Stochastic Generation Of Point Rainfall Data At Sub-Daily Timescales: A Comparison Of DRIP And NSRP

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EXTENDED ABSTRACT

Short duration rainfall data are needed for modelling urban systems. Two models for stochastic generation of point rainfall data at subdaily timescales are compared in this paper: the Disaggregated Rectangular Intensity Pulse (DRIP) model (Figure 1) of Heneker et al. (2001), and the single site version of the Neyman-Scott Rectangular Pulse (NSRP) process model (Figure 2) of Cowpertwait et al. (2002). In Figure 1, t_a, t_d, i, d respectively represent the interstorm duration, the storm duration and the average storm intensity, and the storm depth.

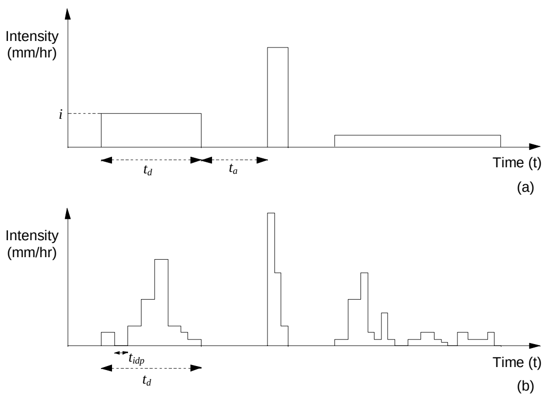


Figure 1. DRIP Model of event series: (a) generation of a time series of rectangular rainfall pulses or events; and (b) a random shaped hyetograph produced by the disaggregation scheme. Figure from Heneker (2001).

These two models are quite different in their conceptualisation of the rainfall process, but have both previously shown good reproduction of statistics not used in calibration – particularly Intensity-Frequency-Duration curves – which are important in hydrological design. The two models were calibrated to 10 major Australian cities/regional centres and sites where there is a relative abundance of pluviograph data for calibration (hence providing a best possible

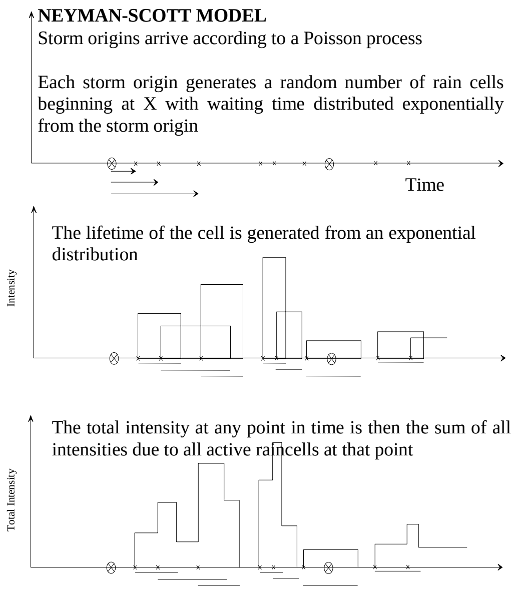


Figure 2. A schematic of the Neyman-Scott model (derived from Cowpertwait (1991)).

scenario in terms of data length for Australian conditions). The purpose of this study was to evaluate and compare the two models for use in the CRC for Catchment Hydrology's modelling toolkit.

The models were evaluated on a monthly basis regarding their ability to reproduce certain 'standard' and extreme rainfall model statistics derived from the pluviograph record over a range of timescales (1, 6 and 24 hr) along with other daily, monthly and annual statistics derived from the longer daily rainfall series. Of the shorter timescale standard statistics, DRIP performed better than the NSRP model. Both models performed adequately at greater timescales of aggregation. The DRIP model has subsequently been incorporated into the CRC's toolkit for use by the practitioner and this study serves as validation for use of that product.

1. INTRODUCTION

Due to the complex and chaotic nature of climate, rainfall is often modelled as a stochastic process. The purpose of such modelling is to produce replicated simulated series of data, which are representative of the range of scenarios that could possibly occur. These simulated series can be used in design in the place of long series of observed data. As there is typically less than 15 years of observed subdaily rainfall at sites throughout Australia, stochastic rainfall models are an important tool in providing input rainfall data for design that requires point rainfall data as an input.

