Encapsulation Of Water Allocation Into A General Equilibrium Model

P. B. Dixon[1], S.Yu. Schreider[2] and G. Wittwer[1]

[1]Centre of Policy Studies, Monash University, PO Box 11E, Vic 3800, Australia, [2]School of Mathematical and Geospatial Sciences, RMIT University, GPO Box 2476V, Melbourne, Vic 3001, Australia E-Mail: Sergei.Schreider@rmit.edu.au

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EXTENDED ABSTRACT

The paper focuses on the idea that water allocation countrywide should be treated as a macroeconomic problem. It describes how the water allocation model based on optimisation technique can be integrated with economic models within the computable general equilibrium (CGE) framework. The paper overviews the theoretical structure of some of hydrology/economy models. These models contain attractive detail for analysing issues in water policy in given hydrological areas. However their partial equilibrium nature is a drawback. They do not provide insights on either the effects of developments in hydrological areas on the rest of the economy or the effects of developments in the rest of the economy on hydrological areas. A general equilibrium perspective is needed to provide such insights. However, current general equilibrium models contain very little water detail. This paper develops mathematic formulations embedding hydrological/economy models in general equilibrium models. A hydrology-enriched general equilibrium model would be used to investigate major economy-wide water issues such as the relationships between water, agriculture, environment, tourism, urban development, population growth, and the potential climatic changes.

An enriched model would also be used to improve the analysis of narrower issues which are currently the province of partial equilibrium hydrology/economy models. Consider, for example, the effects of a drought in a particular hydrological area. A partial equilibrium model will show reduced availability of fodder or irrigated pasture for the dairy sector in the hydrological area, with a consequent reduction in dairy output. A general equilibrium model would include fodder-producing regions outside the hydrological area. It would show possibilities for interregional fodder trade that might mitigate the effects of the drought on the dairy sector in the hydrological area. At the same time, it would show how fodder flows into the drought-stricken hydrological area would affect output and employment in the fodder-supplying regions.

The paper spelled out the mathematical formulation of the integrative concept which unifies optimization-based partial equilibrium models of water allocation in a CGE framework. This framework reduces two different LP formulations, one used for economic gross margin maximisation and another used for minimising the water supply delivery costs, into one Lagrangian formulation common in CGE modelling.

The paper concludes that this way of water allocation modeling has considerable advantages compared to traditional optimisation and provides a better understanding of the driving forces of water allocation processes. A general equilibrium model can provide insights on water issues that are not available in specialist hydrology/economy models. But this cannot be done until general equilibrium models absorb hydrology and water allocation details. One approach to equipping general equilibrium models with hydrology detail is to embed in them hydrology modules. While this would be a challenging task, the analysis provided in this paper suggests that the relevant theory is manageable. With the advances that have taken place in general equilibrium software, it is now routine to solve extremely large general equilibrium models. Thus it is unlikely that computational problems would be a limiting factor in the creation of a general equilibrium model with extensive hydrological detail.

However, to create such a model would require a sustained co-operative effort from specialists in both hydrology/economy modelling and general equilibrium modelling. Compromises and understanding would be required on both sides right from the beginning. Authors are convinced that the CGE approach is substantially superior to alternative ways of integration of water allocation and economic models.

1. INTRODUCTION

This paper briefly overviews the theoretical structure of some of Australia's hydrology/economy models. These models contain attractive detail for analysing issues in water policy in given hydrological areas. However their partial equilibrium nature is a drawback. They do not provide insights on either the effects of developments in hydrological areas on the rest of the economy or the effects of developments in the rest of the economy on hydrological areas. A general equilibrium perspective is needed to provide such insights. However, current general equilibrium models contain very little water detail. The suggestion of this paper is that it would be useful to embed hydrological/economy models in general equilibrium models.

A hydrology-enriched general equilibrium model would be used to investigate major economy-wide water issues such as the relationships between water, agriculture, tourism, urban development, population growth and the environment. An example of such an issue currently under consideration by Australian governments is a change in the diversion threshold from the Snowy River Basin to the Upper Murray to increase the heritage, environmental and recreational values of the Snowy River valley (http://www.parliament.nsw.gov.au/prod/parlment/publications.nsf/).

