Nonlinear Dynamics and Chaos in Hydrologic Systems: Recent Developments and Future Directions

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EXTENDED ABSTRACT

During the last two decades or so, studies on the applications of the concepts of nonlinear dynamics and chaos to hydrologic systems and processes have been on the rise. Earlier studies on this topic focused mainly on the investigation and prediction of chaos in rainfall, river flow and lake volume data, and further advances were made during the subsequent years through applications of the concepts to other problems (e.g. data disaggregation, missing data estimation and reconstruction of system equations) and other processes (e.g. rainfall-runoff and sediment transport). During the same period, some of the important issues in the applications of chaos theory to real hydrologic data (e.g. data size, noise, zeros) were addressed and comparisons with other methods (e.g. stochastic methods, artificial neural networks) as to their applicability and performance for modeling and prediction of hydrologic processes were also made. The outcomes of these studies are certainly encouraging, especially considering the exploratory stage of the concepts in hydrology, although they continue to be under certain criticism mainly on the basis of the possible 'blind' applications of the less-understood' chaos concepts without recognizing their potential limitations for real hydrologic data.

Following up on the earlier reviews by Sivakumar (2000, 2004), this paper highlights some of the latest developments on the applications of nonlinear dynamics and chaos concepts in hydrology and the challenges that lie ahead on the way to further progress. As for their applications, studies in the important areas of scaling, groundwater contamination, parameter estimation and optimization, and catchment classification are reviewed and the inroads made thus far are discussed. In regards to the challenges that lie ahead, particular focus is given to improving our understanding of these largely less-understood chaos concepts and also finding ways to integrate these concepts with the others. With the recognition that none of the existing 'extreme-view' modeling approaches is capable of solving the hydrologic problems that we are faced with, the need for finding a 'middle-ground' approach that can integrate different methods is stressed. To this end, the viability of bringing together the stochastic and deterministic chaotic concepts as a starting point is also discussed.

1. INTRODUCTION

The inherent nonlinear nature of hydrologic systems and the associated processes has been known for long (e.g. Izzard 1966; Amorocho 1967; Amorocho and Brandstetter 1971). However, much of early hydrologic research (1960s-1980s), largely constrained by the lack of data and computational power, resorted to a linear stochastic approach (e.g. Harms and Campbell 1967; Klemes 1978; Salas and Smith 1981). Although the linear stochastic approach continues to be prevalent in hydrology, computational advances during the last two decades have certainly facilitated proposal of a nonlinear approach as a viable alternative. The nonlinear approach includes nonlinear stochastic methods (e.g. Kavvas 2003), artificial neural networks (e.g. Govindaraju 2000), data-based mechanistic models (e.g. Young and Beven (1994), and deterministic chaos theory (e.g. Sivakumar 2000). Among these, chaos theory, with its philosophy that complex and random-looking behaviors could *also* be the result of even simple nonlinear deterministic dynamics with sensitive dependence on initial conditions (Lorenz 1963), seems to be 'the simplest' and yet also 'the most controversial' approach [see, for example, Schertzer et al. (2002) and Sivakumar et al. (2002a) for a debate].

Amid the controversial nature of the concept, chaos theory has been finding increasing applications in hydrology in recent times. Earlier studies on chaos theory applications in hydrology essentially focused on the investigation and prediction of chaos in rainfall, river flow and lake volume data (e.g. Rodriguez-Iturbe et al. 1989; Wilcox et al. 1991; Jayawardena and Lai 1994; Abarbanel and Lall 1996; Koutsoyannis and Pachakis 1996; Puente and Obregon 1996; Porporato and Ridolfi 1997). Subsequent studies attempted chaos theory applications on other hydrologic problems, including data disaggregation, missing data estimation, and reconstruction of system equations (e.g. Sivakumar et al. 2001b; Elshorbagy et al. 2002a; Zhou et al. 2002), and other processes, including rainfall-runoff and sediment transport (e.g. Sivakumar et al. 2001a; Sivakumar 2002; Sivakumar and Jayawardena 2002). They also addressed some of the important issues that have been perceived to significantly influence the outcomes of chaos methods (because of the underlying assumptions of infinite and noise-free data) when applied to real hydrologic data, including data size, noise, zeros, delay time, and neighborhood selection (e.g. Wang and Gan 1998; Sivakumar et al. 1999, 2002c; Jayawardena and Gurung 2000; Sivakumar 2001; Elshorbagy et al. 2002b; Jayawardena et al. 2002). Further, they investigated the 'superiority' of chaos theory, if any, over other theories, such as stochastic methods and artificial neural networks, for prediction purposes (e.g. Jayawardena and Gurung 2000; Lisi and Villi 2001; Sivakumar et al. 2002b, c; Laio et al. 2003). Extensive reviews of these studies are already available in the literature (Sivakumar 2000, 2004a) and, therefore, details are not reported herein.

