Thoughts on VaR and CVaR

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EXTENDED ABSTRACT

Value at Risk (VaR) is an important issue for banks since its adoption as a primary risk metric in the Basel Accords and the requirement that it is calculated on a daily basis. VaR calculates maximum expected losses over a given time period at a given tolerance level. Conditional Value at Risk (CVaR) measures extreme risk. It calculates the risk beyond VaR. Relative industry risk measurement is also very important to Banks in their management of risk, such as for setting risk concentration limits and developing investment and credit policy.

This paper examines market Value at Risk (VaR) and Conditional VaR (CVaR) in Australia from an industry perspective using a set of Australian industries. VaR and CVaR are compared between these industries over time, and a variety of metrics are used including diversified and undiversified VaR, as well as parametric and nonparametric CVaR methods. There has been no prior investigation of industry based VaR metrics in Australia to the authors' knowledge. The relative riskiness of different industry sectors is examined and using diversified VaR, the study finds the highest risk is in the Technology Sectors, whilst the lowest risk is found in the Finance and Utilities Sectors. Composite riskiness is also explored and the existence of correlation between industry risk rankings over time is found to depend on the number of years of data used. There is evidence of rank correlation over time using a 7 year window approach, but not when using 1 year data tranches. This highlights the importance of using both short and long time frames in order to cover different economic cycles as well as consider current conditions.

It is important to note that there is found to be no significant difference between diversified and undiversified industry VaR rankings, or between parametric and nonparametric CVaR approaches. This means that bankers can be reasonably confident of the robustness and consistency of these metrics when calculating and applying them, not only for the purposes of Basel compliance, but also for the determination of industry risk.

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1. INTRODUCTION

VaR models have gained increasing momentum since the VaR concept was first introduced by JP Morgan in 1994. This momentum was spurred by amendments to the Basel Accord in 1996 which required Banks to set aside capital for meeting Market Risk. Market risk arises from factors that affect the whole market. This paper focuses on equities and compares relative VaR and CVaR across 25 Australian industries, based on equity price movements using a parametric distribution, which is the most widely used approach among Banks. VaR has become the recognised standard approach for market risk measurement. VaR calculates maximum expected losses over a given time period at a given tolerance level.

In addition to VaR, this paper examines extreme industry risk using (conditional) CVaR. CVaR considers extreme events, based on losses exceeding VaR. Whilst there have been a wide range of VaR studies in USA and European markets, the vast majority have centred around individual asset or overall portfolio VaR as opposed to adopting a sectoral approach. There is very little study of industry risk using VaR approaches in the Australian market, and even less on CVaR. Indeed, very little research has been undertaken on the uses and applications of VaR or related metrics at all in Australia. (A search of the Australian Prudential Regulatory Authority's (APRA's) website revealed Sy (2006), Engel and Gizycki (1999) and Gizycki and Hereford (1999) as being the only papers considering aspects of VaR).

This paper aims to provide a greater understanding of the VaR and CVaR modelling approaches, as well as industry risk, in an Australian context. Industry market VaR is measured for each industry in Australia based on the variance-covariance parametric model, using both diversified and undiversified approaches. CVaR is measured using both parametric and nonparametric methodology. The study also compares VaR and CVaR changes between industries over time. This comprehensive exploration and application of these various VaR metrics should indicate whether the measures are robust and consistent over time and across industry sectors. The paper is divided into seven sections: section two provides a brief review of the Australian equities market whilst section three reviews the concept of VaR and section four that of CVaR. Section five reviews the data used and the research method, section six presents the results analysis, and section seven concludes.

2. THE AUSTRALIAN MARKET

There has been significant recent growth in the Australian Equities Market. In 1992, the market capitalisation of entities listed on the Australian Stock Exchange (ASX) was $198 billion, and this has since grown to $1.4 trillion. The S&P/ASX 200 is recognised as the investable benchmark for the Australian equity market and comprises 200 stocks selected by the S&P Australian Index Committee and represents approximately 90% of the total market capitalisation of the Australian Market (Standard & Poor's, 2006). The All Ordinaries index (All Ords) is considered to be Australia's market indicator, representing the 500 largest companies listed on the stock exchange (Standard & Poor's, 2006), and is the index used in this paper.

