A Game-Theoretic Approach to Water Quality Management

Schreider, S.[1], P. Zeephongsekul[1] and M. Fernandes[1]
[1]School of Mathematical and Geospatial Sciences, RMIT University, Victoria
Email: sergei.schreider@rmit.edu.au

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EXTENDED ABSTRACT

Phosphorus pollution is a major factor affecting the waterways in Western Victoria and elsewhere in Australia. One of the major strategies in reducing phosphorus contamination is the adoption by farmers of less harmful and more efficient fertilization strategies. The present paper outlines some of the research work and provides some propositions on water quality management in the Glenelg-Hopkins (G-H) catchment region which has as its main objective the development of theoretical and computational models to predict nutrient load and assess its impacts on the health of rivers in this area. The methodology used to achieve this aim relies on the innovative tools of Game Theory.

Specifically, we will formulate a general game-theoretic model, based on information regarding farmers' practices and environmental factors, on application of fertilizer which will assist in reducing agricultural pollution, especially those associated with phosphorus contamination, in the G-H catchment. The two main classes of games, i.e.non-cooperative games and cooperative games (Owen, 2001), will be addressed in this work, although the non-cooperative aspect will be given more prominence. The non-cooperative approach will result in a set of solutions which are Nash Equilibrium and we will hint at the cooperative approach by considering the value of forming coalitions among groups of farmers as well as identifying the Pareto equilibrium. Among the key research questions addressed in this paper will be the benefit of applying game theory for improving the health of waterways in the G-H catchment area.

In this work the game theoretic approach has been employed for modelling the strategies of phosphorus application by farmers of the Hopkins Basin. Game theory may be broadly described as a mathematical theory which was developed to model how rational human beings or organizations make decisions in a competitive environment or conflict situation. It allows researchers to find optimal strategy of behaviour for players involved in the game. In the context of the present work players are farmers (or households) applying fertilisers on their paddocks.

The main advantage of a game-theoretic approach in resource management is that it allows one to consider and compare competitive as well as cooperative actions of agents sharing limited resources. Non-cooperative games are games where each player or groups of players are antagonistic to each other. The main objective of non-cooperative games is to find optimal strategies where players can use against each other to optimize one or more utility functions. In cooperative games, players are able to form coalitions and utilities are transferable (shared) between members of these coalitions. The main objective here is to understand how cooperation could lead to better distribution of utilities to all players, in comparison to players engaging in pure competition between themselves. However, the cooperation can be modelled in the non-cooperative games via the cooperative Pareto equilibrium. This approach was utilised in the present work.

The paper formulates the model composition of the game when players are about 30 households in the Hopkins catchment. The information on their land use structures were taken form the survey specially implemented within the current project. This survey also provided the information on the phosphorus application policies characterising each of these households. The objective function for each of these players has been define as a sum of their crops revenues, total cost of phosphorus used and environmental penalties associated with the current level of pollution, which was calculated using the wide range of data on economic impacts associated to the water quality deterioration.

1 GAME THEORY AND RESOURCE MODELLING

Game theory was first developed by the great mathematician John von Neumann in 1928 and was later applied in modeling economic behaviour and understanding human conflicts, such as warfare. With Oskar Morgenstern, von Neumann co-authored the first seminal book on game theory entitled *The Theory of Games and Economic Behaviour* in 1953. The objective of Game theory is to analyze strategic situations and prescribe actions in an attempt to maximize their returns or minimize their costs.

The application of game theory to natural resource management problems is fairly recent and is at its developmental stage. Most applications appear to be on the cooperative aspect and is applied at different levels, from global to regional and local. The objective is usually to reduce environmental hazards through a detailed analysis of prevailing conditions and availability of resources which allow for a reasonable set of strategies. Lund and Palmer (1997) put up a good case for resolving conflict arising from management of water resources using tools from game theory. Game theoretic approaches in water resource management was further developed by Ratner (1990). That paper presents an analysis of the economic potential in regional cooperation of water usage in irrigation under conditions characterized by a general trend of increasing salinity. Income maximizing solutions for the region were obtained and related income distribution schemes derived using cooperative game theory algorithms and shadow cost pricing. Ray (2000) discussed the significant role that cooperative games and correlated strategies could play in the proper management of our environment. Basaran and Bölen (2005) conducted a case study in northern Turkey using game theory to obtain a better understanding of the decision making process and its consequences on a drainage basin. A similar study was also undertaken, with cooperation at the country level, by Dinar(2004). In Hermans (2004), different experiences from various countries were used to demonstrate limitations and successes of a game -theoretic approach for water resource management.

