Modelling River Pollution and Removal by Aeration

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Keywords: *Pollution, dispersion, aeration, mass-transfer, Michaelis-Menten model*

EXTENDED ABSTRACT

The Tha Chin river is one of the most important rivers in Thailand (see Figure 1 below). It passes through agricultural and industrial areas and through settled communities. These factors have made the Tha Chin river the country's most polluted river. The specific factors causing the river's water quality degradation vary with the pattern of land use. The upper Tha Chin river basin runs through agricultural land where there is heavy use of fertilizer and insecticides. The lower Tha Chin river's runoff comes from urban and industrial areas. The river is polluted from the combined discharges of industrial, domestic and rural inflows before reaching the sea. Moreover, salinity is also a major cause of water degradation.

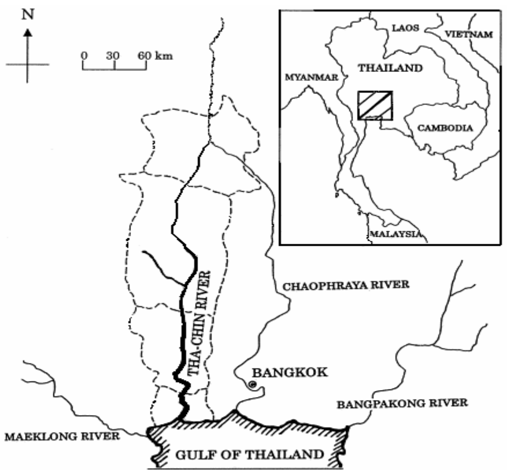

Figure 1. A map of the Tha Chin river.

To help develop a model by which we can manage the river, we need to collect data for precipitation, flow rate, seawater ingress, pollution discharge, etc. The model proposed to do this consists of a pair of coupled reaction-diffusion-advection equations for the pollutant and dissolved oxygen concentrations, respectively. The coupling occurs because the oxygen reacts with the pollutant producing harmless compounds. Boundary conditions are chosen to match the unpolluted upstream conditions and the eventual flow into the open sea. We have considered a steady-state model initially in one spatial dimension and have found analytical solutions for pollutant and oxygen concentrations for some special cases. In view of the rapidity of the chemical reactions and the long-length of the river, it is possible to neglect the effects at the mouth of the river and far upstream.

Such a model and its solutions will aid as a decision support on restrictions to be imposed on farming and urban practices. Analytical solutions of the simplified model and simulations enable scenarios to be tested for fish survival, which is usually taken as above 30% of the saturated dissolved oxygen concentration.

This simplified model forms a basis for modelling the real situation, with the addition of variable pollutant input, tidal flow and the like. The extension to the transient spatial model is relatively straightforward.

The methodology introduced here is generic and can be used with little modification for other rivers.

1. INTRODUCTION

There is increasing concern about water quality worldwide, with increased pollution having a serious impact on the environment. Mathematical models have been used extensively to predict water quality, and to provide reliable tools for water quality management in affected areas. These models simulate the spatial and temporal distribution of various water quality variables in the study area. The principal application that motivated this study is the Tha Chin river in Thailand, one of the most polluted rivers in that country (see Figure 1). However, the modelling approach we have taken here is generic.

Major effort has been made to integrate hydrodynamic and water quality models into some case-studies. The use of mathematical models for the simulations is a very powerful approach that greatly enhances the decision support tools used for water resource management. For these reasons, the use of a deterministic approach, with values of the pollutant sources, and of the biochemical coefficients expressed in the model, is necessary. These can initially be average or bounding values so as to give approximate values of the outputs.

In the present study, a water pollution model composed of two coupled well-known advection-dispersion equations is presented in which the various parameters and the loads of pollutant sources are incorporated. We are interested in a pair of coupled reaction-diffusion-advection equations for the pollutant and dissolved oxygen concentrations, respectively. The problem is treated as being one-dimensional along the length of the river for simplicity. Since the processes of pollution and aeration are sustained, we investigate the occurrence of steady states by considering the removal of pollutant by aeration. We construct analytical and numerical solutions.

