARCH) model. Our proposed models are computationally manageable and are capable of capturing features of volatility asymmetry and the path of time-varying correlations. The issues of conditional heteroskedasticity and volatility asymmetry of business cycles are important because of their implications on macroeconomic and business cycle theory, measurement and forecasting. If business cycles are conditionally heteroskedastic and exhibit volatility asymmetry, then any theory assuming the absence of either of these properties is most likely inadequate. It is crucial to understand the potential macroeconomic policy implications of asymmetric volatility shocks for economies. If negative shocks induce greater future volatilities on IIP than positive shocks of the same magnitude, this might further vindicate the implementation of macroeconomic stabilisation measures by the government in times of recession.

We use monthly data from the OECD website SourceOECD: Main Economic Indicators for the 5 sectoral IIP of the US: the Consumer Good (CG), the Investment Good (IG), the Manufacturing (M), the Non-Durables (ND) and the Raw Materials (RM) with 444 observations.

The results show that negative shocks have a greater impact on future volatilities than positive shocks of the same magnitude for the 5 sectoral IIP series in the U.S. Most parameter estimates of the time-varying conditional correlation coefficient equation are significant at the 5% level, indicating that dynamic correlations probably exist among the 5 main industrial groups/sectors. The estimates of the time-invariant component of the correlation coefficient equation, \( \rho \), are significantly positive and broadly similar to those estimates from the constant conditional correlations models, which is consistent with Lucas’ (1977) observation More importantly, the pattern of conditional correlations and the magnitude of \( \rho \) differ among the 10 sectoral pairs, ranging from a low of 0.2763 (IG-ND pair) to 0.7652 (M-RM pair) and 0.8363 (CG-M pair). This is consistent with results from the VC-LGARCH model. The findings on asymmetric effects have policy implications for government to consider the effective countercyclical measures during recessions.
1. INTRODUCTION

Searching for evidence of asymmetries and non-linearities in the US business cycles has been under extensive empirical examination. Basically there are two major approaches: One focuses on the non-linear behaviour in the conditional mean function, and the other concentrates on the time-varying features of higher moments, particularly the conditional variances. Examples for the first approach include Neftci (1984) and Sichel (1993) who find evidence of asymmetries in the post-war unemployment time series of the United States. Other investigations include Luukkonen and Terasvirta (1991), Terasvirta and Anderson (1992), and Diebold and Rudebusch (1996). Most of these empirical studies are predominantly on the conditional mean equation. For the second approach, a noteworthy example is the seminal paper by Engle (1982), who proposes an autoregressive conditional heteroskedasticity (ARCH) model to capture the time-varying volatility of inflation rates in the United Kingdom. Weiss (1984) finds evidence of ARCH in the US industrial production. The ARCH model is then extended to generalized ARCH (GARCH) models by Bollerslev (1986) and exponential GARCH models by Nelson (1991). Recently Stock and Watson (2002) find that a substantial reduction in the variability of the US output growth since the early 1980s can be explained by a reduction in the variance of macroeconomic shocks. However, a few researchers have attempted to formally model asymmetries in the conditional variance of business-cycle variables (see Brunner 1992, Hamori 2000, and Ho and Tsui 2001, 2003 and 2004). All these studies are confined to univariate GARCH analysis. The primary limitation with the use of the univariate GARCH framework is that it does not capture the co-movement of business-cycle variables, nor analyze the empirical evidence of asymmetric volatilities in the context of multivariate GARCH approach.