Examples of occasions where point rainfall generation (at the subdaily timescale) may be important are especially evident in urban hydrology – particularly in relation to flood estimation (see Kuczera et al., 2003 for a discussion). Coombes et al., (2003) use rainfall generated by a point subdaily rainfall model in testing the implications of using household rainwater tanks for household water use on household demand and household runoff. Cowpertwait et al., (2002) apply a spatial-temporal subdaily rainfall model to hourly data from the Arno catchment, Italy – the resulting simulations being used in flood studies.

Stochastic models provide a relatively simple method of representing the complex rainfall processes occurring. The alternative to stochastic modelling is to use deterministic equations based on physical processes, as is done over large spatial grid scales in General Circulation Models (GCM's). However, this spatial scale (typically $\approx 100km \times 100km$) is too large for many hydrologic design applications. Alternatively, stochastic rainfall generated at a larger timescale can be disaggregated to a smaller timescale (eg. Koutsoyiannis et al., 2003). The strength of the parametric point stochastic approach is its simplicity and ability to limit the extremes that may be produced through choice of distributional form.

The use of point rainfall, as opposed to areal averaged rainfall, as input to hydrological models has been questioned by Jordan and Seed (2002). However, purely stochastic generation of areal averaged rainfall over short timescales and over small grid areas is not yet computationally feasible for long time series – see Seed et al., (2000) for spatial generation of a single storm. Spatial generation of stochastic rainfall at subdaily timescales over a series of sites has been performed by a number of authors (Cowpertwait, et al., 2002, Northrop, 1998). It is the purpose of this study however to test single-site subdaily rainfall models for inclusion into the CRC for Catchment Hydrology's modelling toolkit.

This study selects two of the better performing point subdaily rainfall models, and applies them to 10 Australian capital cities/regional centres. These 10 sites were chosen due to the relative availability of long rainfall records at the 6-min timescale. The application of the DRIP and NSRP demonstrates the current versions of each model under Australian conditions.

2. PREVIOUS ATTEMPTS AT POINT STOCHASTIC SUBDAILY RAINFALL GENERATION

Stochastic modelling has been an area of active research for some years now, and research has generally focussed on two approaches: cluster based and event based models.

Also known as 'alternating renewal' or 'profile based' models, *event based models* break the conceptual rainfall process into a series of events characterised by inter-arrival time, storm duration and mean storm intensity (Eagleson, 1978, Heneker, et al., 2001, Koutsoyiannis and Pachakis, 1996, Menabde and Sivapalan, 2000) – see Figure 1(a). A storm is identified by the occurrence of a dry period (rainfall less than a threshold value, typically zero) that is longer than some specified duration ranging typically from 2-9 hours. These models use various methods to disaggregate a simulated storm event to the desired timescale. Some models disaggregate via a non-dimensionalised storm profiling/scaling (Heneker, et al., 2001, Koutsoyiannis and Foufoula-Georgiou, 1993, Koutsoyiannis and Pachakis, 1996). Others use multi-fractal disaggregation techniques (Menabde and Sivapalan, 2000).

Cluster based models conceptualise rainfall as a series of storm arrivals, with rainfall cells associated with each storm (these rainfall cells usually have a random duration and intensity) such that the total intensity at any time is the sum of the intensities of all cells active at that time – see Figure 2 for an example. Considerable research has been focussed on cluster based modelling, specifically Neyman-Scott (NS) and Bartlett-Lewis (BL) type cluster processes (Cowpertwait, 1991, Cowpertwait and O'Connell, 1997, Onof and Wheater, 1993, Rodriguez-Iturbe et al., 1987). These models differ in the displacement of cell origins relative to storm origins, and there have been several empirical studies comparing the two models (eg. Velghe et al., 1994). Cowpertwait (1998) has shown analytically that BL and NS rectangular pulse rainfall models were equivalent up to second-order properties.

In terms of application to Australian conditions within the literature, DRIP has been the most widely applied event based model (Heneker et al.,2001), with Menabde and Sivapalan (2000) applying their model to a single site only. DRIP was found within the study of Heneker et al. (2001) to reproduce aggregation and IFD statistics not used in calibration well, and hence was chosen for evaluation. Of the cluster based models, the BL rectangular pulses model and subsequent generalisations have been used most widely (Gyasi-Agyei and Willgoose, 1997, Gyasi-Agyei, 1999, Gyasi-Agyei and Willgoose, 1999), and performed well in terms of reproduction of extreme and aggregation statistics. However, a later version of the Neyman-Scott rectangular pulse model was chosen for comparison. This was chosen for the following reasons; the third order properties of the rainfall process have been derived by Cowpertwait (1998) and this model has been generalised for future use in spatial modelling studies (Cowpertwait, et al., 2002).