The realization and recognition, in the aftermath of the encouraging outcomes from most of the above studies, that chaos theory could provide a new perspective towards understanding the workings of hydrologic systems and processes have been important driving forces for its ever-increasing applications, despite the continuing skepticisms being thrown away from some quarters of the hydrologic community largely based on the possible 'blind' applications of the less-understood' chaos concepts without recognizing their potential limitations for real hydrologic data (the result of which could be 'false claims'). While this is indeed heartening, we must not lose sight of the fact that the true potential of chaos theory in hydrology can only be realized when it is attempted to solve the more challenging problems we are faced with (such as scaling and parameter estimation problems), rather than simply chaos identification and prediction problems. Identification of these challenging problems and evaluation of how chaos theory (either independently or in combination with others) can be helpful in solving such problems are crucial for true progress in hydrology. These questions, therefore, form the basis for the present study.

To address the above questions in an effective manner, it is important first of all to be well aware of the latest developments in chaos theory applications in hydrology and the significant inroads we have been able to make thus far. This is done herein through a comprehensive review of some of the important studies carried out in this area during the last few years [especially since the reviews of Sivakumar (2000, 2004a)]. With this status quo, which already identifies some of the challenging problems in hydrology and also hints at the utility and appropriateness of chaos theory (e.g. Sivakumar 2004b), potential scope and directions for further applications are highlighted. A strong case is also made, both from philosophical and from scientific perspectives, for the urgent need of a middle-ground approach (coupling the stochastic approach and the deterministic approach, for example), rather than an extreme approach that seems to prevail in our current research practice.

2. RECENT DEVELOPMENTS IN CHAOS APPLICATIONS IN HYDROLOGY

Since the reviews by Sivakumar (2000, 2004a), ideas gained from nonlinear dynamic and chaotic theories have found their applications in a few other areas of hydrology as well, including scaling, groundwater contamination, parameter optimization and catchment classification.

Regonda et al. (2004) employed the correlation dimension method (Grassberger and Procaccia 1983) to investigate the type of scaling behavior (stochastic or chaotic) in the temporal dynamics of river flow. Analyzing daily, 5-day and 7-day flow data from each of three rivers in the United States, they reported the presence of chaotic scaling behavior in the flow dynamics at the Kentucky River and the Merced River, and stochastic scaling behavior in the flow dynamics at the Stillaguamish River. They also observed that the 'dimensionality' of the flow dynamics increasing with the scale of aggregation; in other words, dynamics changing from a more deterministic behavior to a more stochastic behavior with aggregation in time. Similar results on the effects of data aggregation (i.e. change from determinism to stochasticity with increasing time scale) were also independently observed by Salas et al. (2005) and Sivakumar et al. (2004, 2007), albeit in different contexts and employing different methodologies. The presence of chaotic behavior in flow scaling has important implications in hydrology, since it has been a common practice to employ stochastic (random) cascade approaches in scaling investigations and for data disaggregation.

As noted by Sivakumar (2004a), the field of subsurface hydrology had largely eluded the attention of chaos studies earlier. To this end, especially with the experience gained with the surface hydrologic problems and the encouraging outcomes, Sivakumar et al. (2005) investigated the potential use of chaos theory to understand the dynamic nature of solute transport process in subsurface formations. They analyzed time series of solute particle transport in a heterogeneous aquifer medium (which was simulated using an integrated transition probability/Markov chain model, groundwater flow model, and particle transport model, for varying hydrostratigraphic conditions) using the correlation dimension method. The results generally indicated the nonlinear deterministic nature of solute transport dynamics (dominantly governed by only a very few variables, on the order of 3), even though more complex behavior was found to be possible under certain extreme hydrostratigraphic conditionsns. Later, Hossain and Sivakumar (2006) studied, employing the correlation dimension method, the spatial patterns of arsenic contamination in the shallow wells (< 150 m) of Bangladesh. Particular emphasis was given to the role of regional geology (Pleistocene vs. Holocene) on the spatial dynamics of arsenic contamination. The results, with correlation dimensions ranging between 8 and 11 depending on the region, suggested that the arsenic contamination in space is a medium- to high-dimensional problem. The results were further verified using logistic regression, with an attempt to explore possible (physical) connections between the correlation dimension values and the mathematical modeling of risk of arsenic contamination (Hill et al. 2007).