In this paper, we use the multivariate GARCH framework to investigate the evidence of asymmetric volatility and time-varying conditional correlations between sectors of the monthly US industrial production (IIP) indices from January 1961 through December 1997. Three new multivariate asymmetric GARCH models are proposed, including the VC-Quadratic GARCH (VC-QGARCH) model, VC-Leveraged GARCH (VC-LGARCH) model, and VC-Threshold GARCH (VC-TGARCH) model. These models are developed based on a synthesis and improvement of the methodologies of Ding et al (1993), Sentana (1995) and Tse and Tsui (2002). These specifications are computationally manageable, and permit the simultaneous modelling of conditional volatility asymmetry and time-varying conditional correlations. Furthermore, these models are quite general because they nest various popular versions of asymmetric GARCH models. This in turn permits a systematic comparison of the performance of different asymmetric specifications. The issues of conditional heteroskedasticity and volatility asymmetry of business cycles are important because of their implications on macroeconomic and business cycle theory, measurement and forecasting. If business cycles are conditionally heteroskedastic and exhibit volatility asymmetry, then any theory assuming the absence of either of these properties is most likely inadequate. It is crucial to understand the potential macroeconomic policy implications of asymmetric volatility shocks for economies. If negative shocks induce greater future volatilities on IIP than positive shocks of the same magnitude, this might further vindicate the implementation of macroeconomic stabilisation measures by the government in times of recession.

2. MODEL SPECIFICATION

We first define the GARCH framework for modelling asymmetry volatility and time-varying conditional correlations. There are three components of the bivariate asymmetric GARCH models: the conditional mean equation, the conditional variance equation and the time-varying conditional correlation equation.

Denoting $Y_{it}$ as the $i$th variable of interest, we define $y_{it}$ as the growth rate (in percentage) calculated on a continuously compounding basis:

$$y_{it} = \log\left( \frac{Y_{it}}{Y_{it-1}} \right) \times 100, \ i = 1, 2$$

Assume that the conditional mean equation for each variable is captured by an autoregressive AR(k) filter:

$$y_{it} = \pi_0 + \sum_{j=1}^{k} \pi_j y_{it-j} + \epsilon_{it}, \ i = 1, 2$$

where $\epsilon_{it}$ is the white noise and is assumed to have the following structure:

$$\epsilon_{it} = s_i \epsilon_{it}, \ \epsilon_{it} \sim N(0,1), \ i = 1, 2$$

$s_i$ is the conditional variance with three types of specifications: QGARCH, LGARCH and TGARCH. We choose such specifications as they nest several versions of popular GARCH models.

We adopt the QGARCH(1,1) proposed by Sentana’s (1995) as:
The specifications of LGARCH and TGARCH are based on Ding et al (1993). Depending on the value of the exponent $\delta$, LGARCH and TGARCH share the following structure:

$$s_n^\delta = \eta + \alpha [|\epsilon_{n-1}| - \gamma \epsilon_{n-1}] + \beta s_{n-1}^\delta \quad \delta=1 \text{ or } 2 \quad (5)$$

When $\delta = 2$, this is the LGARCH(1,1) model which nests the Glosten et al’s (1993) GJR model. Alternatively, when $\delta = 1$, it is the TGARCH(1,1) model, which incorporates an asymmetric version of the Taylor/Schwert model and Zakoian’s (1994) Threshold ARCH (TARCH) model.

The major problem with multivariate GARCH models in general is that they inevitably increase the number of parameters to be estimated and complicate the specifications of the conditional variance-covariance matrix. In particular, it could be difficult to verify the condition of positive-definiteness for the variance-covariance matrix of an estimated multivariate (MGARCH) model, not to mention impose this condition during the optimisation of the log-likelihood function. To incorporate dynamic correlations in the MGARCH model and yet satisfy the positive-definite condition, Tse and Tsui (2002) have recently developed the Varying-Correlation (VC) - MGARCH (VC-MGARCH) model.