Section 2 sets out in stylized form the theoretical structure of some hydrology/economy models. We leave out details that may be important for generating realistic results but that are unnecessary for understanding the overall theoretical structure of the models. Section 3 provides a starting point for the embedding of such models by showing how they can be converted into a form that is compatible with a general equilibrium framework. Concluding remarks are in Section 4.

2. THE THEORETICAL STRUCTURE OF AUSTRALIA'S WATER MODELS: BRIEF OVERVIEW

Australia has several well-developed models of economic activities in hydrological areas such as the Murray-Darling Basin or the Southern Murray basin. These models are of two types. The first type explicitly integrates water use with activities such as growing crops. This type of modelling is carried out at the Department of Primary Industry in Victoria (e.g. Eigenram *et al.*, 1999), ABARE (McClintock et al., 2000), Griffith University (Yu *et al.*, 2003), CSIRO (e.g. Qureshi *et al.*, 2004) and Integrated Catchment Assessment and Management Centre (iCAM) at the Australian National University (Scoccimaro *et al.*, 1999). The second type of model focuses on water flows between storage areas (supply points) and users (demand points). Supplies and demands are treated largely exogenously. Models of this type in Australia belong to the REALM family. Versions of REALM are used by the Department of Primary Industry in Victoria (Perera *et al.*, 2005) and the Co-operative Research Centre for Catchment Hydrology at Monash University (Weinmann *et al.*, 2005).

A common feature of both these types of models is that they are presented as optimization problems. They compute the optimal allocation of water between different uses in different regions within a hydrological area and/or the optimal water storage and streamflow management strategies. In stylized single-period form, models of the first type (integrated water use and crop growing) can be represented as:

Choose non-negative values for

$$A(j, r),\ W(j, r),\ L(j, r),\ Z(j, r) \text{ and either } W(\bullet, r) \text{ or } W(\bullet,\bullet) \text{ for all } r, j \qquad (2.1)$$

to maximize

$$\sum_r \sum_j \left[P(j,r) * Z(j,r) - PL(j,r) * L(j,r) - C(j,r) * W(j,r)\right] \qquad (2.2)$$

$$\text{subject to} \quad Z(j,r) = f_{j,r}\{A(j,r), L(j,r), W(j,r)\} \quad \text{for all } r, j \qquad (2.3)$$

$$\sum_j W(j,r) \le W(\bullet, r) \quad \text{for all } r \qquad (2.4)$$

$$\sum_r \sum_j W(j,r) \le W(\bullet,\bullet) \quad \text{and} \qquad (2.5)$$

$$\sum_j A(j,r) \le N(r) \quad \text{for all } r \qquad (2.6)$$

where:

A(j, r) is the quantity of land in region r devoted to crop j;

W(j, r) is the amount of water applied to crop j in region r;

L(j, r) is the amount of other factors (e.g. labour) applied to crop j in region r;

Z(j, r) is the quantity of crop j produced in region r;

$W(\bullet, r)$ is the amount of water available for use in region r;
P(j, r) the price of a unit of crop j from region r;
PL(j, r) is the price of a unit of other factors (e.g. labour) used for crop j in region r;
C(j, r) is the price charged to farmers growing crop j in region r for a unit of water;
$W(\bullet,\bullet)$ is the total amount of water available for use in all regions;
N(r) is the quantity of land available in region r; and
$f_{j,r}$ is a constant-returns-to-scale production function relating output of j in r to inputs.

If $W(\bullet, r)$ is treated as an endogenous choice variable, then (2.1) to (2.6) allocates a given amount of water, $W(\bullet,\bullet)$, across all regions and crops to maximize aggregate profits. There is no explicit modelling of water flows between regions and no account is taken of differences in the costs of transporting water around the system implied by different configurations of water usage. If $W(\bullet, r)$ is set exogenously, then (2.1) to (2.6) allocates this amount of water between crops to maximize profits in region r. There is no connection between the regions and (2.1) to (2.6) could be disaggregated and solved as a series of separate problems, one for each region. Comparison of the solutions in which the $W(\bullet, r)$s are endogenous and exogenous could be used to indicate the advantages of inter-regional water trading.