2. MOTIVATION: THE THA CHIN RIVER

The Tha Chin river is a major river in the Central Plain and also the second significant river in Thailand descending from Chao Phraya River with a catchment covering an area of 13,000 square kilometres where there is a human population of around 2 million. The total length is 325 kilometres (PCD 2000) (See Figure 1).

The Tha Chin river has served the local inhabitants in many aspects ranging from agricultural and industrial use, aquatic life preservation, source of water for domestic consumption, water supply for the Bangkok Metropolitan Area, disposal of waste water and discharge from agricultural activities. The releasing of outflows from water gates is variable. The water budget requirement and the fact that water is needed for agriculture, requires the prevention of sea water intrusion into the Tha Chin river by controlling the salinity, limiting it to not more than one part per thousand (ppt). As for the overall water quality, it is reported that the Tha Chin river is the most highly polluted and degraded river in Thailand (PCD 2000).

The results of Water Quality Monitoring in 2004 indicated that the water quality parameters should contain on average: Dissolved Oxygen (DO) of 0.9 mg/litre, Biological Oxygen Demand (BOD) of 3.7 mg/litre, Total Coliform Bacteria (TCB) of 69,000 MPN/100 ml and Fecal Coliform Bacteria (FCB) of not more than 28,000 MPN/100 ml. However, based on the previously prescribed water sources standards, the water quality of the Lower Tha Chin river is very low due to low DO and high Bacteria pollution.

The pattern of land use in the Tha Chin River basin is mostly agriculture amounting to about 76% of the total, whereas the industrial area is concentrated in the Nakhon Pathom and Samut Sakhon provinces. The usual waste and discharge comes from inhabitants residing along the river banks 30%, industrial 33%, and from the agricultural sector (pig farming) 47% (Simachaya *et al*. 2000).

The problem concerning the water quality in the Tha Chin river includes water quality degradation, toxic dumping, and saltwater intrusion, leading to the need to resolve the conflicts between various water user sectors. Thus, transparent, logical and effective conflict resolution by under-pinning analysis of water usage and wastewater discharge allocation is now needed to ensure the sustainable use of water resources. To resolve the conflict, all stakeholders are participating in establishing an action plan and are involved in activities to restore and implement the integrated management of the Tha Chin river. This research provides a tool for policy-development and mitigation of water quality issues for the river.

3. THE MODEL DESCRIPTION

Consider the coupled equations for the pollutant and dissolved oxygen concentrations. The coupling occurs because the oxygen reacts with the pollutant producing harmless compounds. For simplicity, we assume that diffusion is in one dimension along the river and is accompanied by forced convection and so the concentration $P(x,t)$ (of pollutant) and $X(x,t)$ (of dissolved oxygen) satisfies reaction-diffusion-advection equations. For a list of symbols, their meaning and units, see Table 1. Unless otherwise stated, these will be assumed to be constant for the purposes of this study. To relax this requirement is relatively straightforward.

The system of equations which describes the rate of change of the concentration with position x and time t can be expressed in one dimension as

$$\frac{\partial(AP)}{\partial t} = D_p \frac{\partial^2 (AP)}{\partial x^2} - \frac{\partial(vAP)}{\partial x} - K_1 \frac{X}{X+k} AP + qH(x) , \quad (x < L < \infty, \ t > 0) \tag{1}$$

$$\frac{\partial(AX)}{\partial t} = D_x \frac{\partial^2 (AX)}{\partial x^2} - \frac{\partial(vAX)}{\partial x} - K_2 \frac{X}{X+k} AP + \alpha(S - X) , \quad (x < L < \infty, \ t > 0) \tag{2}$$

These equations are standard (see Chapra 1997). The first equation includes both addition of pollutant (q), and removal by aeration, and the second equation is a mass balance for oxygen, with addition at the surface, and consumption by pollutant.

Here, $H(x)$ is the Heaviside function and

$$H(x) = \begin{cases} 1 , & 0 < x < L \\ 0 , & \text{otherwise} \end{cases} \tag{3}$$

This is used to capture the fact that pollutant is discharged for $x > 0$ only.