Following Tse and Tsui (2002), we adopt an ARMA structure for the conditional correlations equation in a bivariate model:

$$\rho_{ij} = (1-\theta_1)\rho + \theta_1 \rho_{i-1} + \theta_2 \psi_{i,j}$$

where $(1-\theta_1)\rho$ is the time-invariant conditional correlation coefficient, $\theta_1$ and $\theta_2$ are assumed to be nonnegative and sum up to less than 1, and $\psi_{i,j}$ is specified as

$$\psi_{i,j} = \frac{\sum_{n=1}^{2} \epsilon_{i,n} \epsilon_{j,n}}{\sqrt{\left(\sum_{n=1}^{2} \epsilon_{i,n}^2\right)\left(\sum_{n=1}^{2} \epsilon_{j,n}^2\right)}}$$

Assuming conditional normality, the conditional log likelihood function of the sample (ignoring the constant term) is

$$L = -\frac{1}{2} \sum_{t} \left( \log(2\pi \rho_{i,j}^2) + \frac{\tilde{e}_{i,t}^2 + \tilde{e}_{j,t}^2 - 2 \rho_{i,j} \tilde{e}_{i,t} \tilde{e}_{j,t}}{(1-\rho_{i,j}^2)} \right)$$

The total number of parameters to estimate is 11 for a bivariate asymmetric GARCH model with varying correlations, and this number always exceeds that of Bollerslev’s (1990) constant-correlation model by 2, because of the parameters $\theta_1$ and $\theta_2$. Indeed the CC-MGARCH model is nested within the VC-MGARCH model under the restrictions $\theta_1 = \theta_2 = 0$.

3. THE DATA

We use the monthly US industrial production (IIP) indices to study the possible asymmetry and non-linearity of business cycles over the period from January 1961 to December 1997. This is because the IIP demonstrates more cyclical variation than GDP/GNP and constitutes the most cyclical subset of the aggregate economy. GDP inevitably includes both a large service sector which cannot be accurately measured, and a large agricultural sector that is difficult to estimate or dominated by its own meteorological cycle. Such components could tend to obscure business cycles. The preference of IIP to GDP is further corroborated by A’Hearn and Woitek (2001) and the Business Cycle Dating Committee of the National Bureau of Economic Research (see NBER 2002). In particular, the committee considers IIP to be one of the four most important indicators in developing its monthly business cycle chronology. In contrast, it gives relatively little weight to real GDP because this indicator is only measured quarterly and is subject to continuing large revisions.

Nonetheless, analysing the overall IIP alone would not be sufficient in developing a deeper understanding of the properties of the US business cycles, and different sectors would respond with varying degrees of sensitivity to the overall recession. According to Lucas (1977), the production of producer and consumer durables exhibits much more amplitude than the production of non-durables. Also, the production and prices of agricultural goods and natural resources have lower than average conformity. Given these empirical regularities, conducting a sectoral analysis of industrial production would further shed light on the characteristics of the US business cycles. In this study we adopt the 5 sectoral IIP of the US: the Consumer Good (CG), the Investment Good (IG), the Manufacturing (M), the Non-Durables (ND) and the Raw Materials (RM). These series are regularly monitored by the Conference Board’s Business Cycle Indicators to evaluate the future directions and changes of the US business cycles. The 5 (seasonally adjusted) data sets are collected from the OECD website SourceOECD: Main Economic Indicators.
4. RESULTS

We first assess the features of the IIP series. Due to space limitation, the results are not reported but available upon request from the authors. All the IIP growth rate series, which are the differenced logarithmic series, are negatively skewed and leptokurtic, which mirror those of the overall US IIP as reported by Ho and Tsui (2003). This implies that, except for the Non-Durables sector, all the US sectoral IIP series have higher values of standard deviation. Also, for the Manufacturing sector, the values of skewness and kurtosis are higher than the overall US IIP. This could be ascribed to the greater responsiveness of the Manufacturing sector to business cycle shocks, which engender the frequent occurrences of large, extreme observations. Incidentally, the leptokurtic nature of all the sectoral IIP series also lends support to the appropriateness of applying GARCH models to our data sets to accommodate the excess kurtosis.