The focus of this study is on phosphorus pollution of waterways and we will consider the problem mainly from a non-cooperative perspective but will touch upon the cooperative aspect. The type of pollution that is of concern in this paper is an example of non-point source (NPS) pollution which has been discussed in Segerson (1993) and Xepapadeas (1999). The chief characteristic of NPS pollution is the inability of regulators to observe emissions by individual dischargers, leading to games where there is an asymmetric pattern of information. All that the regulators can observe are the ambient concentration of the pollutants without being able to detect the sources of these emissions with full certainty. It is to be noted that the aggregate pollution will also affect the polluters themselves, directly or indirectly, so it is important that some measure of cooperation is achieved between the dischargers of the pollutant.

2 CASE STUDY REGION AND RATIONALE

Providing increased protection to Australian rivers is one of the nation's top research priority and it is justified by the increasing industrial and demographic impacts to the environment, as well as the severe droughts which visited this country on a fairly regular basis. The Hopkins catchment from the G-H basin, situated in Western Victoria (Figure 1), was selected as the case study area because water quality issues are very topical for this region.

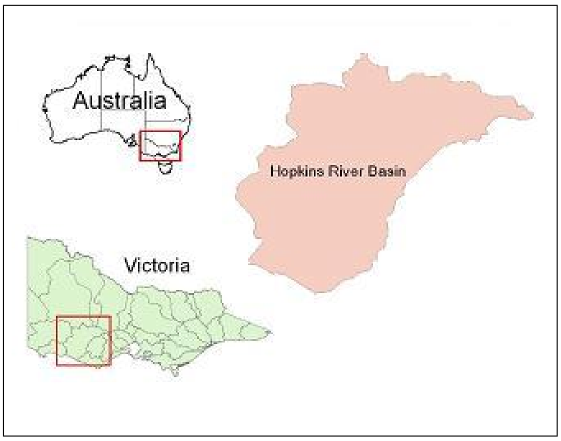

Figure 1. Location of the Hopkins catchment

The pollution of water by nutrients, especially phosphorus, is a major environmental problem in this catchment. Therefore, improvement of water quality in the streams of the Hopkins catchment is one of priorities of the G-H regional strategy indicated in the regional development plan of the local Catchment Management Authorities (Glenelg-Hopkins, 2003).

The Hopkins catchment is predominantly an agricultural area with the wool industry dominating, and some dairy and cereal crop production. The climate is moderately dry with average rainfall of about 700 mm from records kept over the last 120 years. In extremely dry years, annual rainfall could go below 350 mm and in very wet years, annual rainfall has reached 1000 mm (G-H, 2003).

Phosphorus pollution is a major factor causing severe detriment to the waterways of Western Victoria and this is mainly attributable to fertilizer usage. The major source of water pollution in the G-H catchment area is associated with intensive agricultural activities and usage of fertilizers by local farmers. The nutrient load, primarily phosphorus, could be better regulated and controlled by applying more efficient policies. It is our considered opinion that the only way to achieve this would be in careful scheduling of fertilizer applications and monitoring quantities of fertilizers introduced, thus ensuring that demands of farmers' crops are met and pollution limits for the region adhered to.