We consider a river where pollutants are discharges in the form of wastes. It is assumed that these pollutants use dissolved oxygen for various biochemical and biodegradation processes. The discharge of pollutants into the river is at the constant rate q and the rate of depletion of concentration P due to biochemical embodying a "Michaelis-Menten" model is given by interaction involving the concentration of dissolved oxygen as well as the concentration P. For dissolved oxygen, it is assumed that the rate of growth of concentration by movement from the air into the river is proportional to the saturated concentration S less the concentration X, that is $\alpha(S–X)$. Interaction involves the concentration of dissolved oxygen as well as the pollutant concentration P. We consider cases with and without dispersion k negligible ($k \approx 0$) and k non-zero. To simplify the equations, we set the values A, v, q, α and S to be constant.

Table 1. Variables and parameters values**.**

variables	
t is time (day)	
x is the position (m)	
P is the pollutant concentration (kg m^{-3})	
X is the dissolved oxygen concentration (kg m^{-3})	
parameters	**SI units**
L is the length of river (m)	325,000*
D_p is the dispersion coefficient of pollutant in the x direction (m^2 day^{-1})	3,456,000*
D_x is the dispersion coefficient of dissolved oxygen in the x direction (m^2 day^{-1}), taken as the same as D_p	3,456,000
v is the water velocity in the x direction (m day^{-1})	43,200*
A is the cross-section area (m^2)	2100*
q is the added pollutant rate along the river (kg m^{-1} day^{-1})	0.06*
K_1 is the degradation rate coefficient at 20 °C for pollutant (day^{-1})	8.27***
K_2 is the de-aeration rate coefficient at 20 °C for dissolved oxygen (day^{-1})	44.10**
k is the half-saturated oxygen demand concentration for pollutant decay (kg m^{-3})	0.007****
α is the mass transfer of oxygen from air to water (m^2 day^{-1}) ; α = re-aeration rate*A′ (From Chapra (2007), the re-aeration rate = 0.055 day^{-1}. A′ = width of 300*unit length of 1)	16.50**
S is the saturated oxygen concentration (kg m^{-3}) From Chapra (2007), S = 10 mgL^{-1}	0.01**

* PCD (2000) ** Chapra (1997)
*** based on the molecular weights in the chemical reaction $K_1 = (3/16)\, K_2$ **** estimated

For fish to survive we need $X \geq 0.3S$ everywhere (Murphy 2007) and so this implies, via the various models, a limit on q. This is the underpinning constraint from the models, which are shown in Figure 2.

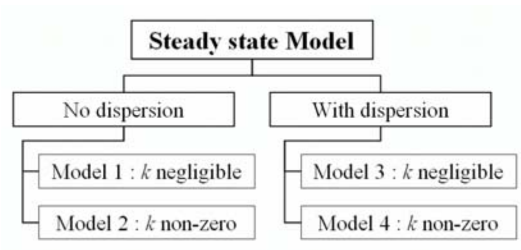

Figure 2. Model special cases.

4. SPECIAL CASES OF THE MODEL

We now consider various special cases from the scheme shown in Figure 2, gradually building up towards the full model. In this paper we consider steady-states, presuming conditions are constant.

Model 1. This model is used for the steady state analysis. In this model we have zero dispersion (D_p= 0, D_x= 0).

$$\frac{d(vAP_s(x))}{dx} = -K_1 AP_s(x) + q, \quad (x > 0,\ t > 0) \tag{4}$$

$$\frac{d(vAX_s(x))}{dx} = -K_2 AP_s(x) + \alpha(S - X_s(x)),\ (x > 0, t > 0) \tag{5}$$

We consider k negligible ($k \approx 0$) and boundary conditions $P_s(0) = 0$ and $X_s(0) = S$. The far downstream pollutant concentration is $P_s(x) = (q/K_1A)(1\text{-}\exp(-K_1x/v))$, and so the downstream limit is given by q/K_1A. This is shown in Figure 3.