3. DRIP MODEL

Following the description given in Heneker et al. (2001), DRIP conceptualises rainfall as a series of storm and interstorm events – see Figure1(a). In terms of the observed rainfall record, a storm is defined as any period of positive rainfall which is separated by a minimum dry period of 2 hours. There are three random event variables: the interstorm duration (t_a), the storm duration (t_d) and the average storm intensity (i), with the storm depth (d) defined as the product of i and t_d. Thus the storm is considered a series of rectangular intensity pulses. The second stage of the model determines the temporal distribution of rainfall within each event using a disaggregation scheme as shown in Figure 1(b).

3.1. DRIP Event Data Calibration

As DRIP breaks the pluviograph data into a series of events, a method of calibration called maximum likelihood can be used to obtain model parameters for simulation. Statistical distributions are fitted to the storm duration, storm intensity and interstorm duration events on a monthly basis.

3.2. DRIP Event Disaggregation

Once DRIP simulates a storm duration and intensity, the event is broken down to reproduce the short-timescale temporal characteristics of the data using a conditional random walk on a dimensionless mass curve.

It is noted here that the calibration of DRIP disaggregation parameters does not currently consider rainfall events with less than 1 hour of cumulative wet bins. This excludes many rainfall events from having an influence on disaggregation. If the assumption used in disaggregation - that the process can be non-dimensionalised – is correct, ignoring these data will increase the sampling variability of the parameter estimates. If however the process is not similar for different storm durations/depths, rejecting these smaller duration storms will bias the parameters towards the longer duration storm characteristics. Refinement of the disaggregation method (and addressing this issue) is a current direction of research of the model authors.

4. NSRP MODEL

The Neyman-Scott rectangular pulse (NSRP) cluster model is the single point version of the spatial NSRP model presented within Cowpertwait et al., (2002). The NSRP model conceptualises rainfall as consisting of a series of storms, with an associated set of rainfall cells with each storm – see Figure 2.

4.1. NSRP Calibration

As the definition of the NSRP allows overlapping of cells and storms, events are no longer defined. This in turn means that the derivation of a likelihood function for the rainfall accumulation amounts is difficult (if not intractable). Therefore, as different form of calibration is used for such models, this method revolves around matching (as closely as possible) rainfall aggregation statistics at various timescales.

5. DATA AND AGGREGATION STATISTICS

Pluviograph data from 10 major cities and regional centres were chosen for this study –Adelaide, Alice Springs, Brisbane, Cairns, Darwin, Hobart, Melbourne, Perth, Sydney and Townsville. These sites were chosen due to the relatively long length of record available, at least 45 years of data (with the exception of Adelaide). Other Bureau of Meteorology pluviograph sites (that have been digitised) throughout Australia have lengths typically in the range 15-25 years. Thus, this can be considered as being the best possible data set for identification of model parameters in Australian conditions.

Details of the pluviograph and daily data used for aggregation statistics can be found in Frost et al. (2004). Over 100 replicates the length of the record used for comparison were generated.

5.1. Validation statistics

The validation statistics used in this study comprised of several 'standard' rainfall model validation statistics, an extreme distribution plot and a plot of the annual rainfall distribution. Standard statistics were used here to describe those statistics which are considered of most importance – and are a requirement if the model is to be considered to be adequate for water resource studies. The standard statistics of previous studies (Cameron et al., 2000, Onof and Wheater, 1993) - dry probability, mean, standard deviation, coefficient of skewness and lag one autocorrelation of rainfall, mean and standard deviation of dryspells and wetspells were used as the primary evidence of a models ability in reproducing observed rainfall characteristics. In this study the skewness statistics were added for further validation, along with the various cross correlation statistics. These 'standard' statistics were calculated at 1, 12 and 24 hour aggregation levels (observed statistics were derived from the pluviograph data).

