Heuristic Methods for Locating Emergency Facilities

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EXTENDED ABSTRACT

Facility location problems form an important class of industrial optimization problems. These problems typically involve the optimal location of facilities. For our purposes, a facility is just a physical entity that assists with the provision of a service or the production of a product. Examples include: schools, ambulance depots, emergency care centers, firestations, workstations, libraries etc. The objective may involve factors such as cost, distance or service utilization. The optimization problems are complicated by the need to meet a number of specified constraints. These constraints may relate to safety, available resources, level of service, time, etc.

The optimization problems are usually grouped into two categories, namely service and manufacturing industries. In the service industries, the location of emergency facilities (ambulance, fire station, emergency centers) affects significantly on the safety and well-being of the community. The safety and well-being of the community depends directly or indirectly on the response time of the emergency facilities. The objective is to locate the facility where the average response time (time between the receipt of a call and the arrival of emergency vehicle) is minimized. The minimization of the response time measures the performance of emergency facilities. The performance of these facilities can be improved by either moving the existing locations of the emergency facilities or increasing the number of facilities. However, increasing the number of facilities is generally limited or impossible due to capital constraints. It is, therefore, important to locate emergency facilities effectively and efficiently.

One way to measure the efficiency and effectiveness of emergency facilities is by evaluating the average distance between the customers and the facilities. When the average distance decreases, the accessibility of the facilities increase and average response times decrease. This is known as the *p*-median problem, which was introduces by Hakimi (1964). It is defined as: determine the location of *p* facilities to minimize the average (total) distance between demands and their closest facility.

The *p*-median problem is computationally difficult to solve by exact methods because it is *NP*-hard on general networks (Kariv and Hakimi 1979). However, solutions from the *p*-median model are considered efficient since they bring the facility locations into closer proximity of the users. The difficulty of solving the *p*-median problem using exact methods has led researchers to consider sub optimal solutions generated by heuristic approaches. Heuristics for solving the *p*-median problem have been discussed in Daskin (1995), Maranzana (1964), Teitz and Bart (1968) and Densham and Rushton (1992).

This paper discusses three new heuristic methods for solving the *p*-median problem. These methods are motivated by the desire to eliminate outliers from having strong influence over the final solution given by the heuristics. These heuristics will also improve the delivery of emergency medical care by properly locating emergency facilities in an area.

In these heuristics, the facility location problem is formulated as a network optimization problem as follows. The geographical region is partitioned into a number of subregions and a corresponding graph is constructed. Each node of this graph represents a subregion and each link of the graph represents the fact that the corresponding regions share a boundary. This gives us a structural model. Non-structural information is added as weights on the nodes (reflect expected demand in region) and the links (reflect travel time). Usually the nodes of the network represent possible locations of facilities. An efficient reduction method is then used to address the problem of outliers.

Computational results, based on 400 random uniformly generated problems, show that the heuristics perform well in terms of quality of solution and computational time. Our best heuristic is compared with the well known existing *p*-median heuristics. Better solutions are achieved in most cases.

1. INTRODUCTION

The provision and utilization of effective and efficient emergency services is an important optimization problem encountered in all parts of the world. Integer programming problems and, specifically, facility location models have real application in the service and manufacturing industries. Facility location models are used extensively in solving optimization problems, which attempt to choose the 'best' location for facilities such as warehouses, schools, hospitals, ambulance stations, fire stations etc. In this paper, we develop a number of new heuristic algorithms and test them on simulated data and data from the literature.

2. THE *P*-MEDIAN MODEL AND EMERGENCY FACILITIES

The criterion for finding a good location for emergency facilities requires the improvement of the response times. The response time depends on the distance between the emergency facilities and the emergency sites. The aim is to locate these facilities such that the average (total) distance traveled by those who visit or use these facilities is minimized. This measures the effectiveness and efficiency of the emergency facilities. It is clear that people tend to travel to the closest facility regardless of the distance or time travelled. A good way to achieve this is by solving the p-median problem.

The *p*-median problem consists of determining the location of *p* emergency facilities to minimize the weighted distance between emergency (demand) points and their closest new emergency facility. A number of authors, such as Berlin *et a (*19760, Mirchandani (1980), Carson and Batta (1990), Serra and Marinov (1998), Paluzzi (2004), use the *p*-median problem solution to locate emergency facilities.