An important part of the empirical content of (2.1) to (2.6) is the specification of the production functions $f_{j,r}$. A relatively simple specification, used by Eigenraam *et al.* (1999) rests on yield functions of the form

$$Y(j, r) = \alpha(j, r) + \beta(j, r) * W(j, r) / A(j, r) \quad \text{for all r and j} \tag{2.7}$$

where:
Y(j, r) is the output of crop j in region r per unit of land devoted to j in region r; and
$\alpha(j, r)$ and $\beta(j, r)$ are parameters.

Underlying (2.7) is the notion that a fraction of the land in region r devoted to crop j is fully supplied with water and the remainder is not supplied at all. As more water is used [i.e. as W(j,r) increases] the watered fraction of the (j, r) land increases. To obtain (2.7) we let $A_W(j, r)$ and $A_{NW}(j, r)$ be the watered and non-watered quantities of (j, r) land. Then

$$A(j, r) = A_W(j, r) + A_{NW}(j, r) \tag{2.8}$$

$$W(j, r) = \Pi(j, r)*A_W(j, r) \quad \text{and} \tag{2.9}$$

$$Z(j, r) = \delta(j, r)*A_{NW}(j, r) + \gamma(j, r)*A_W(j, r) \tag{2.10}$$

where:
$\Pi(j, r)$ is water used per unit of watered (j, r) land;
$\delta(j, r)$ is output per unit of non-watered (j, r) land; and
$\gamma(j, r)$ is output per unit of watered (j, r) land.

This leads eventually to (2.7) with

$$\alpha(j, r) = \delta(j, r) \tag{2.11}$$

and

$$\beta(j, r) = [\gamma(j, r) - \delta(j, r)] / \Pi(j, r) \tag{2.12}$$

More complicated forms for yield functions can be found in Qureshi *et al.* (2004). In effect, these allow for partial watering of land, that is they avoid the assumption built into (2.8) to (2.10) that each unit of land is either fully watered or not watered.

As already indicated, models of the form (2.1) to (2.6) give little emphasis to water flows between regions. By contrast, interregional water flows are the main emphasis of the REALM family of models, the second type of water model used in Australia. In stylized, form REALM models can be represented as:

Choose non-negative values for

$$F(i, r, t), \quad \text{for } i \in D,\ r \in D \cup E,\ t = 1, 2, \ldots, T \tag{2.13}$$

$$S(e, t), \quad \text{for } e \in E,\ t = 1, 2, \ldots, T \tag{2.14}$$

and

$$W(i, t) \quad \text{for } i \in D,\ t = 1, 2, \ldots, T \tag{2.15}$$

to minimize

$$\sum_{i \in D} \sum_{r \in D \cup E} \sum_{t} c_{i,r}(t) * F(i, r, t) + \tag{2.16}$$

$$\sum_{e \in E} \sum_{t} \beta_e(t) * \left| d(e,t) - S(e,t) \right| + \sum_{i \in D} \sum_{t} g_{i,t} [W(i,t) - W_{min}(i,t)]$$

subject to

$$W(i, t+1) \le W(i, t) - \sum_{r \in D \cup E} F(i, r, t) + \tag{2.17}$$

$$\sum_{k \in D} F(k, i, t) * [1 - l(k, i, t)] + X(i, t) - \theta_{i,t} [W(i, t)]$$

for $i \in D$, t = 1, 2, …, T

$$W(i,t) \le C(i) \quad \text{for } i \in D,\ t = 1, 2, \ldots, T \qquad (2.18)$$

and

$$S(e,t) = \sum_{i \in D} F(i,e,t) * \left[1 - l(i,e,t)\right] \qquad \text{for } e \in E,$$

t = 1, 2, …, T (2.19)

where:

D is the set of dams. Dams include not only water storage facilities but also junctions in the water network. A junction has either more than one inlet or more than one outlet. It can be treated as a dam with zero capacity;

E is the set of end users;

F(i, r, t) is the flow in period t from dam i to dam or end-use r;

T is the last period of interest. If the model were solved for one year with periods of one month, then T =12;

W(i, t) is the amount of water in dam i at the beginning of period t. W(i,0) is exogenous;

S(e, t) is the amount of water supplied to end-user e in period t;

d(e, t) is the exogenously determined ideal water requirements of end-user e in period t;