Intensity-Frequency-Duration plots were produced to examine the ability of the models to reproduce the distribution of annual extremes. The IFD curves were derived from the maximum intensity in each year for a given duration.

A plot of the annual rainfall amounts (versus exceedance probability) provided a test of the model ability to produce annual variability within the observed record.

As there were many plots produced as part of this study, not all plots will be presented or discussed here – see Frost et al. (2004) for further detail. Discussion of results here will focus on the model validation statistics on occasions where a particular model does not reproduce a given statistic (there is an obvious bias across all sites). Also, if a model reproduces an observed statistic poorly for a single site, it will also be discussed. A few plots will be presented here for one site only (Brisbane) – which was chosen as it was considered to represent the overall results well.

6. RESULTS

For all statistics plotted, the observed values were plotted as a point value. Simulated median (a thick line) and 90% confidence limits (thin lines) were also plotted. Models will be judged on their performance over all sites; however some individual sites may provide exceptions.

6.1. 1, 6 and 24hr standard validation statistics

Dry probability

Seasonality of dry probability was reproduced well by DRIP. Seasonality of dry probability was not as well produced by the NSRP (eg. 24 hr Brisbane). Neither constant underestimation nor overestimation was observed across all sites. However, the NSRP simulated series does vary around the observed values to a greater degree than for DRIP (eg. Hobart). This variability is surprising considering the dry probability was used in calibration of the NSRP.

Mean, Standard deviation, Skewness and Autocorrelation

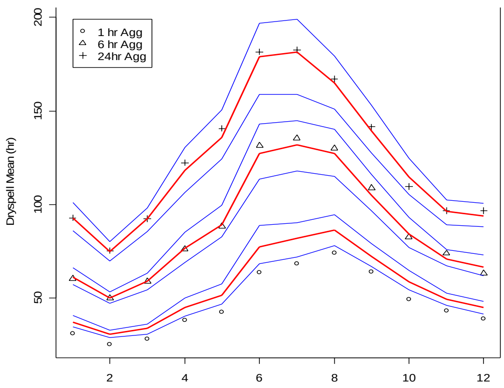
The observations lie close to the simulated DRIP and NSRP median (and within the 90% confidence limits) for the majority of sites/months. Although generally within the confidence limits, the DRIP simulations tended to show overestimation of 1 hour lag one autocorrelation, and underestimation of 24 hour autocorrelation. NSRP reproduced the observed values well, with the 1hr simulation occasionally significantly overestimating that observed (eg. Hobart).

Dryspell duration mean and standard deviation

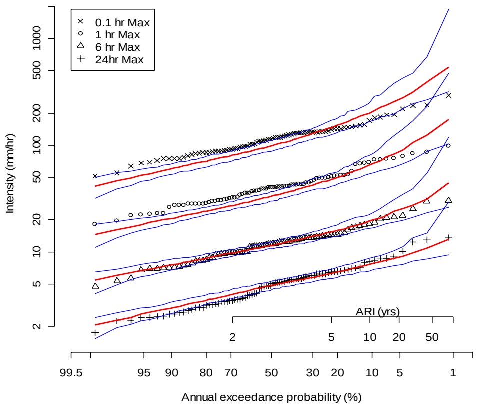
These were reproduced very well by DRIP for the 6 and 24 hour aggregation levels. DRIP 1 hour dryspell median mean duration was consistently above that observed – see Figure 3(a) for Brisbane. This is possibly a result of dryspells terminating/starting with negative values in the record being used in the observed calculation. This on average would shorten the observed mean dryspell length. The dryspell mean was reproduced very poorly by the NSRP model, especially at the 24 hour level (eg. Brisbane, Cairns, Hobart, Melbourne, Sydney) – displaying constant overestimation – see Figure 4(a) for an example. Marked overestimation of dryspell duration standard deviation occurred at all aggregation levels for the NSRP model.