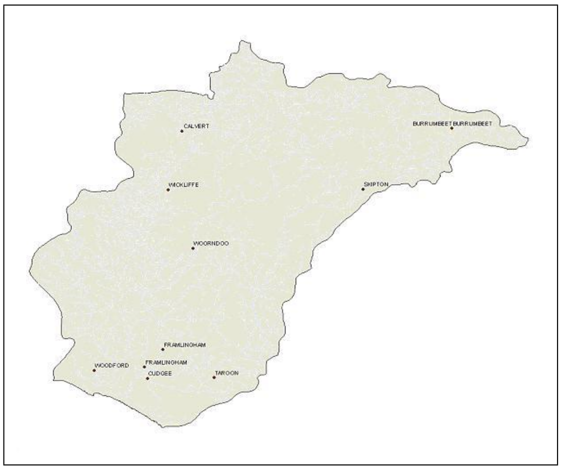

Figure 2. Phosphorus measurement locations in the Hopkins River Basin

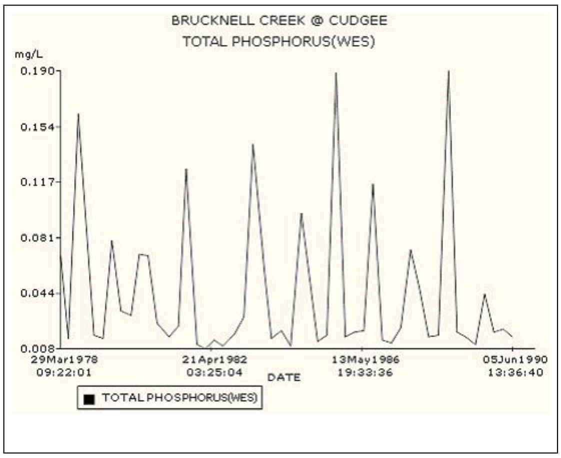

Figure 3. Phosphorus concentration in the Hopkins River measured at Cudgee (see location in Figure 3)

There is not a great deal of data on phosphorus pollution in the region. However, regular phosphorus measurements are taken from eleven stations in the Hopkins river basin, three of which are located on the Hopkins River (Figure 2). This information is very useful in estimating the phosphorus load in surface water, thereby providing an indirect measure of phosphorus usage through farming activities. Figure 3 is an example of phosphorus measurements taken from a location (Cudgee) in the region. (Source: Victorian Resources Data Warehouse: http://www.vicwaterdata.net/vicwaterdata/.)

3 JUSTIFICATION OF SELECTED APPROACH

One of the most common approach for resource management modeling in environmental economics is to apply various optimization techniques, especially linear programming, to appropriate objective (revenue) functions. These optimization techniques assume that the major driving force of all economic agents, farmers in our case, is revenue maximization (or cost minimization). Environmental parameters can also be incorporated in these objective functions after being expressed in some monetary equivalent. The relevant references can be found for instance in Scoccimarro (1999). As the objective of each agent is to increase its own revenue, often at the expense of others, this approach meant that all farmers (or groups of farmers) are posited in a competitive framework. The theory of non-cooperative games are especially suited to model such framework, where group of agents are intent on maximizing their own revenue. However, sometimes cooperation, rather than pure competition, plays a more significant role in resource management because most community members share the same environmental concerns on issues such as water quality, soil salinity, biodiversity, etc.

In the next section, we propose a non-cooperative game - theoretic approach to model the source of phosphorus pollution in the G-H Catchment. The players' optimal strategies in case of competition are given by the set of *Nash Equilibrium Profiles (NEPs)*. NEPs are combinations of strategies, or profiles, which, if adopted by all players, will render it infeasible for anyone to gain by unilaterally deviating from adopting them, i.e. they are *stable* (Pindyck and Rubinfeld, 2001). This type of equilibrium does not necessarily provide the best outcome to all players, but they are stable with respect to the behaviors of others. Collusion between agents, i.e. cooperation, can lead to a *Pareto equilibrium*, which is a combination of strategies where there is no other combination which is preferred by *all players* and strictly preferred by *at least one player* (Owen, 2001). In Figure 4, we show an example, known in game theory literature as the *"Prisoners' Dilemma"* (Luce and Raiffa, 1957), of these two types of equilibria. This is a two-person, non-zero sum game where each player has exactly two pure strategies. The values in the payoff matrix are purely hypothetical, with the first number the payoff to Player 1 and the second number payoff to Player 2. The NEP consists of both players adopting Strategy 1, since a player deviating from this will improve the other player's payoff. Notice that players will do much better if they both adopt Strategy 2, which is the Pareto Equilibrium. However, this combination is not stable, i.e. the revenue of a player decreases significantly if the other player decides to unilaterally deviate and adopt Strategy 1 instead. In order to achieve this Pareto Equilibrium, the two players will have to sign a binding agreement accepting the condition that they will both adopt Strategy 2.

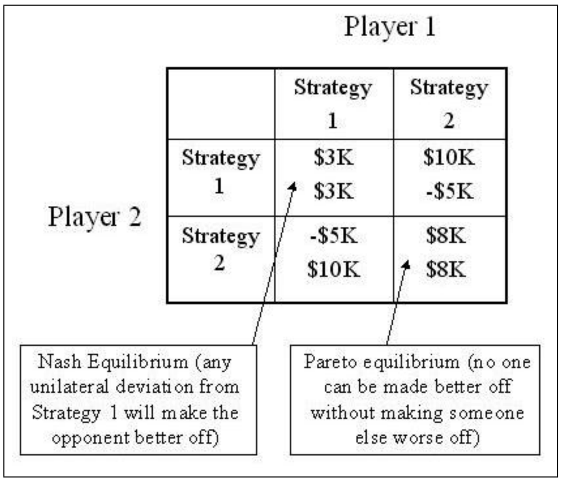

Figure 4. Prisoners' Dilemma in economics: illustration of Nash and cooperative Pareto Equilibrium. The objective function is the expected outcome accrued to each player.