$c_{i,r}(t)$ is the cost of sending a unit of water from dam i to dam or end-user r in period t. If it is physically impossible to send water from i to r, then $c_{i,r}(t)$ can be set at an arbitrarily large number;

$\beta_e(t)$ is the penalty or cost per unit of shortfall in meeting the water demands of end-user e in period t;

$W_{min}(i, t)$ is the minimum level of water for dam i that is desirable from an environmental or aesthetic point of view;

$g_{i,t}$ is a penalty function. It takes positive values if $W(i, t) – W_{min}(i, t)$ is negative;

l(k, i, t) is losses per unit of flow from dam k to dam or end-user i in period t (exchange losses);

X(i, t), specified exogenously, is the natural inflow to dam i in period t;

$\theta_{i,t}$ is a function giving evaporation from dam i in period t;

C(i) is the capacity of dam i. If dam i is a junction then C(i) = 0.

Models such as REALM can be used to plan flows in a hydrological area and to decide how these flows should be varied in response to changes in rainfall [reflected in X(i, t)], changes in demands [d(e, t)], and changes in a myriad of technical and cost coefficients.

3. MOVING FROM PARTIAL EQUILIBRIUM TO GENERAL EQUILIBRIUM

The strength of models such as (2.1) to (2.6) and (2.13) to (2.19) is their ability to encapsulate relevant detail concerning water technology and costs. However, they are incomplete and partial equilibrium.

They are incomplete in that they are missing potentially important relationships between prices and quantities. For example, in model (2.1) to (2.6), product prices [P(j, r)], prices of non-water inputs [PL(j, r)] and water charges [C(j, r)] are treated as exogenous. However, all of these variables could be expected to react to developments within the hydrological area. For example, changes in outputs [Z(j, r)] could be expected to affect product prices; changes in input demands [L(j, r)] could be expected to affect input prices; and changes in the budgetary situation of the water authority could be expected to affect water charges.

The models are partial equilibrium in that they represent a hydrological area as if it were not connected with the rest of the economy. This means that the models cannot give insights about the effects of developments within the hydrological area on the rest of the economy or the effects of developments in the rest of the economy on the hydrological area. Policy makers need to know the effects that droughts, technological changes and changes in water charges and other costs have not only directly on the hydrological area but indirectly on Australia's regional economies. They also need to know how the hydrological area is likely to be affected by developments in the mining sector, for example. Such developments affect the hydrological area through the exchange rate and through economy-wide competition for scarce resources including water.

Potentially, a general equilibrium model can provide the missing price/quantity relationships and can link hydrological areas to the rest of the economy. However, to date, most general equilibrium models have included no water detail.

At the Centre of Policy Studies, we have been developing a general equilibrium model, TERM-water, in which there are up to 16 irrigation plus 32 other industries in up to 12 Irrigation and 6 other regions (Wittwer, 2003; Horridge *et al.,* 2005). Each of these regional irrigation industries is specified as using water as an input. TERM-water has been used to simulate the regional and economy-wide effects of changes in water availability to irrigators. Scenarios modelled in TERM include a diversion of 500 Gigalitres of water from the Murray-Darling Basin (MDB) to the environment. This is based on The Living Murray Project of the MDB Commission (http://www.thelivingmurray.mdbc.gov.au) which deals with water allocation to six significant ecological assets of the Basin.

At this stage, water-related technological and behavioural specifications in TERM-water are not informed by the detail that is available in hydrological models. Sensible, but largely arbitrary, assumptions are made about the extent to which water can substitute for other inputs. Consequently, TERM-water can give no more than broad insights. To go beyond this will require the introduction of considerable water detail. For TERM-water to deal convincingly with water trading, for example, it will necessary to introduce estimates of the scarcity value of water in each region. These estimates are available in hydrological models as shadow prices on constraints such as (2.4). TERM-water would also need to embed technological information on water flows and exchange losses, information that is available in REALM.

Can we build general equilibrium models that embed genuine hydrological detail?