We now present the model for the *p*-median problem. We start with some notation: $I = \{1,....,m\}$ is the set of demand locations, $J = \{1,...,n\}$ is the candidate sites for facilities, d_{ij} is the shortest distance between location *i* and location *j*, $x_{ij} = 1$ if the customer at location *i* is allocated to the facility at location *j and* 0 otherwise, $y_j = 1$ if a facility is established at location *j and* 0 otherwise, p is the number of facilities to be established, and a_i is the population at the demand node *i.* The mathematical formulation is

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} a_i d_{ij} X_{ij}, \qquad (1)$$

subject to

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \qquad (2)$$

$$\sum_{j \in J} y_j = p \qquad (3)$$

$$x_{ij} \le y_j \quad \forall i \in I,\ \forall j \in J \qquad (4)$$

$$y_j \in \{0,1\}\ ,\ x_{ij} \in \{0,1\} \qquad (5)$$

The objective (1) is to minimize the total distance from customers or clients to their nearest facility. Constraint (2) shows that the demand of each customer or client must be met. Constraint (3) shows the number of facilities to be located is *p.* Constraint (4) shows that customers must be supplied from an open facility, and constraint (5) restricts the variables to 0, 1 values.

Several extensions have been proposed for the p-median model, which improves its efficiency (Daskin *et al*., 1988). Extensions to the *p*-median problem that account for its stochastic nature have been given by Fitzsimmons (1973), Weaver and Church (1985) and Swoveland *et al.* (1973).

3. SOLUTION METHODS FOR THE *P*-MEDIAN PROBLEM

The *p*-median problem is a computationally difficult problem to solve (the problem is *NP*-hard on general networks). Most solution methods are heuristic based because of the large number of variables and constraints that arise for a medium sized network. The heuristics are based on: genetic algorithms, simulated annealing, tabu search, node partitioning, node insertion, node substitution and various hybrids (Hosage and Goodchild (1986), Golden and Skiscism (1986), Glover (1990)). Some of these heuristics, together with Lagrangian relaxation, which is one of the most successful exact methods, are briefly discussed below.

3.1 Lagrangian Relaxation

Lagrangian relaxation is based on the principle that removing constraints from a problem makes the problem easier to solve. Generally, Lagrangian relaxation removes a constraint and solves the revised problem, which introduces a penalty for violating the removed constraint. The solution procedure for solving the problem is stated below.

The Lagrangian relaxation for the *p*-median is given as

$$L(\lambda) = min \sum_i \sum_j d_{ij} x_{ij} + \sum_i \lambda_i \left(1 - \sum_j x_{ij} \right) \quad (6)$$

subject to constraints (3)-(5).
The expression

$$r_j = \sum_i min\{0, d_{ij} - \lambda_i\} \quad (7)$$

is used to minimize the objective function (6) for the fixed values of the Lagrange multipliers. We then set

$$x_{ij} = \begin{cases} 1 \; if \; y_j = 1 \; and \; d_{ij} - \lambda_i < 0 \\ 0 \; otherwise \end{cases} \quad (8)$$

The lower and upper bounds of the objective function are determined by using the variables of modified and unmodified problems respectively. The next step involves the use of subgradient optimization to update the value of the Lagrange multipliers by using the equation below (Daskin 1995):

$$\lambda_i^{m+1} = max\left\{ 0, \lambda_i^m - t^m \left(\sum_j x_{ij}^m - 1 \right) \right\} \quad (9)$$

$$t^m = \frac{A^m (UB - L^m)}{\sum_i \left\{ \sum_j x_{ij}^m - 1 \right\}^2} \quad (10)$$

Where A^m is a constant on the m^{th} iteration, t^m is the stepsize at the m^{th} iteration of the Lagrangian procedure, UB is the best (smallest) upper bound on the *P*-median objective function, L^m is the value of the objective function using the solution obtained from the relaxed problem and x_{ij}^m is the optimal value of the allocation variable at the m^{th} iteration.

An optimal solution is found if the lower bound is equal to the upper bound. Narula *et al.* (1977) and Galvao (1980) and Beasley (1993) have successfully applied the subgradient optimization to solve a number of problems. However, for the larger problems tested, the computational time is excessively large.