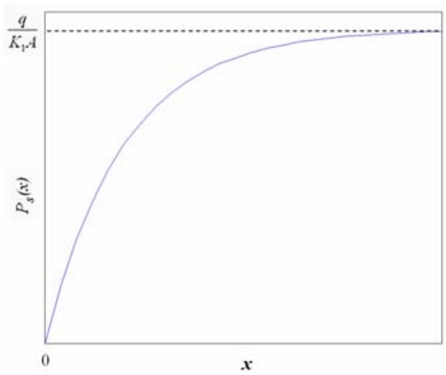

Figure 3. The steady state solution for P with no dispersion in the case k negligible.

Upstream there is no pollution as there is no dispersion. For the dissolved oxygen concentration, the solution is

$$X_s(x) = S - \frac{K_2 q}{K_1}\left(\frac{1}{\alpha} - \left(\frac{1}{\alpha - K_1 A}\right)e^{\frac{-K_1 x}{v}}\right) - \left(\frac{K_2 qA}{\alpha(\alpha - K_1 A)}\right)e^{-\frac{\alpha x}{vA}} \tag{6}$$

and downstream the oxygen level decreases due to interaction with the pollutants.

$$\lim_{x\to\infty} X_s(x) = S - \frac{K_2 q}{\alpha K_1} \tag{7}$$

This is shown in Figure 4. If the discharge from inhabitants residing and farming along the river q is such that X is less than 30% of the saturated values S, the fish do not survive (PCD 2000).

So the requirement for q is $q < 0.7\alpha K_1\ S/K_2$

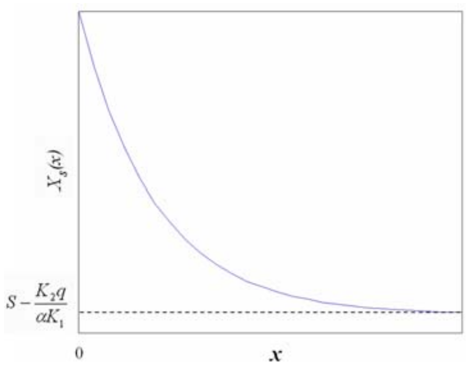

Figure 4. The steady state solution for X with no dispersion in the case k negligible.

Model 2. The previous model simplified because the half-saturated oxygen demand concentration for pollutant decay is negligible ($k \approx 0$). If instead it is significant then

$$\frac{d(vAP_s(x))}{dx} = -K_1 \frac{X_s(x)}{X_s(x)+k} AP_s(x) + q,\ (x > 0,\ t > 0) \tag{8}$$

$$\frac{d(vAX_s(x))}{dx} = -K_2 \frac{X_s(x)}{X_s(x)+k} AP_s(x) + \alpha(S - X_s(x)),\quad (x > 0, t > 0) \tag{9}$$

With the same boundary conditions $P_s(0) = 0$ and $X_s(0) = S$, the far downstream solutions for pollutant and dissolved oxygen concentration are given, respectively.

$$(P_s(x), X_s(x))\big|_{\text{large } x} = \left(\frac{q}{K_1 A} + \frac{\alpha kq}{\alpha K_1 S - qK_2}, S - \frac{qK_2}{\alpha K_1}\right) \tag{10}$$

The steady far-downstream solution depends therefore on the parameters k and q. A computation was performed using MATLAB to find the steady state solution for various k and q in the phase plane with x as an independent parameter (see Figure 5).

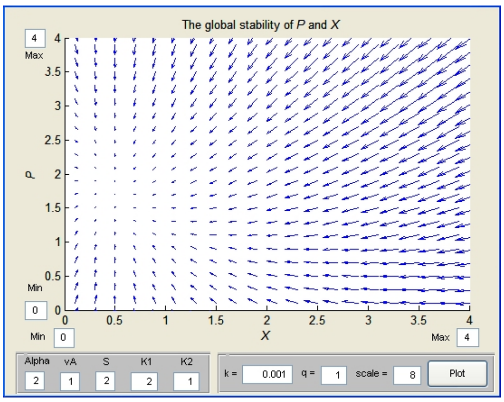

Figure 5. The global stability in the P–X plane with no dispersion for k non-zero.