3. VALUE AT RISK

The use of VaR has become all-pervasive in a relatively short period of time despite its conceptual and practical shortcomings. VaR received its first broad recommendation in the Group of Thirty Report (1993). Subsequently its use and recognition have increased dramatically, particularly when the Basel Committee on Banking Supervision adopted the use of VaR models, contingent upon certain qualitative and quantitative standards. VaR has subsequently become one of the most important and widely used measures of risk. As a risk-management technique VaR describes the loss that can occur over a given period, at a given confidence level, due to exposure to market risk. The appealing simplicity of the VaR concept has lead to its adoption as a standard risk measure for financial entities involved in large scale trading operations, but also retail banks, insurance companies, institutional investors, and non-financial enterprises. Its use is encouraged by the Bank for International Settlements, the American Federal Reserve Bank and the Securities and Exchange Commission.

The groundbreaking Basel Capital Accord, originally signed by the Group of Ten (G10) countries in 1988, but since largely adopted by over 100 countries, requires Authorised Deposit-taking Institutions (ADI's) to hold sufficient capital to provide a cushion against unexpected losses. Value-at-Risk (VaR) is a procedure designed to forecast the maximum expected loss over a target horizon, given a (statistical) confidence limit. Initially, the Basel Accord stipulated a standardized approach which all institutions were required to adopt in calculating their VaR thresholds. This approach suffered from several deficiencies, the most notable of which were its conservatism (or lost opportunities) and its failure to reward institutions with superior risk management expertise. Following much industry criticism, the Basel Accord was amended in April 1995 to allow institutions to use internal models to determine their VaR and the required capital charges. However, institutions wishing to use their own models are required to have the internal models evaluated by the regulators using the back-testing procedure. The Basel Accord (BA) was adopted by the Australian government in 1988, with the Australian Prudential Regulatory Authority (APRA) as the national regulator of financial markets. According to APRA, Australia is now fully compliant with 11 BA principles, largely compliant with 12, and materially non-compliant with 2. Importantly, Australia is compliant with Principle 12, which states that:

"Banking supervisors must be satisfied that banks have in place systems that accurately measure, monitor and adequately control market risk; supervisors should have the powers to impose specific limits and/or a specific capital charge on market risk exposures, if warranted."

A description of the various methodologies for the modelling of VaR can be seen at the gloriamundi.org website. The predominant approaches to calculating VaR rely on a linear approximation of the portfolio risks and assume a joint normal (or log-normal) distribution of the underlying market processes. There is a comprehensive survey of the concept by Duffie and Pan (1997), and discussions in Jorion (1996), Pritsker (1997) RiskMetricsTM (1996) , Beder (1995), and Stambaugh (1996).

Despite its universal adoption and promotion by the regulatory authorities and its embrace by the financial services industry there are a number of theoretical and practical difficulties associated with the use of VaR as a risk metric. A standard procedure, in terms of the practical implementation of VaR metrics, if the portfolio of concern contains non-linear instruments such as options, is to make recourse to historical or Monte-Carlo simulation based tools. See the discussions in Bucay and Rosen (1999), Jorion (1996), Mauser and Rosen (1999), Pritsker (1997), RiskMetricsTM (1996), Beder (1995), and Stambaugh (1996). The optimisation problems associated with calculating VaR are discussed in papers by Litterman (1997a) and (1997b), Kast et al (1998), and Lucas and Klaussen (1998).

Nevertheless, despite its popularity, VaR has certain undesirable mathematical properties; such as lack of sub-additivity and convexity; see the discussion in Arztner et al (1999; 1997). In the case of the standard normal distribution VaR is proportional to the standard deviation and is coherent when based on this distribution but not in other circumstances. The VaR resulting from the combination of two portfolios can be greater than the sum of the risks of the individual portfolios. A further complication is associated with the fact that VaR is difficult to optimize when calculated from scenarios. It can be difficult to resolve as a function of a portfolio position and can exhibit multiple local extrema, which makes it problematic to determine the optimal mix of positions and the VaR of a particular mix. See the discussion of this in Mckay and Keefer (1996) and Mauser and Rosen (1999).