3.2 Heuristics

In this section, we start our discussion by observing that it is an easy task to assign a set of m clients to p facilities J' with fixed locations. We just determine

$$d_{ij_i^*} = \min\{d_{ij}\}, \; 1 \le i \le m, j \in J' \quad (11)$$

and assign customer i to facility j_i^*. This gives us a tool for generating possible solutions. The procedure is also useful for determining alternative solutions through exchange of facility locations. We now use the idea above to describe three simple heuristics, which are competitive with other methods.

3.2.1 Myopic Algorithm (MA)

The myopic heuristic is a greedy type, which works in the following way. First, a facility is located in such a way as to minimize the total cost for all customers. Facilities are then added one by one until p is reached. For this heuristic, the location that gives the minimum cost is selected. The main problem with this approach is that once a facility is selected it stays in all subsequent solutions. Consequently, the final solution attained may be far from optimal.

3.2.2 Neighborhood Search Heuristic (NS)

Maranzana (1964) proposed this heuristic, which is described as follows. We begin with any set of p facility nodes. The demand nodes are then divided into p subsets and, for each subset, a demand node is allocated to the nearest facility node. The node giving the optimal for each subset is found, which results in a new pattern of facility nodes. This process is repeated until the facility nodes pattern remains the same as that in the previous step.

3.2.3 Exchange Heuristic (EH)

This is one of the early heuristics developed by Teitz and Bart (1968) for the p-median problem. The heuristic starts by choosing an initial set of p number of nodes as the solution, and then a node, which is not in the current solution, is selected to substitute for each of the p nodes in turn. We find the objective value in each case and compare the changes in the objective function. The substitution leading to the biggest decrease in the objective function is selected and is exchanged for a node in the current solution. This exchange of nodes results in a new solution configuration and this process continues until there is no further improvement in the objective value.

4. NEW *P*-MEDIAN HEURISTICS FOR LOCATING EMERGENCY FACILITIES

4.1. Reduction Heuristics (RH1, RH2, RRH)

In the previous section, the discussion of some of the heuristics (myopic in particular) for the p-median problem uses all the values of the distance matrix without any modification to solve the problem of extreme values (outliers). In this section, we tried to eliminate the problem of outliers by using a reduction technique. Outliers can have a strong influence over the final solution. We also eliminate the uncertainty of choosing a good initial solution in the case of the Neighborhood search and Exchange heuristics by using a specific and efficient way of selecting the initial solution for the three new heuristics.

We obtained the initial solution set for the heuristics by first eliminating the outliers and then sum the columns. We then choose the nodes corresponding to the first *p* nodes of the totals arrange in ascending order. The initial set is the first *p* nodes corresponding to the first *p* total, which is arranged in ascending order. The aim of the heuristics is to eliminate the outliers before using the data. This will enhance a facility to be located at nodes that are not far away from all customers, so the cost of using these facilities is minimized.

We use the initial solution to reduce the distance matrix by setting the nodes that corresponding to the initial set for both rows and columns to zero. This is done with the assumption that customers at those nodes are not charged to uses the facilities. For *RH1*, the columns of the resulting distance matrix are added and the minimum value is chosen for substituting into the initial solution. We finally choose the set with the minimum objective value. In the case of *RH2*, all the nodes not in the initial solution are exchanged one-by-one for the nodes in the initial solution. We then choose the facility set with the minimum objective value as the final solution. However, for both heuristics, we choose the initial set as the final solution if there is no improvement in the objective value after the swapping procedure.

Motivated by the performance of the two new heuristics (*RH1* and *RH2*), we extend *RH2* and propose a new heuristic, which we call Repeated Reduction Heuristic (*RRH*). The process of reducing the matrix is similar to *RH2* but, in this case, the reduction is done repeatedly until there is no improvement in the final solution.

We describe the three new reduction heuristics for the *p*-median problem below.

4.2 Reduction Heuristic One (RH1)

Step 1: Set the number of nodes and facilities to be equal to *n* and *p* respectively.

Step 2: Arrange the *n* values for each column in ascending order and delete the last α number of values from each column. Next, let the resulting number of nodes be equal to n' (i.e. $n' = n - \alpha$ where α is *p* for less than twenty nodes, 2*p* for less than thirty nodes, 3*p* for less than forty nodes etc.)

Step 3: Sum the first n' values for each column, arrange the values in ascending order, and choose the first *p* nodes as the initial set.

Step 4: Set the columns and rows corresponding to the initial set to zero and sum the columns of the resulting distance matrix.

Step 5: Choose the node or nodes corresponding to the minimum value and substitute for the nodes in the initial set.