We note the downstream solution above does not exist if $q \geq \alpha K_1 S/K_2$ and in that case, $X_s(\infty) = 0$. Figure 6 shows the numerical solution as k varies.

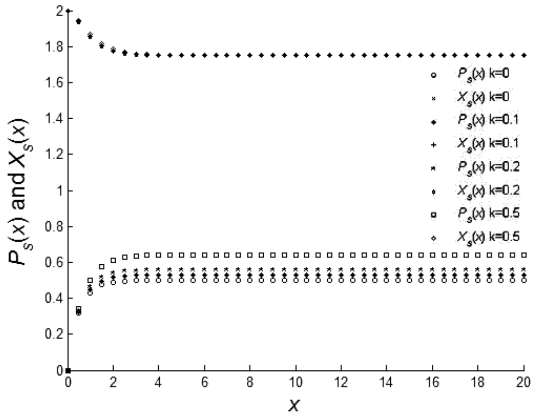

Figure 6. The steady state numerical solutions for P and X as k varies.

Model 3. We now consider the steady state case with dispersion terms (let $D_p \neq 0$, $D_x \neq 0$).

$$D_p \frac{d^2(AP_s(x))}{dx^2} - \frac{d(vAP_s(x))}{dx} - K_1 AP_s(x) + qH(x) = 0\ , \quad (x > L, t > 0) \tag{11}$$

$$D_x \frac{d^2(AX)}{dx^2} - \frac{d(vAX_s(x))}{dx} - K_2 AP_s(x) + \alpha(S - X_s(x)) = 0\ , \quad (x > L, t > 0) \tag{12}$$

In this model, k is assumed to be negligible ($k \approx 0$) and the solution below is obtained.

$$P_s(x) = \begin{cases} \dfrac{q}{K_1 A}\left(1 - (\dfrac{\delta+\beta}{2\beta})e^{(\delta-\beta)x}\right) & , x \geq 0 \\ \dfrac{q}{K_1 A}\left(\dfrac{\beta-\delta}{2\beta}\right)e^{(\delta+\beta)x} & , x < 0 \end{cases} \tag{13}$$

where $\delta = v/2D_p$ and $\beta = (\sqrt{v^2 + 4D_p K_1})/2D_p$ (14)

In this model we used the conditions $P_s(\infty) < \infty$ and $P_s(-\infty) < \infty$. We also require $P'_s(x)$ and $P_s(x)$ to be continuous at $x = 0$. There are no point sources of pollutant (only distributed sources), which makes $P_s(x)$ continuous. Since the dispersive flux $DP'_s(x) - vP_s(x)$ is also continuous this implies that $P'_s(x)$ is also continuous. For dissolved oxygen, we find

$$X_s(x) = \begin{cases} S - \dfrac{K_2 q}{K_1 \alpha} + \dfrac{K_2 q}{K_1}\left[\left(\dfrac{\gamma+\eta}{2\eta\alpha} - \dfrac{\delta+\beta}{4\beta\eta A^*} + \dfrac{\delta-\beta}{4\beta\eta B^*}\right)e^{(\gamma-\eta)x} - \dfrac{\delta+\beta}{2\beta A^*} x e^{(\delta-\beta)x}\right], & x \geq 0 \\ S + \dfrac{K_2 q}{K_1}\left[\left(\dfrac{\gamma-\eta}{2\eta\alpha} - \dfrac{\delta+\beta}{4\beta\eta A^*} + \dfrac{\delta-\beta}{4\beta\eta B^*}\right)e^{(\gamma+\eta)x} - \dfrac{\delta-\beta}{2\beta B^*} x e^{(\delta+\beta)x}\right], & x < 0 \end{cases} \tag{15}$$

where

$$\gamma = \frac{v}{2D_x}\ ,\ \eta = \frac{\sqrt{v^2 + \dfrac{4\alpha D_x}{A}}}{2D_x}\ ,\ A^* = 2AD_x(\delta - \beta) - vA\ ,$$
$$B^* = 2AD_x(\delta + \beta) - vA \tag{16}$$

Again we use the initial conditions $X_s(\infty) < \infty$ and $X_s(-\infty) = S$. Also $X_s(x)$ and $X'_s(x)$ are continuous at $x = 0$. The pollutant concentration $P_s(x)$ is relatively smooth with a discontinuity of q/D_pA in the second derivative at $x = 0$. The dissolved oxygen concentration $X_s(x)$ has a discontinuity in the fourth derivative at $x = 0$. To test our model we set the parameters A, v, q, D_p, D_x and K_2 to be 1; α, S and K_1 to be 2. For the above set of parameters the graph of P versus X is shown in Figure 7 as below.