This paper features the exploration and application of an alternative to VaR: CVaR – Conditional-Value-at-Risk. Pflug (2000) proved that CVaR is a coherent risk measure with a number of desirable properties such as convexity and monotonicity w.r.t stochastic dominance of order 1, amongst other desirable characteristics. Furthermore, VaR gives no indication on the extent of the losses that might be encountered beyond the threshold amount suggested by the measure. By contrast CVaR does quantify the losses that might be encountered in the tail of the distribution. This is because a portfolio's CVaR is the loss one expects to suffer, given that the loss is equal to or larger than its VaR. A number of recent papers apply CVaR to portfolio optimization problems; see for example Rockafeller and Uryasev (2002; 1999), Andersson et.al (2000), Alexander et al (2003), Alexander and Baptista (2003) and Rockafellar et al (2006). However, there has been no prior use or application of CVaR in an Australian setting and its use, properties and applications are still in the early stages of their development.

There are 3 methods of calculating VaR. The Variance-Covariance method estimates VaR on assumption of a normal distribution. The historical method groups historical losses in categories from best to worst and calculates VaR on the assumption of history repeating itself. The Monte Carlo method simulates multiple random scenarios. The Variance-Covariance approach is the most widely used approach, and is the method we use in this study. To obtain VaR for a single asset X, all that needs to be calculated is the mean and standard deviation. Given the normal distribution assumption, we know where the worst 1% and 5% lie on the curve. VaR at 95% confidence level = 1.645 x σ_x and at 99% confidence level = 2.330 x σ_x. When calculating VaR, it is usual practice to not use actual asset figures, but the logarithm of the ratio of price relatives, which is the method used by RiskMetrics (J.P. Morgan & Reuters, 1996). This is obtained by using the following equation:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

i.e. the logarithm of the ratio between today's price and the previous price. The standard deviation is annualised by multiplying it by the square root of the number of trading days per annum (usually taken to be 250).

When additional assets are introduced into the portfolio, we need to account for correlations between the assets. Portfolio variance is calculated as follows, with w being the relative weighting of the assets:

$$V_{port} = w^2{}_x\sigma^2{}_x + w^2{}_y\sigma^2{}_y + 2w_x w_y \sigma_x \sigma_y \rho_{xy} \quad (2)$$

When dealing with multiple assets, variance-covariance matrix multiplication is used. The portfolio standard deviation is the square root of the variance multiplied by the square root of 250.

4. CONDITIONAL VALUE-AT-RISK

CVaR is closely related to VaR. CVaR is equal or greater than VaR. It is the conditional expected loss under the condition it exceeds VaR. CVaR is also called mean excess loss, mean shortfall, or tail VaR. β-VaR is a value with probability β the loss will not exceed β-VaR. CVaR is the mean value of the worst 1- β*100% losses (Uryasev & Rockafellar, 1999). For instance, if we are measuring VaR at a 95% confidence level (β=0.95), CVaR is the average of the 5% worst losses. CVaR can be calculated using the actual 5% worst losses (nonparametric), or using a normal distribution (parametric) approach, as follows (Huang, 2000):

$$CVaR_\alpha = \frac{\exp(-\frac{q_\alpha^2}{2})}{\alpha\sqrt{2\pi}}\sigma \quad (3)$$

Where qα is the tail 100α percentile of a standard normal distribution (e.g. 1.645 as obtained from standard distribution tables for 95% confidence).

5. METHODOLOY

5.1. Data

We use the All Ords index and obtain daily share prices for the last 15 years (which is the maximum available) from Datastream. For market VaR, Basel requires 250 days data. This is only 1 year, and we are more concerned with a longer term perspective, spanning different economic conditions. We follow the Basel requirement for 7 years data for the advanced credit approach (Bank for International Settlements, 2004). For comparison purposes, and to meet our requirement for longer market perspectives, we also use 7 year windows for calculating market VaR. This allows 9 years of comparative data (the first tranche being years 1-7, second tranche years 2-8, and so on until the 9th tranche which represents the 7 years from 9 – 15 of our data sample). We recognise that the longer sample may have different results to a shorter sample, and so we also do an historical comparison using 250 day windows. Industry codes are obtained from the ASX website and Market Capitalisation (for weighting of market VaR company data) is obtained from Datastream.