Step 6: Choose the set corresponding to the minimum objective value after the substitution procedure reaches the final solution. Otherwise, go to step 3 and choose the initial set as the final solution if that value is lower.

4.3 Reduction Heuristic Two (RH2)

For *RH2,* Steps 1 to 4 is the same as *RH1* and the remaining steps are outlined below.

Step 5: Substitute all the nodes not in the initial set with the nodes in the initial set.

Step 6: Choose the set corresponding to the minimum value as the final solution. Otherwise, we choose the initial set as the final solution if that is lower

We note that the different swapping procedure lead to an improved final solution as compared with *RH1* (Section 5).

4.4 Repeated Reduction Heuristic (RRH)

In this heuristic, we repeatedly use the final solution of *RH2* as the initial set and use step 4 of *RH1*, and steps 5 and 6 of *RH2*. We continue this until there is no improvement in the final solution. We note that the repeated reduction incorporated in *RRH* has increased its performance as compared with *RH2*.

The proposed heuristics are unique in three different ways. First, the methodology is simple and tractable. Second, the elimination of outliers gives a good initial solution. Third, the determination of swapping a node or nodes and the swapping procedure gives a good final solution. We also note that an improvement procedure can be further introduced to reduce the response time.

4.5 Illustrative Example

0	82	37	51	100
67	0	78	93	97
74	18	0	20	49
20	87	27	0	66
62	37	51	87	0

We use the data above to illustrate the three new heuristics. To locate two facilities, we eliminate the two greatest values in each column. Hence, we eliminate 67 and 74 in column 1, 82 and 87 in column 2, 51 and 78 in column 3, 87 and 93 in column 4 and 97 and 100 in column 5. Summing the remaining values and arranging them in ascending order gives the following: 2 (55), 3 (64), 4 (71), 1 (82) and 5 (115). We choose nodes 2 and 3 as the initial solution for *RH1*, *RH2* and *RRH*. We, therefore, set rows and columns 2 and 3 of the data to zero and we have the following table.

0	0	0	51	100
0	0	0	0	0
0	0	0	0	0
20	0	0	0	66
62	0	0	87	0

The resulting totals for the non-zero columns give node 1 with the minimum value, so, for *RH1*, we substitute nodes 2 and 3 with node 1, which results in the possible solution sets of {1,3} and {1,2}. We choose {1,2} since that gives an optimal value of 75.

In the case of *RH2* and *RRH,* we use all the nodes not in the initial solution for substituting for nodes in the initial solution. This gives the possible solution set as follows: {1,2}, {1,3}, {2,4}, {3,4}, {2,5} and {3,5}. We choose {1,2} as the final solution since it gives an optimal value of 75. We continue the same process repeatedly for *RRH* and now use {1,2} as its initial solution, which finally yield {1,2} as the final solution.

We use the same data to locate three facilities. In this case, we eliminate the three greatest values in each column and sum the values of the remaining columns. This gives the initial solution of 1, 2 and 4. Going through the same process, and setting the rows and columns 1, 2 and 4 to zero, we have the following table.

0	0	0	0	0
0	0	0	0	0
0	0	0	0	49
0	0	0	0	0
0	0	51	0	0

For *RH1,* node 5 has the minimum value, so we substitute node 5 for nodes 1, 2 and 4. Thus, we have the possible sets of {2,4,5}; {1,4,5} and {1,2,5}. We choose {1,2,5} as the final solution, which has an optimal value of 38. In the case of *RH2* and *RRH*, we use nodes 3 and 5, which are not in the initial solution for substituting into nodes 1, 2 and 4. This gives the possible solution of {2,3,4}, {1,3,4}, {1,2,3}, {2,4,5}, {1,4,5} and {1,2,5}. We finally choose {1,2,5} as the final solution, which has an optimal value of 38. For *RRH,* we again use {1,2,5} as the initial solution and continue the process repeatedly. The final solution is {1,2,5}.

For the Myopic heuristic, we eliminate any extreme values, which gives the following table.

0	82	37	51	100
67	0	78	93	97
74	18	0	20	49
20	87	27	0	66
62	37	51	87	0

When we sum all the columns, node 3 has the minimum value of 193. Therefore, one facility is located at node 3. We note that, for the *p*-median problem, a demand is allocated to the nearest facility. We, therefore, adjust the distance matrix, which gives the following table.