With the ever-increasing complexities of hydrologic models, which require more details about processes and more parameters to be calibrated, parameter estimation and optimization has become an extremely challenging problem [see, for example, Beven (2002) for details]. In an attempt towards simplifying this problem, Sivakumar (2004b) proposed an approach that incorporates and integrates the chaos concept with expert advice and parameter optimization techniques. The simplification is brought out essentially through the determination (using the correlation dimension method) of the 'number' of dominant variables governing the system under study, with the use of only a limited amount of data (often data corresponding to a single variable) representing the system. Hossain et al. (2004), in their study of Bayesian estimation of uncertainty in soil moisture simulation by a Land Surface Model (LSM), presented a simple and improved sampling scheme to the Generalized Likelihood Uncertainty Estimation (GLUE) by explicitly recognizing the nonlinear deterministic behavior between soil moisture and land surface parameters in the stochastic modeling of the parameters' response surface. They approximated the uncertainty in soil moisture simulation (i.e. model output) through a Hermite polynomial chaos expansion of normal random variables that represent the model's parameter (model input) uncertainty.

The realization that hydrologic models are often developed for specific situations and thus that their extensions and generalizations to other situations are difficult has recently motivated researchers to call for a catchment classification framework (Woods 2002; Sivapalan et al. 2003; McDonnell and Woods 2004). According to them, identification of dominant processes may help in the formation of such a classification framework. With this idea, Sivakumar (2004b) introduced a classification framework, in which the extent of complexity or ‘dimensionality’ (determined using nonlinear tools, such as the correlation dimension method) of a hydrologic ‘system’ is treated as a representation of the (number of) dominant processes. Following up on this, Sivakumar et al. (2007) explored the utility of the phase-space reconstruction (a fundamental step in the applications of chaos methods), in which the ‘region of attraction of trajectories’ in the phase-space is used to identify the data as exhibiting ‘simple’ or ‘intermediate’ or ‘complex’ behavior and, correspondingly, classify the system as potentially low-, medium- or high-dimensional. The utility of this reconstruction concept was first demonstrated on two artificial time series possessing significantly different characteristics and levels of complexity, and then tested on a host of river-related data representing different geographic regions, climatic conditions, basin sizes, processes and scales. The ability of the phase-space to reflect the river basin characteristics and the associated mechanisms, such as basin size, smoothing, and scaling, was also observed.

There have also been several other studies that have, in one way or another, looked into the applications of chaos theory in hydrology. These include applications to understand the processes, such as rainfall-runoff (e.g. Dodov and Foufoula-Georgiou 2005) and soil nutrient cycles (e.g. Manzoni et al. 2004), and those to investigate the reliability of the chaos methods for hydrologic data, such as data size (e.g. Sivakumar 2005a) and others (e.g. Khan et al. 2005). Due to space constraints, details of these studies (and many others) are not reported herein.

3. ACHIEVEMENTS THUS FAR AND CHALLENGES THAT LIE AHEAD

It must be clear by now that we have made some sincere efforts to explore the potential of nonlinear dynamic and chaos concepts for modeling and prediction of hydrologic systems. The outcomes of these efforts are certainly encouraging, considering the fact that we are still in the exploratory stage in regards to these concepts, as opposed to the much more established and prevalent stochastic concepts (wherein also, by the way, we are struggling with the interpretation of the methods and outcomes). The additional inroads we have made in recent years in the areas of scaling, groundwater contamination, parameter estimation and optimization, and catchment classification, among others, are significant, albeit their preliminary nature, since these are arguably some of the most important topics in hydrology at the current time.

With these positives, however, we must not forget the challenges that lie ahead on our way to progress. Among such challenges, two are noteworthy: (1) improving our understanding of the largely less-understood chaos concepts; and (2) finding ways to integrate these concepts with the others, either already in existence or emerging in the future. The former is important for avoiding ‘blind’ applications of the related methods (simply because the methods exist) and ‘false’ claims (either in support of or against their utility); and the latter is important for taking advantage of the merits of the different approaches for their ‘collective utility’ to solve hydrologic problems rather than their ‘individual brilliance’ as perceived. In what follows, some examples are provided as to the potential limitations of the above studies and possible ways to improve them.

The studies by, for instance, Regonda et al. (2004) and Salas et al. (2005) provide interesting insights into the problem of scaling and effects of data aggregation. Their message, in essence, is that the ‘complexity’ of the system dynamics increases (from more determinism to more stochasticity) with aggregation in time scale. While this may indeed be the case in certain situations, its generalization is often difficult to make, since the system’s dynamic complexity depends on the catchment characteristics and the inputs. For example, the catchment area (and, hence, the time of concentration, not to mention the rainfall characteristics) plays a vital role in defining the relationship between data aggregation and dynamic complexity. In fact, depending on the catchment, the dynamic complexity may increase with aggregation in time up to a certain point (probably, somewhere close to the concentration time) and then decrease with further aggregation [see Sivakumar et al. (2004) for an example in a disaggregation context].