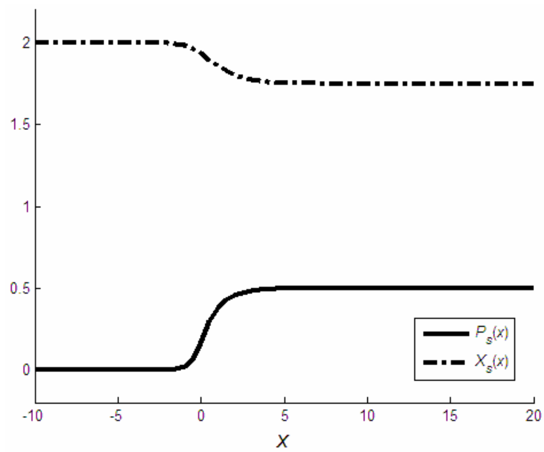

Figure 7. The analytical steady state solution with dispersion for P and X.

Model 4. The last model includes dispersion terms (let $D_p \neq 0$, $D_x \neq 0$). In this case k is non zero. The following system of nonlinear second order differential equations is obtained:

$$D_p \frac{d^2(AP_s(x))}{dx^2} - \frac{d(vAP_s(x))}{dx} - K_1 \frac{X_s(x)}{X_s(x)+k} AP_s(x) + qH(x) = 0\ , \quad (x > L, t > 0) \tag{17}$$

$$D_x \frac{d^2(AX_s(x))}{dx^2} - \frac{d(vAX_s(x))}{dx} - K_2 \frac{X_s(x)}{X_s(x)+k} AP_s(x) + \alpha(S - X_s(x)) = 0\ , \quad (x > L, t > 0) \tag{18}$$

Boundary concentrations for P and X are still given by $P_s(-\infty) = 0$ and $X_s(-\infty) = S$ far upstream and far downstream, respectively.

$$P_s(\infty) = \frac{q}{K_1 A}\left(1 + \frac{k}{X_s(\infty)}\right), \tag{19}$$

and $X_s(\infty) = S - \dfrac{K_2 q}{K_1 \alpha}$ (20)

Furthermore, we obtain flux conditions by mathematical analysis directly,

$P'_0(0) \leq P'_k(0) \leq q/Av$ and $X'_0(0) \leq X'_k(0) \leq 0$.

A numerical solution will be used for the governing equations (17, 18).

5. NUMERICAL PROCEDURE

We have developed a numerical routine to search for the solution to the nonlinear equations. We integrate from a grid of initial values at $x = 0$ and refine this grid to find the solution to the boundary conditions. Figure 8 illustrates this procedure. In Figure 9 we explain the steps involved in the algorithm by means of a flow diagram for the numerical computation.

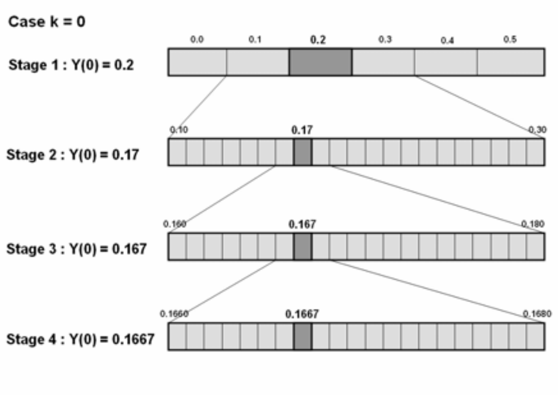

Figure 8. The numerical computation for finding the initial condition.