0	37	37	37	37
67	0	78	78	37
0	0	0	0	0
20	27	27	0	27
51	37	51	51	0

Node 2 has the minimum value of 101 when the columns of the above matrix are added, so, for two facilities, we have nodes 2 and 3 with an objective value of 101.

Similarly, we have adjusted the above matrix after the two facilities were located, as shown below.

0	37	37	37	37
0	0	0	0	0
0	0	0	0	0
20	27	27	0	27
37	37	51	37	0

Node 1 has the minimum value when all the columns are added, so, for three facilities, we have nodes 1, 2 and 3 with an objective value of 57. We present, in Table 1, the results of the example of the three heuristics and Myopic Algorithm. The three heuristics give better results than myopic algorithm.

Table 1: Results for RH1, RH2, RRH and Myopic

P	**Solution**			
	RH1, RH2, RRH		**Myopic**	
	Fac.	**Obj.**	**Fac.**	**Obj.**
2	{1,2}	75	{1,3}	101
3	{1,2,5}	38	{1,2,3}	57

5. COMPUTATIONAL RESULTS

The three new heuristics are implemented in C++ and tested on sets of 20 randomly generated data for a [10, 100] matrix with n ranging from 10 to 50 in steps of 10 and p ranging from 2 to 5. The statistic used to measure the quality of the solution is given as $\frac{H-O}{O}\times 100$ where H is the value given by the implementation of the heuristic and O is the optimal value determined by the enumeration method. The value of 0% is considered to be optimal. A small deviation results in a better solution than a large deviation.

Table 1 gives the performance of the three new heuristics for locating 2, 3, 4 and 5 facilities. In Table 2 below, we have the average values for using ten, twenty, thirty, forty and fifty nodes.

Table 2: Average Values for the New Heuristics

Number of Nodes (n)	**Average Values (%)**		
	RH1	***RH2***	***RRH***
10	2.22	0.79	0.32
20	4.87	1.96	0.72
30	4.38	1.65	0.66
40	4.60	2.27	0.87
50	3.04	1.00	0.49

From Table 2, the average values for *RH1* ranges from 2.22% to 4.87%, *RH2* ranges from 0.79% to 2.27% and *RRH* ranges from 0.32% to 0.87%. The values of *RRH* are almost optimal, which is good for locating emergency facilities and might give rise to acceptable response times.

5.1 Comparison of the Repeated Reduction Heuristic (RRH) and some *P*-Median Heuristics

Motivated by the performance of *RRH,* we compare the heuristic using data from the literature. We compare this heuristic using the 55-node network data (Swain 1971). The data are given in Colome *et al.* (2003). The data has been used by authors such as Daskin (1982, 1983), Colome *et al.* (2003) and Church and Gerrard (2003) for testing location problems. The 55-node data set represents 55 communities in the Washington D.C (USA) area. Demands for each node were generated in pseudo-random manner with most large demands at the center of the region and most small demands at the outer region.

We compare *RRH* with the Myopic algorithm (MA), Exchange heuristic (EH) and Neighborhood search (NS) heuristic. We coded the Repeated Reduction Heuristic (*RRH*) in C++ while the results of the other heuristics were obtained using the SITATION software (Daskin, 1995). The solutions of the heuristics were compared with the optimal solutions, which were determined using Lagrangian Relaxation (Daskin, 1995).

Table 3: Comparison Performance of RRH and Existing Heuristic using 55-node Data

Number of Facilities (p)	**MA**	**NS**	**EH**	**RRH**
	$\frac{H-O}{O}\times 100$			
1	0	0	0	0
2	0	0	0	0
3	0	0	0	2.3
4	4.0	0	0	0
5	3.5	3.5	0	0
6	5.3	5.3	2.4	2.4
7	6.9	3.1	0	0
8	7.7	0.2	1.4	0
9	7.0	0.6	0.4	0

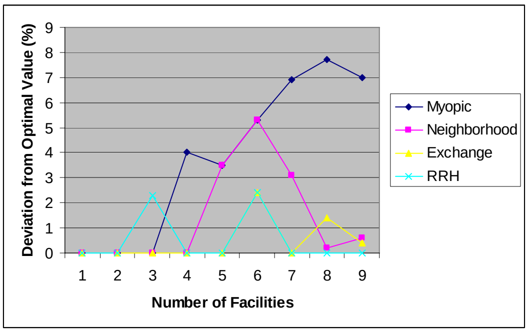

Figure 1: Comparison Performance of Heuristic using 55-node Data

Table 3 and Figure 1 show the performance of the new heuristics and the existing ones for the 55-node literature test problem. From Table 2 and Figure 1, the performance measured in terms of the number of optimal solutions gives the rank (from the best to the worst) of *RRH*, Exchange heuristic, Neighborhood Search heuristic and Myopic heuristic. The *new* heuristic *RRH* performs better in the location of all facilities with the exception of the location three and six facilities.