General equilibrium models are formulated as a series of equations rather than as constrained optimization problems. Our plan is to embed a hydrological model in a general equilibrium model by including in the general equilibrium model equations that can be derived from the first-order conditions for a solution of the hydrological model. Thus, if we wished to embed model (2.1) to (2.6), for example, we would start by specifying the Lagrangian function:

$$\begin{aligned}\mathcal{L} = & \sum_r \sum_j [P(j,r) * Z(j,r) - PL(j,r) * L(j,r) - C(j,r) * W(j,r)] \\ & - \sum_r \sum_j \Lambda(j,r) * [Z(j,r) - f_{j,r}\{A(j,r), L(j,r), W(j,r)\}] \\ & - \sum_r Q(\bullet,r) * [W(j,r) - W(\bullet,r)] \\ & - Q(\bullet,\bullet) * \left[\sum_r \sum_j W(j,r) - W(\bullet,\bullet)\right] \\ & - \sum_r PA(r) * \left[\sum_j A(j,r) - N(r)\right] \qquad (3.1)\end{aligned}$$

where:

$\Lambda(j,r)$, $Q(\bullet,r)$, $Q(\bullet,\bullet)$ and $PA(r)$ are Lagrangian multipliers.

Assuming (without significant loss of generality) that all water and land resources are used so that (2.4) to (2.6) hold as equalities, we obtain the first-order conditions as:

$$P(j,r) - \Lambda(j,r) = 0 \quad \text{for all j and r} \qquad (3.2)$$

$$-PL(j,r) + \Lambda(j,r)\frac{\partial f_{j,r}}{\partial L(j,r)} = 0$$

for all j and r (3.3)

$$-C(j,r) + \Lambda(j,r)\frac{\partial f_{j,r}}{\partial W(j,r)} - Q(\bullet,r) - Q(\bullet,\bullet) = 0$$

for all j and r (3.4)

$$\Lambda(j,r)\frac{\partial f_{j,r}}{\partial A(j,r)} - PA(r) = 0 \quad \text{for all r} \qquad (3.5)$$

$$Z(j,r) - f_{j,r}\{A(j,r), L(j,r), W(j,r)\} = 0$$

for all r and j (3.6)

$$\sum_j W(j,r) - W(\bullet,r) = 0 \quad \text{for all r} \qquad (3.7)$$

$$\sum_r \sum_j W(j,r) - W(\bullet,\bullet) = 0 \qquad (3.8)$$

$$\sum_j A(j,r) - N(r) = 0 \quad \text{for all r and j} \qquad (3.9)$$

$Q(\bullet,\bullet) = 0$ if $W(\bullet,\bullet)$ is treated endogenously (non-trading), and (3.10)

$Q(\bullet,r) = 0$ if $W(\bullet,r)$ is treated endogenously (trading). (3.11)

Equivalently, (3.2) to (3.11) can be written as

$$PL(j,r) = P(j,r)\frac{\partial f_{j,r}}{\partial L(j,r)} \quad \text{for all j and r} \qquad (3.12)$$

$$PW(j,r) = P(j,r)\frac{\partial f_{j,r}}{\partial W(j,r)} \quad \text{for all j and r} \qquad (3.13)$$

$$PA(r) = P(j,r)\frac{\partial f_{j,r}}{\partial A(j,r)} \quad \text{for all r} \qquad (3.14)$$

$$Z(j,r) - f_{j,r}\{A(j,r), L(j,r), W(j,r)\} = 0$$

$$\text{for all j and r} \qquad (3.15)$$

where

$$PW(j,r) = C(j,r) + Q(\bullet,r) + Q(\bullet,\bullet)$$

$$\text{for all j and r} \qquad (3.16)$$

$$\sum_j W(j,r) - W(\bullet,r) = 0 \quad \text{for all r} \qquad (3.17)$$

$$\sum_r \sum_j W(j,r) - W(\bullet,\bullet) = 0 \qquad (3.18)$$

$$\sum_j A(j,r) - N(r) = 0 \quad \text{for all r} \qquad (3.19)$$

$Q(\bullet,\bullet) = 0$ if $W(\bullet,\bullet)$ is treated endogenously (non-trading), and (3.20)

$Q(\bullet,r) = 0$ for all r if $W(\bullet,r)$ is treated endogenously (trading). (3.21)

In a general equilibrium model, (3.12) to (3.21) would be represented as shown in Table 1. In this table, (T1.1) to (T1.3) are input-demand equations derived from (3.12) to (3.15). For general equilibrium modellers, a more familiar but equivalent derivation would invoke the cost-minimizing problem:

Choose A(j, r), L(j, r), W(j, r) (3.22)

to minimize

$$PA(r) * A(j,r) + PL(j,r) * L(j,r) + PW(j,r) * W(j,r) \qquad (3.23)$$

subject to

$$Z(j,r) = f_{j,r}\{A(j,r), L(j,r), W(j,r)\} \qquad (3.24)$$

Whereas in (2.1) to (2.6), the entire hydrological area is treated as though it is controlled by a single profit-maximizing agent that owns all the land, in the general equilibrium approach, we assume that there are many agents. One agent produces crop j in region r. This agent rents land from the landowning agent in region r. Whatever level of output [Z(j, r)] is produced by the (j, r) agent, we obtain (T1.1) to (T1.3) by assuming that inputs of land, water, and other are chosen to minimize the cost of producing that output.

(T1.4) to (T1.6) simply repeat (3.17) to (3.19). In general equilibrium language, (T1.4) equates the total demand for water in region r with the supply in region r. (T1.5) equates the total demand for water in the hydrological area with the supply in the hydrological area. (T1.6) equates the demand for land in region r with the supply of land in region r.

(T1.7) is a repeat of (3.16). It defines the price of water to growers of j in region r as the sum of three components. The first, C(j, r), is a charge specific to growers of crop j in region r. This charge might be used by governments to allow for negative externalities in the use of water on insecticide-intensive crops such as cotton. The second component, $Q(\bullet,r)$, reflects the scarcity of water in region r. It will be non-zero only if water trading is ruled out and $W(\bullet,r)$ is exogenously given. In this case $Q(\bullet,r)$ will adjust to ensure that water demands in region r given by (T1.3) are compatible with the exogenously given supply, $W(\bullet,r)$. If water trading is allowed then $Q(\bullet,r)$ is zero. In this case there is no water scarcity specific to region r and $W(\bullet,r)$ is simply the endogenously determined sum over water uses in region r. The third component, $Q(\bullet,\bullet)$, reflects the scarcity of water in the entire hydrological area. It will be non-zero only if water trading is allowed. If water trading is not allowed, then there is no overall scarcity of water. Instead, there is scarcity in each region, which is already accounted for by $Q(\bullet,r)$. When water trading is allowed, $Q(\bullet,\bullet)$ adjusts to ensure that water demands in the whole hydrological area are compatible with the exogenously given supply.

The final equation in Table 1, (T1.8) imposes zero pure profits in all crop-growing activities in all regions. In general equilibrium modelling we assume that if revenue from activity (j, r) exceeds costs, then this activity will expand, forcing up prices of scarce factors (land and water). Similarly, if revenue is less than costs then the activity will contract leading to reductions in the prices of scarce factors. We can derive (T1.8) from (3.12) to (3.15) by: multiplying both sides of (3.12) by L(j, r); multiplying both sides of (3.13) by W(j, r); multiplying both sides of (3.14) by A(j, r); adding over the three resulting equations; and invoking Euler's Theorem for functions that are homogeneous of degree one, that is

Table 1. General equilibrium representation of hydrological model

Equations

Input-demand equations

No. of equations

$$A(j,r) = Z(j,r) * A_{j,r}\left(PL(j,r), PA(r), PW(j,r)\right) \quad \text{for all j and r,} \quad J*R \qquad (T1.1)$$

$$L(j,r) = Z(j,r) * L_{j,r}\left(PL(j,r), PA(r), PW(j,r)\right) \quad \text{for all j and r,} \quad J*R \qquad (T1.2)$$

$$W(j,r) = Z(j,r) * W_{j,r}\left(PL(j,r), PA(r), PW(j,r)\right) \quad \text{for all j and r,} \quad J*R \qquad (T1.3)$$

Market-clearing conditions

$$\sum_j W(j,r) = W(\bullet, r) \quad \text{for all r} \quad R \qquad (T1.4)$$

$$\sum_r \sum_j W(j,r) = W(\bullet,\bullet) \quad 1 \qquad (T1.5)$$

$$\sum_j A(j,r) = N(r) \quad \text{for all r} \quad R \qquad (T1.6)$$

Zero-pure-profits and water pricing

$$PW(j,r) = C(j,r) + Q(\bullet, r) + Q(\bullet,\bullet) \quad \text{for all j and r,} \quad J*R \qquad (T1.7)$$

$$P(j,r) * Z(j,r) = PA(r) * A(j,r) + PL(j,r) * L(j,r) + PW(j,r) * W(j,r) \quad \text{for all j and r,} \quad J*R \qquad (T1.8)$$