4 NON-COOPERATIVE GAME THEORY FORMULATION TO PHOSPHORUS POLLUTION OF WATERWAYS

In this section, we formally present a static, non-cooperative game model of phosphorus pollution in the G-H catchment region, which we will refer to as the *G-H project*. As a preliminary remark, we define a *strategic form non-cooperative game* Γ as the system represented by

$$\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$$

where $N = \{1, 2, \ldots, n\}$ represents the set of players, S_i is the set of *pure strategies* available to Player i and $u_i(s)$ is a function defined on the Cartesian product set $S = \prod_{i\in N} S_i$ which represents the payoff or *utility* to Player i when a combination of strategies, or *profile*, $s \in S$ is selected by the players. If chance is involved in a game, i.e. a lottery is played, then the payoff is an *expected value*, as commonly defined in probability theory.

4.1 The Players

In the G-H project, each player would represent a group of farmers which are "similar" in some sense, e.g. they reside in the same geographical region, use similar methods of cultivation, have similar water usage pattern or derive similar incomes from their properties. Each group of farmers representing a player forms a cooperative unit and these groupings could be decided by applying well known statistical methods such as *cluster analysis* to our empirical or survey data. The number of players, n, should not be too large. Otherwise, the analysis would be intractable. We will denote Player i by P_i.

4.2 The Strategies

The strategy set S_i, $i = 1, 2, \ldots, n$, available to each player, consists of tuples

$$s_i = (\alpha_i, t_i) = (\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^R, t_i^1, t_i^2, \ldots, t_i^R)$$

where R is the number of crops fertilized using phosphorus and

$$\begin{aligned} \alpha_i^r &= \text{the amount of phosphorus used} \\ &\quad \text{by } P_i \text{ for crop } r \text{ per unit area} \\ t_i^r &= \text{the scheduling of the application of} \\ &\quad \text{phosphorus by } P_i \text{ for crop } r, \\ &\quad r = 1, 2, \ldots, R. \end{aligned}$$

That is, each member of S_i consists of the amount and time of application of phosphorus to crop r planted by the farmers, $r = 1, 2, \ldots, R$. We allow α_i^r to vary continuously within the interval $A_r = [A_{r1}, A_{r2}]$, i.e., irrespective of the player, there is a minimum quantity A_{r1} and a maximum quantity A_{r2} of phosphorus that can be applied to crop r. Similarly, the time of application, t_i^r, also takes values in an interval $T_r = (t_{r1}, t_{r2}]$ where t_{r1} is the minimum time and t_{r2} the maximum time of application. Note that it is sometime more realistic for T_r to be a finite set, e.g. $T_r = \{1, 2, \ldots, 12\}$ when we have once - a -month application; however, we have chosen an interval for mathematical convenience and tractability.

Thus, $s_i \in S_i = \prod_{r=1}^R A_r \times \prod_{r=1}^R T_r$ for any i and $s = (s_1, s_2, \ldots, s_n) \in S = \prod_{i\in N} S_i$ represents a profile adopted by all players. In the sequel, we also use the notation $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ to represent a profile adopted by all players *except* P_i.

4.3 The Pay-off Function

This will be measured by a *profit function* which has as its components the price obtained for the farm produce and the negative impact of environmental degradation. Before displaying the payoff function, we first define the following terms:
For each strategy $(\alpha_i, t_i) \in S_i$ executed by P_i, let