Wetspell duration mean and standard deviation

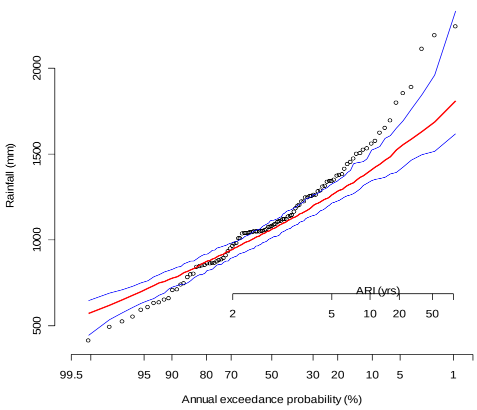
Seasonality of the wetspell mean/standard deviation was reproduced well by DRIP. Slight overestimation occurs at both the 1 and 24 hour levels. NSRP tended to overestimate the 24 hour wetspell mean, and underestimate the 1hr wetspell mean. The NSRP wetspell standard deviation showed a greater degree of variability for the majority of sites, whilst not matching the seasonality as well. Occasional months occurred which show huge overestimation (eg. Brisbane, Hobart).



(a) Monthly mean dry spell



(b) Intensity-Frequency_Duration

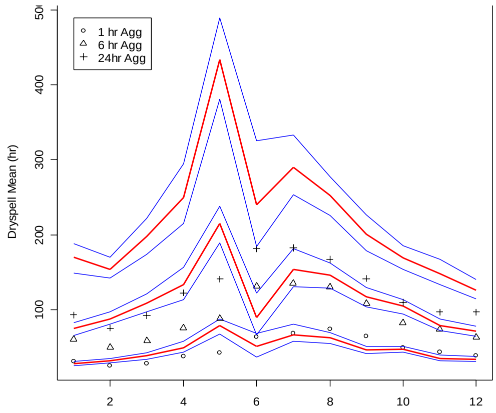


(c) Annual rainfall distribution

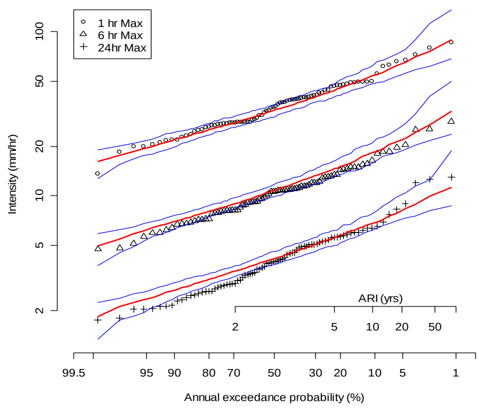
Figure 3. Simulated versus observed for DRIP

6.2. Intensity-Frequency-Duration curves

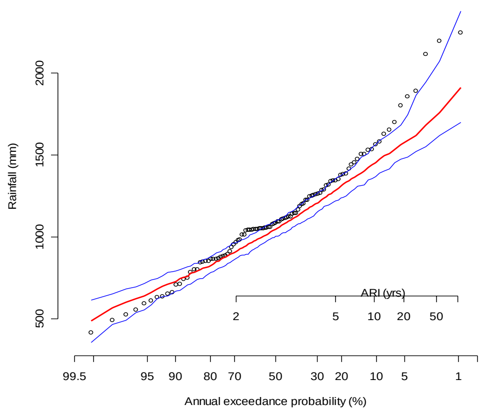
The annual maxima for a range of durations were extracted from the observed and simulated records. DRIP used a 6 minute moving bin width, while NSRP used a 1 hour bin width. This resulted in the annual maximum being greater on average for DRIP for a given duration, given the same observed record. This is not of consequence regarding the comparisons as consistent binning techniques for each model and data set were used – and it is relative differences between that observed and simulated that are important (rather than DRIP simulated versus NSRP simulated).



(a) Monthly mean dry spell



(b) Intensity-Frequency-Duration



(c) Annual rainfall distribution

Figure 4. Simulated versus observed for NSRP

The IFD curves are presented for DRIP at the 0.1, 1, 6 and 24 hour levels. For the 1, 6 and 24 hour aggregation levels, observed values for the majority of sites fell within the simulated bounds (even at high ARI's eg. 50yrs). Some notable exceptions were the 1 hour Melbourne, Sydney and Brisbane (Figure 3(b)) simulations showing underestimation for ARI's in the range 2-10yrs. DRIP's performance over the 1, 6 and 24 hour timescales was considered to be satisfactory. DRIP's 0.1hr simulations gave varied results. For the tropical sites (Darwin, Cairns, Townsville and to a lesser extent Brisbane) the upper tail of the distributions was markedly overestimated.

