Testing Uncertainty In A Model Of Stream Bank Erosion

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EXTENDED ABSTRACT

Sediment and nutrient loads in Australian rivers are a significant management concern. The National Land and Water Audit (2002) identified bank erosion as a major source of sediment, particularly in southern Australian systems. This paper tests a method of incorporating uncertainty into and the up-scaling of a cross-section scale stream bank erosion model. The cross-section scale model is based on an understanding of fluvial erosion and mass failure processes in which fluvial erosion is estimated using an excess shear stress approach while mass failure is estimated using a limit equilibrium analysis at the cross-section scale. Figure 1 shows a schematic of the model. A Monte-Carlo framework is used to propagate input uncertainty to output uncertainty in the model and to scale up to the reach scale.

Widely available databases are used to estimate variables for the two model components. A range of spatial information (GIS layers) is used to describe spatial variations in general properties such as soil type and catchment area. These are considered to be relatively well known (compared with cross-section geometry, geotechnical properties of the bank materials, riparian tree density, and hydrologic variables), although spatially coarse. A variety of empirical models and assumptions are used to transform the spatial information into model parameters, which are considered to be relatively poorly known. Two major challenges, which are related, involve incorporating the effects of natural variability along a river reach and estimating the uncertainty in the model inputs and the effect that this has on uncertainty in the model prediction. A Monte Carlo framework is used to achieve this. This involves developing a series of statistical models to predict the erosion model inputs and their (co)variability.

A hierarchical approach is used to develop these input models. An attempt is first made to construct a statistical model that predicts each model parameter from available spatial information using multiple regressions. Uncertainty in these parameters is incorporated using the regression error statistics. Where cross-correlations were found to be important, these were incorporated in the generation models. Where it was not possible to develop empirical relationships with available spatial data sets, a suitable parametric distribution is fitted for those input variables for which some data is available. Where no data were available for fitting a distribution, a distribution was assumed with a shape and parameters based on heuristic consideration of the relevant processes. Once both the erosion model and the various input models were established, the Monte Carlo technique was applied. This involves generating sets of the input variables of the model from the respective stochastic input models and the running the erosion model. This allows the probability distribution for the model output to be estimated for a location in the stream network. The model is tested using historical records of plan form change from a 40km reach of the Goulburn River downstream of Eildon Dam in Victoria, Australia. The results obtained from the model are promising; with bank erosion rates being predicted within a factor of two without calibration.

A series of sensitivity analyses (detail sensitivity analysis, scenario analysis, and advance sensitivity analysis) were conducted to identify key variables for predicting bank erosion rates using this particular bank erosion model. This suggested that bank angle, bank material physical characteristics, stream bed slope, and the high-flow flow regime (bankfull duration) control the behaviour of the model for loam bank materials.

1. INTRODUCTION

Streambank erosion and associated sedimentation and land loss hazards are a resources management problem of global significance. Streambank erosion is a dominant source of sediment in many river systems [e.g. 37% in the River Ouse, UK (Walling et al 1999); 50% in the Midwestern streams, USA (Wilkin and Hebel 1982); 78% in the Gowrie Creek, Murray Darling Basin, Australia (Howard et al 1998), 80% in the loess area of Midwest United States (Simon et al 1996); and up to 92 % (including channel scour) in Gelbaek stream, Denmark (Kronvang et al 1997)]. Streambank erosion is a nuisance in that it rapidly shifts stream courses laterally in numerous drainage basins [e.g. 14 m/year in the Cimarron River, Kansas, USA (Schumm and Lichty 1963), 50 m/year in the Gila River, Arizona and 100 m/year in the Toutle river system, Washington, USA (Simon 1992), and up to 824 m/year in the lower Meghna River, Bangladesh (Banglapedia, 2005).

In Australia, Streambank erosion is a major source of stream sediment loads, especially in the eastern Australian Rivers, in the Murray-Darling basin, in South Australia and in the south-western regions of Western Australia (National Land and Water Resources Audit 2002). The rate of stream bank erosion has increased markedly since European settlement (Land and Water Australia 2002). Sediment loads in streams have generally increased by 10 to 15 times in comparison with pre-European loads in intensively used river basins (National Land and Water Resources Audit 2002). Streambank and gully erosion have resulted in sand and gravel accumulation in 30,000 km of streams, including 30% of southern Western Australia and 20% of the Murray-Darling basin stream length, to such an extent that in-stream ecological health has been significantly impaired (National Land and Water Resources Audit 2002).

The above examples illustrate an overall picture of the worldwide bank erosion problem. There is a need for further development of simulation tools for analysing bank erosion to assist with management resource prioritization between various bank erosion management options and between bank erosion management and other catchment management priorities.

This paper tests a method of incorporating uncertainty into and up-scaling of a cross-section scale stream bank erosion model. It also identifies and presents an estimate of the contribution of various sources of input data and model parameter uncertainty to the model predictions. This will assist in both identifying key information requirements and gaps for predicting bank erosion and in improving parameterisation of catchment scale models.

2. THE BANK EROSION MODEL

We have developed a reach-scale stochastic model for predicting a long term average bank erosion rate based on an understanding of the fluvial erosion and mass failure processes at a cross-section and the variability of properties along a reach. This model assumes fluvial erosion and bank failure the key processes controlling bank erosion and would not be applicable where subaerial processes dominate. Figure 1 shows the structure of the model. Bank retreat is due to a combination of lateral erosion due to fluvial scour followed by mass failure where banks become unstable as lateral erosion, particularly at the bank toe, progresses.