The appropriateness of GARCH models is further corroborated by the possible existence of conditional heteroskedasticity, as manifested by various tests for non-linear dependencies such as the McLeod-Li and the ARCH LM (Engle 1982) tests. In fact, most of the McLeod-Li and ARCH LM test statistics are significant at least at the 5% level. The non-parametric BDS and runs tests further support the existence of non-linear dependencies. Also, the runs test statistics based on the squared and absolute series of the growth rates are generally significant and demonstrate the possible presence of conditional heteroskedasticity. The Sentana’s (1995) QARCH(q) LM test statistics are significant at the 5% level, suggesting the presence of asymmetric conditional volatilities. Sentana (1995) has also suggested a one-sided version of the QARCH(q) LM test, which is based on the summation of the square of the t-ratios of the coefficients, for greater power. To ensure consistency, we have also implemented this test version and the results are similar to the aforementioned findings.

Another noteworthy issue is the need to ensure the stationarity of our data sets. We apply the Augmented Dickey Fuller (ADF) Tests to our data sets in accord with the procedure stipulated by Enders (1995). Additionally, to ensure consistency of our results, the Phillips-Perron (PP) test statistics are calculated. All test statistics are statistically significant at the 1% level, therefore indicating stationarity. Additionally the Ljung-Box Q-statistic and the Breusch-Godfrey (BG) test statistics for the ADF test are statistically insignificant at the 5% level. This implies that the residuals obtained from the ADF test equations are approximately white noise.

We finally estimate the various asymmetric GARCH models using the Maximum Likelihood Estimation (MLE) sub-routines in GAUSS. We adopt the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton optimisation algorithm for most of our estimation results. Additionally, we have set the convergence criterion (tolerance level) to the default level of $10^{-5}$. To ensure that the parameter estimates obtained are stable, we have also used stricter criteria (up to $10^{-9}$) and the estimates are found to be consistent. In fact, estimation results are generally invariant to the choice of the initial values.

The conditional mean, variance/covariance matrix, and the conditional correlations are simultaneously estimated in one step assuming normality. For the conditional mean specification, an AR(12) filter is chosen. This specification follows Schwert (1989) applying a 12th-order autoregression to monthly US macroeconomic data to model macroeconomic volatility. Nelson and Foster (1994) also suggest that “mis-specifying the conditional means adds only trivially (at least asymptotically) to measurement error”, whereas other factors such as capturing the “leverage effect” and modelling the fat-tailed nature of residuals are potentially more important. In fact, we have alternatively estimated an AR(6) model to filter the series and the results are consistent with those of AR(12).

Given that there are many parameters to be simultaneously estimated and that we are using macroeconomic data sets that are relatively small compared with financial time series, difficulties in attaining convergence are sometimes encountered. To facilitate convergence, we adopt the following strategies during the estimation: first, we apply a two-step approach by estimating an AR(12) filter for the conditional mean and then use the residuals to estimate the 3 asymmetric GARCH models. Second, we adopt an incremental approach by first estimating the most restrictive model and then proceeding gradually to the least restrictive one.

Conditional Variance and Correlations

Our results show that the parameter estimates for the conditional variance equations in the CC-QGARCH, CC-LGARCH, and CC-TGARCH models are similar to those in the VC-QGARCH, VC-LGARCH, and VC-TGARCH models, respectively. This is consistent with the findings of Tse and Tsui (2002), who note that incorporating time-varying correlations does not have much effect on the conditional variance estimates.
Tables 1 and 2 show the estimation results of the VC-QGARCH and VC-TGARCH models for the 5 sectoral IIP series respectively. Due to the space limitation, the results for VC-LGARCH are not reported here. First, the coefficient of volatility asymmetry is generally significant at least at the 5% level for all the 5 series in the three models. In particular, the coefficient suggests that negative shocks have a greater impact on future volatilities than positive shocks of the same magnitude. This is consistent with the findings of Ho and Tsui (2001, 2003 and 2004), who have detected significantly negative volatility asymmetry in the real GDP and overall IIP of the US.

Next, most parameter estimates of the time-varying conditional correlation coefficient equation are significant at the 5% level, indicating that dynamic correlations probably exist among the 5 main industrial groups/sectors. Additionally, the estimates of the time-invariant component of the correlation coefficient, \( \rho \), are significantly positive and broadly similar to those estimates from the constant conditional correlations models. The finding of positive correlations is consistent with Lucas’ (1977) observation that output changes across broadly defined sectors move together in the sense that they exhibit high conformity.