$$\begin{array}{rcl} \gamma & = & \text{Cobb-Douglas Constant} \\ q_i^r(\alpha_i^r, t_i^r) & = & \text{proportion of phosphorus} \\ & & \text{that is released into farmland} \\ & & \text{devoted to crop } r; \\ 1 - q_i^r(\alpha_i^r, t_i^r) & = & \text{proportion of phosphorus} \\ & & \text{that flow into the effluent} \\ & & \text{river systems as a} \\ & & \text{consequence thereof;} \\ E(t_i^r) & = & \text{(negative) environmental} \\ & & \text{impact manifested as cost} \\ & & \text{per unit application} \\ & & \text{of phosphorus;} \\ A_i^r & = & \text{total quantity of land devoted} \\ & & \text{to crop } r \text{ by } P_i; \\ W_i^r(t_i^r) & = & \text{amount of water available at} \\ & & \text{time } t_i^r; \\ Q^r(t_i^r) & = & \text{quantity of crop } r \text{ produced} \\ & & \text{per unit area per unit of} \\ & & \text{phosphorus}^\gamma \text{ per unit of} \\ & & \text{water}^{1-\gamma}; \\ p_r & = & \text{price (revenue) obtained per} \\ & & \text{unit of crop } r \text{ sold;} \\ \alpha_i^0 & = & \text{base quantity of phosphorus} \\ & & \text{in soil of user } i; \\ F & = & \text{price per unit of phosphorus} \\ & & \text{fertilizer ;} \\ \text{and} \quad L & = & \text{toxicity threshold level, i.e.} \\ & & \text{the amount of phosphorus in} \\ & & \text{the effluent river systems} \\ & & \text{above which there will be a} \\ & & \text{negative environmental impact.} \end{array}$$

The payoff function accrued by Player i if all players adopted the profile

$$s = ((\alpha_1, t_1), (\alpha_2, t_2), \ldots (\alpha_n, t_n))$$

is given by (1) where $0 \leq \beta_{ij} \leq 1$ are constants and $I(A)$ refers to the indicator of the set A, i.e. $I(A) = 1$ if even A has occurred, and equals to 0 otherwise.

$$\begin{aligned} u_i(s) = & \\ \sum_{r=1}^{R} \Bigg[& p_r Q^r(t_i^r) A_i^r \left[\alpha_i^r q_i^r(\alpha_i^r, t_i^r) + \alpha_i^0)\right]^\gamma W_i^r(t_i^r)^{1-\gamma} \\ & - \sum_{j=1}^{N} \Big[\beta_{ij} E(t_j^r) A_j^r (\alpha_j^r (1 - q_j^r(\alpha_j^r, t_j^r)) - L) \times \\ & I(\alpha_j^r (1 - q_j^r(\alpha_j^r, t_j^r)) > L) \Big] - F A_i^r \alpha_i^r \Bigg] \end{aligned} \quad (1)$$

The rationale for (1) is as follows: not all phosphorus that were used are released into farmland, a proportion of this flowed into the effluent river systems, producing a negative environmental impact if the total amount released exceeded a toxicity threshold level. The term in β_{ii} represents Player i's own impact with constants $\beta_{ii} = 1 \; \forall \, i \in N$. Fixed proportions $\beta_{ij} \mid j \neq i$ of this environmental influence are indirectly induced by other players on $P_i's$ domain and add a further negative impact on $P_i's$ payoff. Note that the functions $E(.)$ and $Q^r(.)$ depend only on the time of application t_i^r and not on α_i^r. This assumption is not unreasonable since these quantities, expressed as amounts per unit application, should be independent of the total amount of phosphorus being applied.

4.4 Optimal Strategies

The G-H project as outlined above is an example of a non-zero sum, n-person game. Therefore, the competitive optimal solutions, if they exist, can be expressed as *Nash Equilibrium Profiles (NEPs).* A profile

$$s^* = (s_1^*, s_2^*, \ldots, s_n^*) \in S$$

is a NEP if it satisfies the following property:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \;\; \forall s_i \in S_i \;\text{ and }\; \forall i \in N. \quad (2)$$

Thus, a NEP is stable for all players since any unilateral deviation from equilibrium by a player, given that all other players adhere to their best strategies, will result in a possible decrease of its revenue.
If the various functions forming $u_i(.), i = 1, 2, \ldots, n$, render them *concave* with respect to the variables α_i^r and t_i^r, and, in addition, the optimum occurs in the interior of S, then the NEP of the game is the solution of the following systems of equations involving first-order partial derivatives:

$$\begin{aligned} \left.\frac{\partial u_i}{\partial \alpha_i^r}\right|_{s^*} &= 0 \\ \left.\frac{\partial u_i}{\partial t_i^r}\right|_{s^*} &= 0 \\ r &= 1, 2, \ldots, R; i = 1, 2, \ldots, n. \end{aligned} \quad (3)$$

On the other hand, a strategy profile s^* is a *Pareto Optimum* if there exists no profile s such that

$$u_i(s) \quad \geq \quad u_i(s^*) \qquad \forall i \in N$$

with at least one $i \in N$ such that

$$u_i(s) \quad > \quad u_i(s^*). \tag{4}$$

A Pareto optimum is achievable only if players enter into a binding cooperative agreement. We remark that the Nash and Pareto optima in Figure 5 satisfy (2) and (4) respectively.