6. CONCLUSION

In this paper, we introduced three new heuristic methods to locate emergency facilities. These heuristics are based on the p-median problem and were tested using about 400 random data. The performance of our new heuristics compared with the optimal solution and existing heuristics is encouraging. The best heuristic among the three is within 1% of the optimal, and, when compared with other heuristics, it performs better.

7. REFERENCES

Beasley, J. (1993), Lagrangean heuristics for location problems, *European Journal of Operational Research,* 65, 383-399.

Berlin, G., C. Revelle, and J. Elzinga (1976), Determining ambulance-hospital locations for on-scene and hospital services, *Environment and Planning A*, 8, 553-561.

Carson, Y. and R. Batta, (1990) Locating an ambulance on Amherst campus of State University of New York at Buffalo, *Interfaces,* 20, 43-49.

Colome, R., H.R. Lourenco, and D. Serra (2003), A new chance-constrained maximum capture location problem, *Annals of Operations Research*, 122, 121-139.

Daskin, M.S. (1982), Application of an expected covering location model to EMS design, *Decision Sciences*, 13(3), 416-439.

Daskin, M.S. (1983), A maximum expected covering location model: formulation, properties and heuristic solution, Transportation Science, 17, 48-70.

Daskin, M.S. K. Hogan, and C. ReVelle, (1988) Integration of multiple, excess, backup, and expected covering models, *Environment and Planning B,* 15, 15-35.

Daskin, M.S. (1995), *Network and Discrete Location: Models, Algorithms and Application,* John Wiley and Sons, Inc., 498 pp., New York,

Densham, P.J., and G. Rushton (1992), A more efficient heuristic for solving large *p*-median problems *Papers in Regional Science* 71, 307-329.

Fitzsimmons, J.A. (1973), A Methodology for Emergency Ambulance Deployment, *Management Science* 19, 627-636.

Galvao, R.D. (1980), A dual-bounded algorithm for the *p*-median problem, *Operations Research,* 28, 1112-1121.

Golden, B.L., and C. Skiscism (1986) using simulated annealing to solve routing and location problems, *Naval Logistics Quarterly*, 33, 261-279.

Glover F. (1990), Tabu search: a tutorial, *Interfaces*, 20, 74-94.

Hakimi, S.L. (1964), Optimisation locations of switching centres and the absolute centres and medians of a graph, *Operations Research,* 12, 450-459.

Hosage, C.M., and M.F. Goodchild (1986), Discrete space location–allocation solutions from genetic algorithms, *Annals of Operations Research*, 6, 35-46.

Kariv, O. and S.L. Hakimi (1979), An algorithmic approach to network location problems II: the *p*-medians, *SIAM Journal on Applied Mathematics*, 37, 539-560.

Maranzana, F.E. (1964), On the location of supply points to minimize transport costs, *Operational Research Quaterly*, 15, 261-270.

Mirchandani, P.B. (1980), Locational decisions on stochastic networks, *Geographical Analysis,* 12, 172-183.

Narula, S.C., V.I. Ogbu and H.M. Samuelson (1977), An algorithm for the *p*-median problem, *Operations Research,* 25, 709-712.

Paluzzi, M. (2004), Testing a heuristic *p*-median location allocation model for siting emergency service facilities, *Paper presented at annual meeting of association of American Geographers,* Philadelphia, PA.

Serra D., and V. Marianov (1998), The *p*-median problem in a changing network: the case of Barcelona, *Location Science,* 6, 383-394.

Swain, R. (1971*), A decomposition algorithm for a class of facility location problems*. Unpublished Ph.D. dissertation, Cornell University, Ithaca, NY.

Teitz, M. B., and P. Bart (1968), Heuristic methods for estimating generalized vertex median of a weighted graph, O*perations Research,* 16, 955-961.

Weaver, J.R., and R.L. Church (1985), A median location model with nonclosest facility service, *Transportation Science,* 19, 58-74.