For initial testing of the algorithm we have used the case where k is negligible. The Euclidean norm is used for measuring the agreement of trial initial conditions with pollutant and dissolved oxygen concentration far upstream and downstream. We have called this "score". This is shown in Figure 10. The lowest "score" corresponds to the best numerical solution.

There is a pair of coupled nonlinear differential equations in this model. Some parameter values have been assumed for testing the MATLAB computation. Figure 11 illustrates how transient solutions approach asymptotically to the downstream solution of pollutant and dissolved oxygen in the case when k is negligible ($k \approx 0$).

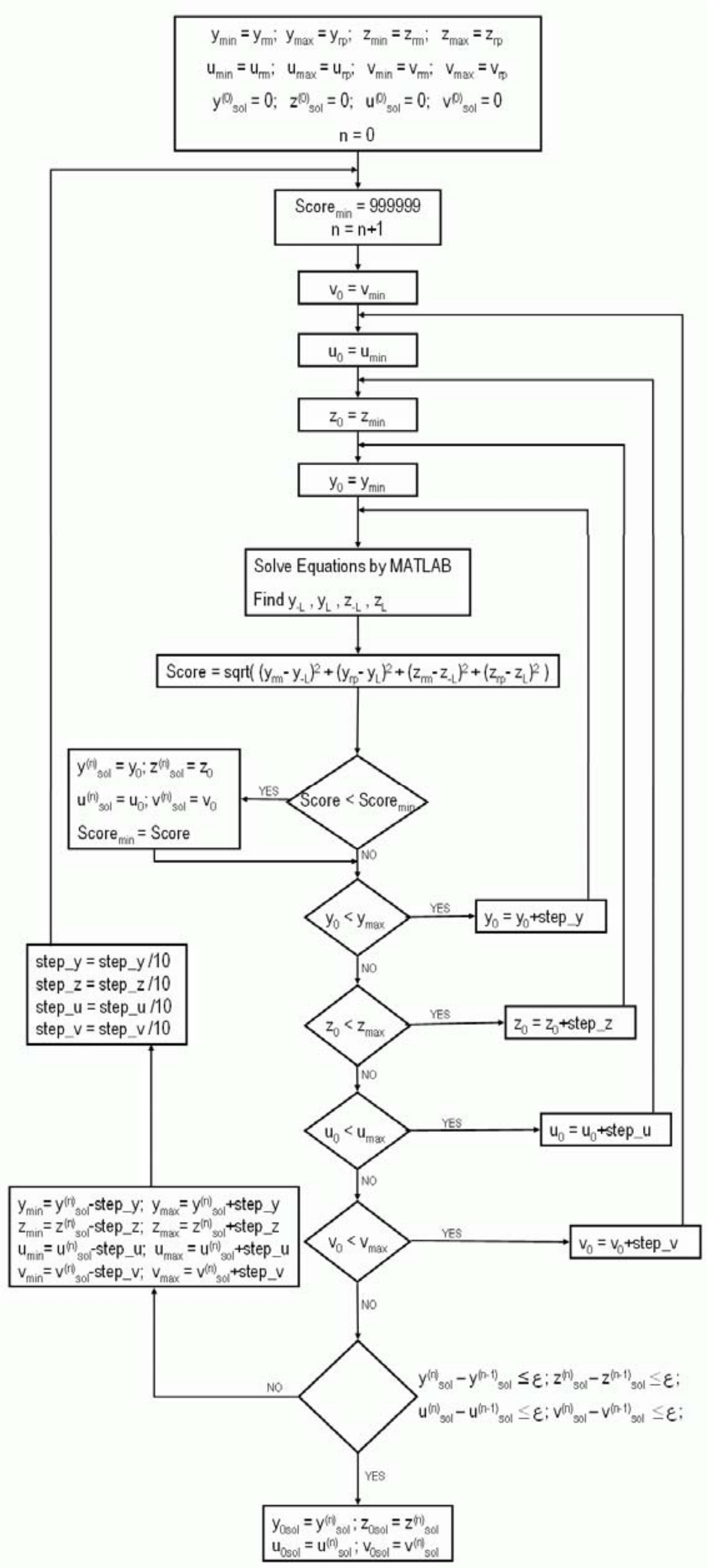

Figure 9. The flowchart of the numerical procedure.