To ensure accuracy of industry classification, industry codes obtained from Datastream were all re-classified to the Global Industry Classification System (GICS) used by ASX. To ensure a meaningful quantity of data, Sectors with less than 5 companies, and companies with less than 12 months data have been excluded. The remaining companies represent 93% of the All Ords Index by both number and market capitalisation. As the All Ords represent more than 90% of the value of listed Australian companies, we consider 5 entities to be sufficient to provide meaningful conclusions.

Survivorship bias can occur when an index only includes current surviving companies and excludes failed entities (Brailsford & Heaney, 1998). We obtained data from Datastream for companies placed in administration or receivership and delisted over the past 3 years (maximum available). To test for survivorship bias we ran our model with these companies included, and compared industry VaR rankings to results excluding failed companies, testing for significance using the Spearman Rank Correlation Test (refer Section 5.4). Changes were found to be not significant at the 95% level and we concluded that survivorship bias does not have a significant impact on our study.

Thin trading problems can occur, especially with daily price data, when infrequently traded companies are included in a time series analysis. Thin trading problems can be reduced by avoiding thinly traded assets. In our case we are using the All Ords index which consists of the top 500 companies on the ASX, thus avoiding the most thinly traded assets. We further account for thin trading by applying an adjustment factor as proposed by Miller, Muthuswamy, and Whaley (1994) who suggest that a Moving Average model reflecting the number of non-trading days should be used to adjust returns. Due to difficulty in identifying non trading days, the approach shows that this is equivalent to estimating an AR (1) model from which the required adjustment can be determined. Their model involves the regression equation (4), with the residual then used to estimate the adjusted return in equation (5):

$$R_t = a_1 + a_2 R_{t-1} + \varepsilon_t \qquad (4)$$

$$R_t^{adj} = \frac{\varepsilon_t}{(1 - a_2)} \qquad (5)$$

5.2. VaR Calculation

We calculated VaR using the methodology described in Section 3. We begin by calculating the standard deviation of the logarithm of the daily price relatives. Weightings are calculated for each company according to market capitalisation. Undiversified VaR is obtained by multiplying the weighted undiversified standard deviation by 1.645 (as obtained from standard normal distribution tables for 95% confidence level). Diversified VaR is obtained through construction of a weighted variance-covariance matrix for each rolling 7 year period, and multiplying the portfolio standard deviation by 1.645. Both undiversified VaR and diversified VaR are annualised by multiplying by the square root of 250.

5.3. CVaR calculation

We use a parametric approach to calculate VaR, therefore intuitively it makes sense to use this approach for CVaR. However this approach has some limitations. It will yield a ranking spread for CVaR that is the same as VaR, which may not highlight the extreme returns. We therefore use both parametric and nonparametric approaches.

We use equation (3) to calculate parametric CVaR. As we have calculated VaR based on a 95% confidence level, CVaR is based on the worst 5% of losses. Nonparametric CVaR is calculated as the weighted average of returns beyond VaR.

5.4. Testing for significance

Hypotheses were formulated to test for association in industry risk between VaR and CVaR, diversified and undiversified VaR, parametric and nonparametric CVaR, and VaR and CVaR over time. We used nonparametric testing, as this is particularly suitable for testing ranking and for smaller data samples (we have 25 industries and 9 time periods). The Pearson Rank Correlation Test to was used test for ranking association between diversified and undiversified VaR, VaR and CVaR, parametric and nonparametric CVaR. The Kruksal-Wallis Test was used to test for ranking association over time. The details of these testing methods is beyond the scope of this paper but can be found in statistical textbooks such as Siegel & Castellan (1988) and Lee, Lee & Lee (2000). We tested for significance at a 95% level of confidence.

6. RESULTS

Table 1 provides a summary of industry rankings using our various VaR and CVaR metrics. A ranking of 1 is the highest risk and 25 the lowest.

Table 1 Results Summary

The table shows VaR on both a diversified and undiversified basis. The undiversified approach being the weighted average of all the individual company VaRs and the diversified approach including the correlation of all the entities in the industry with each other. It should be noted that the table only includes the most recent 7 year rolling window. CVaR is obtained using both the parametric approach and the nonparametric approach. The parametric approach uses equation 3 and the nonparametric approach is calculated as the weighted average of the actual returns beyond VaR.