A closely related equilibrium concept to the NEP is the concept of a *Strong Nash Equilibrium* (SNE) introduced by Aumann (1959). This generalizes the NEP concept in the sense that instead of just looking at the negative effect to the individual deviating, one now considers group of individuals,i.e. a *coalition*, deviating from adopting the optimum strategy, given that the remaining players adhere to theirs. For any $K \subseteq N$, let us first define

$$S_K = \prod_{i \in K} S_i \quad \text{and} \quad S_{-K} = \prod_{i \ni K} S_i$$

and use the notation $s_K \in S_K$ and $s_{-K} \in S_{-K}$. A profile s^* is a SNE if there exists no coalition $K \subseteq N$ such that

$$u_i(s_K, s^*_{-K}) \quad \geq \quad u_i(s^*) \qquad \forall i \in K$$

with at least one $i \in K$ such that

$$u_i(s_K, s^*_{-K}) \quad > \quad u_i(s^*). \tag{5}$$

Thus, no group of individuals would deviate from adopting SNE since each individual in that group would do no better, and, some would do worse, if those not in the group adhere to their SNE.

Note that by letting $K = \{i\}$ and $K = N$ in (5) respectively, it follows that SNE is also a NEP and a Pareto optimum point.

5 APPLICATIONS OF THE MODEL

For this project, a special survey of farmers in the Hopkins River catchment region was conducted. This allows us to learn about attitude of landowners to phosphorus application and to collect information about timing and quantity of phosphorus application in this region. Detailed description of this survey is beyond the scope of the present paper and is described in a separate work (Schlapp and Schreider, 2007).

Results from this survey can be used in applications of the model. In a first run of the model the time component of the strategy was not considered and thus the strategy set is $s_i = (\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^R)$. So the variables to be determined are the α_i^r, phosphorus application by player i on crop r over the season. As a consequence,

$$\begin{aligned} E(t_i^r) &= E \\ Q^r(t_i^r) &= Q^r \\ W_i^r(t_i^r) &= W_i^r \end{aligned}$$

become exogenous parameters. Furthermore for simplicity sake we remove the α_i^r dependence from the absorbtion constant $q_i^r(\alpha_i^r, t_i^r))$. That is $q_i^r(\alpha_i^r, t_i^r)) = q_i^r$ is an exogenous parameter. Then (1) becomes,

$$\begin{aligned} u_i(\alpha_i^r) = \sum_{r=1}^{R} \Big[& p_r Q^r A_i^r [\alpha_i^r q_i^r + \alpha_i^0)]^\gamma (W_i^r)^{1-\gamma} \\ & - \sum_{j=1}^{N} \beta_{ij} E A_j^r (\alpha_j^r (1 - q_j^r) - L) I(\alpha_j^r (1 - q_j^r) > L) \\ & - F A_i^r \alpha_i^r \Big] \end{aligned} \tag{6}$$

In order to find the Nash Equilibrium we will need to solve

$$\frac{\partial u_i}{\partial \alpha_i^r} = 0 \qquad r = 1, 2, \ldots, R; i = 1, 2, \ldots, N$$

That is solve $R \times N$ equations for the $R \times N$ variables α_i^r. The solutions yielded are,

$$\alpha_i^r = \frac{W_i^r \left[\frac{F + E(1-q_i^r) I(\alpha_i^r (1-q_i^r) > L)}{\gamma p_r Q^r q_i^r} \right]^{1/\gamma - 1} - \alpha_i^0}{q_i^r}$$

6 DISCUSSION AND CONCLUSIONS

This paper formulates the concept of a game theoretic approach in developing the optimal strategies for water quality management associated with phosphorus pollution in an agricultural region in eastern Victoria, Australia. However, as in all conceptual works, a bridge must be established between the concept that was developed and the decision support tool that can be used by resource managers in the region considered. The key requirement here is that all parameters included in the developed model are measurable, thereby allowing appropriate data to be collected.

One important outcome of the survey conducted during the project implementation is that the model described in the present paper cannot be realized to its fullest extent as yet because not all parameters included in the model are easily measurable. This means that some of the equations presented in Section 5 may have to be modified. For example, from the survey, it appears that no detailed nor precise documentation of fertilization schedule is available and this is mainly due to farmers having very vague recollections of when they applied phosphorus. The responses to this question, when they are available, are usually within a margin of error of plus or minus two weeks. Hopefully, such problem could be resolved in future by implementing a more stringent and accurate survey.

7 ACKNOWLEDGEMENT

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