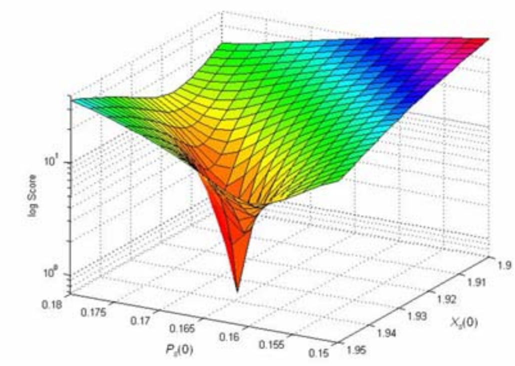

Figure 10. Existence of a minimum value of score for $P_s(0)$ and $X_s(0)$**.**

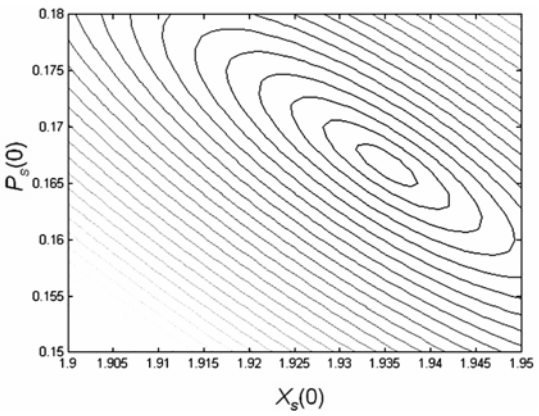

Figure 11. The contour plot of $P_s(0)$ and $X_s(0)$**.**

6. DISCUSSION AND CONCLUSIONS

We have proposed a mathematical model for river pollution comprising a coupled pair of nonlinear equations and investigated the effect of aeration on the degradation of pollutant. The results from numerical calculations (Figure 12) agree with the analytical solution under the conditions of no pollution and saturated dissolved oxygen far upstream, tending to a steady state far downstream for a long (considered infinite) river. Using this technique we will be able to obtain the steady state solution for the nonlinear model.

The actual rate of pollutant insertion is q = 0.06 kg m^{-1} day^{-1}. This makes X_s = 0 for large x, that is, the river is ecologically dead. From Model 1, the fish survival constraint on q for the Tha Chin river is $q < 0.015$.

However, with the value of q = 0.06 kg m^{-1} day^{-1}, subsequent investigations (to be published later) show that for a river of length of that in this study (the lower 325 km of the Tha Chin river), the oxygen level fortunately remains above the critical value of 30% of the saturated oxygen concentration and reaches zero far beyond 325 km. Thus, for an infinite river, pollutant being inserted into the river is about 4 times the ideal rate for fish to survive (Murphy 2007). This implies that the total BOD rate for the river should have a maximum insertion of 5000 kg BOD day^{-1}. However, this constraint is not reached due to the finite length over which pollution is actually discharged and the oxygen concentration which remains above the critical threshold value.

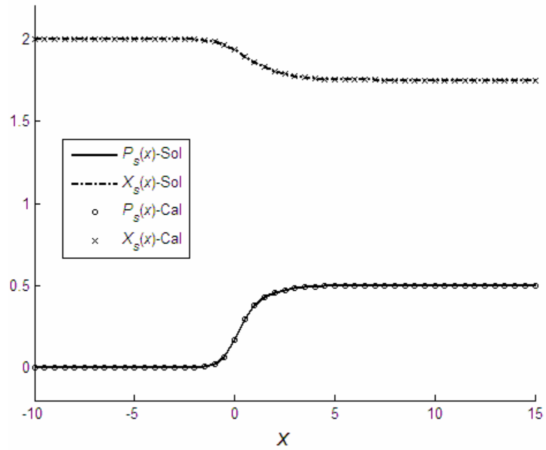

Figure 12. The numerical steady state solutions of $P_s(0)$ and $X_s(0)$ for negligible k.

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