	Undiversified Standard Deviation	Annual Undiversified 95% VaR	Diversified Standard Deviation	Diversified Portfolio 95% VaR	Daily Undiversified VaR	Parametric CVaR	Nonparametric CVaR
Automobiles & Components	7	7	12	12	7	7	7
Banks	25	25	21	21	25	25	25
Capital Goods	15	15	18	18	15	15	15
Chemicals	18	18	11	11	18	18	17
Commercial Services & Supplies	8	8	17	17	8	8	8
Construction Materials	17	17	9	9	17	17	19
Consumer Durables & Apparel	10	10	3	3	10	10	10
Diversified Financials	19	19	24	24	19	19	18
Energy	5	5	13	13	5	5	6
Food & Staples Retailing	23	23	15	15	23	23	24
Food Beverage & Tobacco	20	20	23	23	20	20	21
Healthcare Equipment & Services	11	11	20	20	11	11	11
Hotels Restaurants & Leisure	12	12	10	10	12	12	9
Insurance	9	9	8	8	9	9	5
Media	16	16	19	19	16	16	16
Metals & Mining	6	6	7	7	6	6	12
Paper & Forest Products	4	4	5	5	4	4	4
Pharmaceuticals & Biotechnology	3	3	4	4	3	3	3
Real Estate	21	21	25	25	21	21	20
Retailing	13	13	14	14	13	13	13
Software & Services	2	2	2	2	2	2	2
Technology Hardware & Equipment	1	1	1	1	1	1	1
Telecommunication Services	24	24	6	6	24	24	23
Transportation	14	14	16	16	14	14	14
Utilities	22	22	22	22	22	22	22

Using undiversified VaR, the model rates the technology sectors as having the highest risk, with Technology Hardware & Equipment and Software & Services having the highest VaR scores. This is not surprising given the well known high volatility experienced in the technology sector over the past 7 years. Also ranked in the top risk quartile are Pharmaceuticals & Biotechnology, Paper & Forest Products, Energy, and Metals & Mining. Lowest risk ranking is accorded to the Banking Sector. This is followed by Telecommunications, Food & Staples Retailing, Utilities, Real Estate, and Food, Beverage & Tobacco. The results generally tend to show a lower VaR in essential / staple industries (e.g. food & beverage, staples retailing, utilities, banking) as opposed to discretionary and high technology ones (e.g. software, technology hardware, other retailing).

CVaR must always exceed VaR, as CVaR is based on the worst 5% of returns, and this is reflected in the results shown. Parametric CVaR has exactly the same ranking as VaR (CVaR is the tail end of the normal distribution). Nonparametric CVaR is the average of the actual returns beyond VaR, and tends to be slightly higher than parametric CVaR.

Significant association was found in industry rankings between VaR and CVaR, diversified and undiversified VaR, and parametric and nonparametric CVaR. When testing for association over time, association was found in VaR and CVaR rankings when using 7 year rolling windows, but not when using 1 year data tranches.

7. CONCLUSIONS

The objectives of the study were to provide market industry VaR and CVaR measurements, to compare VaR and CVaR rankings between industries over time, to compare diversified (correlated) and undiversified industry VaR rankings, to compare parametric and nonparametric CVaR rankings for each industry. We find the Technology Sectors to show the highest risk, and lowest risk in the Financial and Utility Sectors. Although some industries show differences between diversified and undiversified risk (such as Telecommunications showing a much higher risk ranking on a diversified basis), overall there is found to be significant association between diversified and undiversified VaR.

There are some ranking differences between VaR and (nonparametric) CVaR, such as Insurance showing relatively higher CVaR than VaR, but overall CVaR rankings show significant similarities to VaR rankings. There is found to be significant association between parametric and nonparametric CVaR. There is significant ranking correlation over time for both VaR and CVaR using our 7 year rolling windows but not when using 1 year data frames. This highlights the importance of using both short and long time frames in order to cover different economic cycles as well as consider current conditions.

Using the 7 year time frames shows significant association between the outcomes of all the metrics used in this study. We conclude that, provided sufficiently lengthy time periods are used, these metrics show robustness and consistency over time and across industry sectors.

With the increased momentum in risk modelling brought about by the Basel II Accord, and the relative lack of VaR and CVaR studies in Australia, there is significant scope for additional studies on this topic, particularly with regards to CVaR for both market and credit risk.

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