Variables

Description	*Number*	*Status*[*] *No trading*	*Trading*
A(j, r), Land used for j in r	J*R	N	N
W(j, r), Water used for j in r	J*R	N	N
L(j, r), Other inputs used for j in r	J*R	N	N
PA(r), Rental price of land in r	R	N	N
PW(j, r), Price of water for j in r	J*R	N	N
PL(j, r), Price of other inputs for j in r	J*R	XN	XN
Z(j, r), Output of j in r	J*R	N	N
P(j, r), Price of output of j produced in r	J*R	XN	XN
$W(\bullet, r)$, Water used in region r	R	X	N
$W(\bullet,\bullet)$, Total water used	1	N	X
$C(j,r)$, Component of water price specific to (j, r)	J*R	XN	XN
$Q(\bullet, r)$, Component of water price specific to region r	R	N	X
$Q(\bullet,\bullet)$, Non-specific component of water price	1	X	N
N(r), Land in r	R	X	X

N= endogenous, X = exogenous and XN = exogenous to the hydrological module but endogenous to the whole general equilibrium system.

Number of equations

5J*R + 2R + 1

Number of endogenous variables determined in the hydrological part of the general equilibrium model

5J*R + 2R + 1

$$f_{j,r}\{A(j,r), L(j,r), W(j,r)\} = \frac{\partial f_{j,r}}{\partial L(j,r)} * L(j,r) + \frac{\partial f_{j,r}}{\partial W(j,r)} * W(j,r) \frac{\partial f_{j,r}}{\partial A(j,r)} * A(j,r) \qquad (3.25)$$

Altogether, the system (T1.1) to (T1.8) contains 5J*R + 2R + 1 equations where J is the number of crops and R is the number of regions. If there is no water trading, these equations can be thought of as determining the 5J*R + 2R + 1 variables marked N (for endogenous) in the *No trading* column in Table 1. Similarly, if there is water trading then (T1.1) to (T1.8) can be thought of as determining the 5J*R + 2R + 1 variables marked N in the *Trading* column. As we go from the no trading column to the trading column, total water used, $W(\bullet,\bullet)$, moves from endogenous (merely a sum of regional uses) to exogenous (the constraint on total water usage). At the same time, $Q(\bullet,\bullet)$ moves from exogenous (set at zero) to endogenous (to play the role of equating demands with exogenously given overall supply). $W(\bullet, r)$ moves from exogenous (reflecting regional availability of water) to endogenous. Correspondingly, $Q(\bullet, r)$ moves from being endogenous to being exogenously set at zero.

Three variables [P(j, r), PL(j, r) and C(j, r)] are marked in Table 1 as having status XN with either trading or no trading. These are variables that we could expect to be endogenous in a general equilibrium model, but determined outside the equations of the hydrological module. In a general equilibrium model, product prices and prices of other inputs are determined by demands and supplies throughout the economy. As mentioned earlier, C(j, r) might be partly determined by government budgetary policy.

4. CONCLUDING REMARKS

A general equilibrium model can potentially provide insights on water issues that are not available in specialist hydrology/economy models. But this cannot be done until general equilibrium models absorb hydrology detail.

One approach to equipping general equilibrium models with hydrology detail is to embed in them hydrology modules. While this would be a challenging task, the analysis in section 3 suggests that the relevant theory is manageable. With the advances that have taken place in general equilibrium software, it is now routine to solve extremely large general equilibrium models. Thus it is unlikely that computational problems would be a limiting factor in the creation of a general equilibrium model with extensive hydrological detail.

However, to create such a model would require a sustained co-operative effort from specialists in both hydrology/economy modelling and general equilibrium modelling. Compromises and understanding would be required on both sides right from the beginning. For example, one of the very early decisions would be the definition of regions. Should these reflect natural hydrological areas or should they be defined according to the boundaries in published economic statistics?

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