More importantly, however, the pattern of conditional correlations and the magnitude of \( \rho \) differ among the 10 sectoral pairs. For instance, in the case of the VC-QGARCH model, \( \rho \) ranges from a low of 0.2763 (IG-ND pair) to 0.7652 (M-RM pair) and 0.8363 (CG-M pair). This is consistent with results from the VC-LGARCH model. Also, the VC-TGARCH model suggests that the correlation between Investment Good and Raw Materials is stronger than that between the Investment Good and the Non-Durables.

One possible explanation is the differences in the type and nature of output from the various industry groups/sectors. For instance, the output of
Investment Good is usually considered durable, and thus output fluctuations in this sector do not correlate closely with those of the Non-Durables sector. As highlighted earlier, Lucas (1977) observes that the production of producer and consumer durables exhibits much more amplitude than the production of non-durables. This could partly explain the lower correlation between investment and non-durable goods. In contrast, the Manufacturing and Raw Materials exhibit appreciably higher correlation, probably because the latter sector is a major source of intermediate inputs to the former. As such, when there is a decline in the Manufacturing sector, the derived demand for factor inputs from the Raw Materials subsequently falls. In the case of Consumer Goods demand for factor inputs from the Raw Materials decline in the Manufacturing sector, the derived inputs to the former. As such, when there is a downturn across different industry groups/sectors. In particular, increases in the conditional deviation usually occur after the contractions (recessions) in the US economy. According to the NBER, for the period from January 1961 to December 1997, recessions have occurred in 1970, 1975, 1982, and 1990-91. The results indicate that, during or shortly after these recessions, the conditional standard deviations have been significantly higher for sectors such as the Consumer Good, Investment Good and Manufacturing sectors. This result is consistent with Ho and Tsui (2003), who observe that in the period after the 1973/74 and 1979 oil price shocks, the world economy plunged into a global recession and the conditional standard deviation of the overall US IIP is relatively higher. Indeed, Engle (1982) has noted that, in the chaotic 1970s when economies were plagued by stagflation, estimated variances of inflation increase substantially.

Another noteworthy feature is that the level of IIP conditional volatility is generally lower in the late 1980s and the 1990s. This could be partly ascribed to the generally accepted view that the US economy is more stable in the years after World War II than in the pre-war period (Diebold and Rudebusch 1992). This consensus is reinforced by formal examinations of postwar stabilisation, such as DeLong and Summers (1986). These studies have focused on the changing volatility of business fluctuations, and they have uniformly concluded that the variability of various macroeconomic aggregates about trend have diminished during the post-war period. Also, according to the NBER’s Business Cycle Dating Committee, the period from March 1991 onwards marks the beginning of a 10-year expansion that is the longest in the NBER’s chronology (NBER 2002). This protracted period of expansion, partly spurred by substantial productivity gains arising from advances in technological advances, has probably helped to reduce economic uncertainty and contributed to the low conditional standard deviations of the sectoral IIP series.

5. CONCLUSION

In this paper, we formulate three new multivariate asymmetric GARCH models under a synthesis and improvement of the methodologies of Ding et al (1993), Sentana (1995), and Tse and Tsui (2002). We apply these models to five main sectoral indices of Industrial Production (IIP) of the United States, including the Consumer Good (CG), the Investment Good (IG), the Manufacturing (M), the Non-Durables (ND) and the Raw Materials (RM). A major advantage of the proposed models over the existing specifications is that they are computationally manageable and are capable of capturing the properties of volatility asymmetry and time-varying correlations concurrently. Our study demonstrates that asymmetric conditional volatility is present in many of the sectoral IIP series, and that the conditional correlations are significantly time-varying. The finding of asymmetric volatility shocks has important policy implications. If negative shocks induce greater future volatilities on IIP than positive shocks of the same magnitude, this might further vindicate the implementation of macroeconomic stabilisation measures by the government in times of recession.

6. REFERENCES