The attempts by Sivakumar et al. (2005) and Hossain and Sivakumar (2006) to search for possible nonlinear deterministic dynamics in solute transport in a heterogeneous aquifer and arsenic contamination in shallow wells are certainly interesting. However, these studies are crude one-dimensional simplifications, at best, to the complex three-dimensional groundwater flow and transport phenomena. They only consider the time or space (as the case may be), but what is actually needed is a spatio-temporal perspective. Moreover, while there is no ‘mathematical’ constraint, the ‘philosophical’ merit behind the use of phase-space reconstruction concept in a spatial context (with its delay parameter defined in space), as is done in Hossain and Sivakumar (2006), remains an issue to ponder.

The proposal by Sivakumar (2004) on the integration of different concepts/methods to deal with the workings of hydrologic systems, more specifically to simplify the existing parameter optimization procedures, is a notable move forward, as different concepts/methods possess different advantages and limitations. However, the utility and effectiveness of this proposal are yet to be seen through implementation. Similarly, the proposal on the use of a data-based reconstruction approach for 'system classification' and its effective testing on river-related data, as presented by Sivakumar et al. (2007), seem to provide strong clues to the potential of such an approach for formulation of a catchment classification framework. What remains to be studied, however, is how to incorporate the catchment characteristics into this classification framework and how to establish connections between data (usually at the catchment scale) and the actual catchment physical mechanisms (at all scales) for this classification framework to be successful.

These are some of the important questions that need attention to advance further our understanding of the role nonlinear dynamic and chaos concepts can play in hydrology. While there is every positive indication that this will be done, sooner rather than later, such alone may not be sufficient to solve the hydrologic problems that we are faced with today and will be facing in the future. What is required, as experiences suggest, is a change in our research paradigm and attitude [see, for example, Gupta et al. (2000) for a similar opinion, albeit from a different perspective].

4. CONCLUSION – STRIVING FOR A MIDDLE-GROUND

Every human being has his/her own perceptions about the workings of nature, which, to a great extent, are influenced by his/her societal, cultural, economic and environmental backgrounds, among others. The authors believe that these perceptions are generally the driving force for selecting his/her field of study and research and also for identifying the methods for applications, although it is becoming increasingly difficult these days with our society's lifestyles and pressures, largely driven by the economy.

We, researchers in hydrology, are also starting to put more emphasis on applications of specific concepts and methods rather than on addressing the most challenging hydrologic problems affecting us all, if literature is any indication [see, for example, Sivakumar (2005b) for details]. We are also increasingly realizing that none of the tools that are currently available (linear or nonlinear, stochastic or chaotic) is adequate by itself for solving our hydrologic problems to our satisfaction (in other words, 'accurate' modeling and prediction). The following are only a few examples of the numerous questions that need to be asked about our existing research approaches and methods to tackle the challenges in hydrology: How are we going to incorporate the nonlinear deterministic components that are inherent in hydrologic systems and processes in our linear (and nonlinear) stochastic approaches? How are we going to address the property of sensitive dependence of hydrologic system dynamics on the initial conditions, when the initial conditions themselves cannot be known? How are we going to explain the 'random' and unpredictable system behavior using our (nonlinear) deterministic approaches? How are we going to estimate the uncertainty in the parameters that serve as important inputs to our complex models, and even worse how are we going to define uncertainty? How are we going to establish the 'connections' between our 'data-based' approaches and the 'process-based' approaches, especially when there are 'disconnections' (either intended or unintended) in our research approaches?

These are difficult questions to answer, and the only way, in the opinion of the authors, is to find 'common grounds' in our approaches to research. This does not mean that someone has to 'give up' his/her ideas to make way for others', but this certainly requires some kind of 'compromise' and 'sacrifice' for the betterment of the hydrologic community. To this end, bringing together the linear (and nonlinear) stochastic concepts and nonlinear deterministic chaotic concepts could be a good starting point. The stochastic concepts have already been well established in hydrology, although improvements are needed for their more effective and efficient applications. On the other hand, the nonlinear dynamic and chaos concepts still remain largely unexplored, although we are certainly starting to see encouraging signs of their potential, as the review herein indicates. Let us hope for a new chapter in hydrology, one that finds a 'middle-ground' and yet capture the 'extremes.'

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