IFD curves were derived for NSRP at 1, 6 and 24 hour durations – eg. for Bribane in Figure 4(b). It was considered that the maxima distributions are preserved very well. IFD statistics at durations smaller than 1 hour are not presented as this was the minimum bin width for the model.

6.3. Annual rainfall distribution

Annual rainfall distribution: DRIP reproduced annual rainfall distributions satisfactorily for such a model, with the exception of Hobart and Perth. For the remaining sites the distributional shape was matched reasonably; however underestimation of annual variance is evident – see Figure 3(c). The NSRP performs better than DRIP in terms of annual distributional shape, with the majority of observations lying within the confidence limits (eg. Figure 4(c)). One exception to this quality of fit for the NSRP was for Perth, where the distribution is markedly underestimated. This could plausibly be due to the prevalence of missing values, possibly causing bias in the mean for particular months.

7. DISCUSSION

The study concentrated on evaluating two sub-daily rainfall generation models with respect to:

- preserving various 'standard' 1, 6 and 24 hour rainfall model aggregation statistics using pluviograph data as input;
- preserving extreme rainfall statistics - Intensity-Frequency-Duration curves;
- preserving other statistics at greater timescales (daily, monthly and annual) using daily data as input;

DRIP and NSRP used different calibration techniques, which focus on reproduction of different variables from the observed record. Given that subdaily modelling is the focus of this paper, it is imperative that short timescale 'standard' statistics are reproduced well. Reproduction of statistics at greater timescales is less important, but desired. Finally, the IFD curves provide a good indication of the models ability of simulation of extremes.

The results indicated that both models adequately preserve the mean, standard deviation and skew of historical rainfall at 1, 6 and 24 hour time scales. The two models did differ in the quality of fit to the dry probability; DRIP followed the seasonality displayed in the historical record closely, whereas the NSRP model showed an inferior fit. DRIP showed a good fit to the mean and standard deviation of dry spell durations. NSRP again shows an inferior fit to these statistics. For wetspell mean durations, both the models were evenly matched, however DRIP produced wetspell standard deviations more consistent with those observed than the NSRP. NSRP outperformed DRIP in terms of serial autocorrelation of rainfall.

Importantly, NSRP reproduced the IFD curves very well for the timescales presented (1, 6 and 24hr). DRIP allowed testing to a finer timescale (0.1, 1, 6 and 24 hrs). The majority of sites showed satisfactory reproduction of observed values over the 1, 6 and 24hr range – with the notable underestimation of Sydney and Melbourne 1hr maxima for ARI's from 2-10yrs. DRIP also severely overestimated the upper tail for the short durations (0.1hrs) for some tropical sites.

NSRP reproduced the annual rainfall distribution more satisfactorily than DRIP, with DRIP tending to underestimate the annual variance.

8. CONCLUSION

Two stochastic rainfall models were applied to pluviograph data obtained from 10 sites within Australia. The two models chosen for comparison were the Disaggregated Rectangular Intensity Pulse (DRIP) model of Heneker et al. (2001), and the single site version of the Neyman-Scott Rectangular Pulse (NSRP) process model of Cowpertwait et al. (2002). The models were evaluated on a monthly basis regarding their ability to reproduce certain 'standard' and extreme rainfall model statistics derived from the pluviograph record over a range of timescales (1, 6 and 24 hours). Other daily, monthly and annual statistics derived from the longer daily rainfall series were also compared.

Of the shorter timescale standard statistics, DRIP performed more adequately. The NSRP model performed poorly for wet- and dryspell statistics. DRIP's superior performance is attributed to DRIP being calibrated to interstorm durations and storm duration, which are closely related to mean wet and dryspell lengths. NSRP on the other hand is calibrated to a range of statistics, some of which may hinder the reproduction of these spell statistics.

Both models performed adequately at greater timescales of aggregation (for the statistics discussed). The NSRP performs better in regard to Intensity-Frequency-Duration curves. DRIP performing generally well with some exceptions for 1hr maxima for ARI's from 2-10 years (Sydney, Melbourne), and overestimating the upper tail significantly at short timescales (0.1hr) for some tropical sites. Further details and discussion can be found in Frost et al. (2005). Overall both the models performed equally well and the DRIP model has subsequently been incorporated into the CRC's Catchment modelling toolkit because it can generate sub-hourly rainfall and extensively tested in Australia.

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