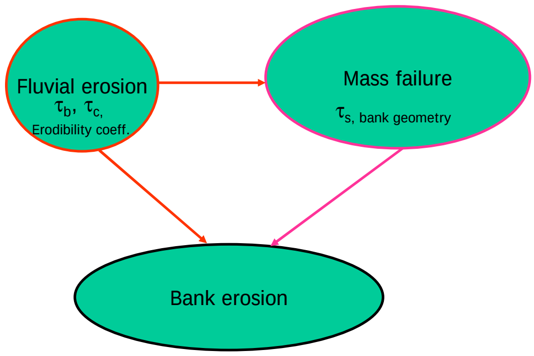

Figure 1. Structure of the bank erosion model

In the model, fluvial erosion is estimated using an excess shear stress approach (Foster et al 1977) and this continues until a mass failure occurs. The occurrence of mass failure is estimated based upon a limit equilibrium analysis assuming a planar failure mode. This is the typical failure mode in stream banks (Darby and Thorne 1997) and evident in field observations of Victorian Rivers, although rotational mass failure has also been found to be common in Australia (Abernethy and Rutherfurd, 1999). The influence of pore water pressure in banks, hydrostatic pressure on banks, riparian vegetation (apparent root cohesion and tree weight), and tension cracks are incorporated in the stability analysis. Increased shear strength due to matric suction (Fredlund et al 1978), while acknowledged, is not accounted for during the simulation. The volume of bank material entering the stream due to the fluvial erosion and mass failure processes is calculated separately. For shallow bank angles (less than the effective internal friction angle of the material) it is assumed that only fluvial erosion occurs. For steeper bank angles (greater than the effective internal friction angle) both fluvial scour and mass failure erosion are considered possible. The total bank erosion rate (*BE*, m/year) is simply the sum of the fluvial erosion rate and mass failure volume divided by the time taken for fluvial erosion to cause a mass failure.

3. MONTE CARLO SIMULATION

Because streams are highly variable and some of the processes involved are highly non-linear (eg the threshold interaction between mass failure and fluvial erosion), the variability of the various model inputs along a stream reach needs to be incorporated. This is done using the Monte Carlo simulation technique and the @RISK 4.5 software (Palisade 2002). The overall model development process involves a number of steps. 1) The cross-section scale fluvial erosion and mass failure model was developed (summarised above). 2) Stochastic models for simulating each of the erosion model inputs and parameters were developed (details are described below). 3) A Monte Carlo simulation was run.

In all Monte Carlo simulations, 65,000 sample sets were used, which led to minimal sampling uncertainty in the Monte Carlo results. This results in both an estimate of the long term bank erosion rate at the reach-scale and of the variability of the cross-section scale erosion rates. This variability represents an estimate of the real input and parameter variation combined with estimation uncertainty arising from the input parameter and variable generation models. Note that GIS layers of input variables were held constant while generating each realization.

4. STUDY AREA

The Goulburn River basin was used as a case study. The Goulburn River Basin has an area of about 17,000 km^2, (7.1 % of Victoria) and produces a mean annual discharge of 3,040,000ML (Department of Water Resources, Victoria, 1991). It drains north from headwater in the Great Dividing Range. Terrain varies from the mountains of the Great Dividing Range (Mt. Buller 1804 m) to the Riverine Plains of the lower Goulburn and Murray Valleys. Erskine et al (1993) discuss bank erosion in the Goulburn River catchment.

5. DATA SOURCES

Several data sources from both the state and national levels were explored when developing the stochastic input/parameter models. The most important were:

- Victorian stream flow records (Victoria Water Resources Data Warehouse)
- Digital geological information along the Victorian streams (the Department of Primary Industries, Victoria)
- Digital soil information from the Victorian Land System mapping (1:100,000 and 1:250,000 scale).
- The Victorian Statewide Assessment of Physical Stream Condition (SOS): Phase 1, (Ian Drummond & Associates Pty Ltd, 1985)
- Tree 25 for identifying vegetative banks in stream networks (the Department of Primary Industries, Victoria).
- The 9” (250m) AUSLIG DEM
- The 1:25000 scale digital stream networks.
- Data from a variety of geotechnical site investigations (Atterberg’s, limit) (limited)

6. TESTING DATA

The testing of model predictions is limited in the sense that no systematic databases on bank erosion rate exist, although global reviews by Rutherfurd (2000) and Walker and Rutherfurd (1999) suggest that bend migration rates for Australian Rivers are at the lower end of Global rates. Most available data comes from sporadic bank erosion measurements and from historical studies (Prosser et al 2003). An historical time series of channel plan forms and inferred bank movement rates for a 57 km section (divided into six reaches) of the Goulburn River below Eildon Dam (Michael Stewardson, pers. Comm.) is used here for testing the model. These data cover the period between 1935 and 1979 and are a revised version of those of Wilson et al (2005). One reach was excluded due to erroneous results. It should be noted that model performance in low-order streams is not tested.

7. TESTING RESULTS

The long term average annual rate of bank erosion (*BE*, m/year) was simulated for the river reaches two, three, four, five, and six (in order downstream from Lake Eildon). Reach 1 was excluded due to an obvious error in the observed data (negative erosion). Figure 2 shows a comparison of the simulated bank erosion rates with the bank retreat rate observations. No calibration has been undertaken. Clearly the results are of similar magnitude as the observations and the ranking of the reaches is well predicted. However there is some bias. Figure 3 gives an indication of the variability of simulated bank erosion (BE_s) by reach. The mean, the mean + 1 standard deviation, and the 95th percentile range are shown (the mean - 1standard deviation and the 5th percentile are always zero). Note that the plot is truncated at zero and the 5th percentile is always zero. Figure 3 indicates that reaches five and six have both much lower mean erosion rates and a smaller range in erosion rate variations.

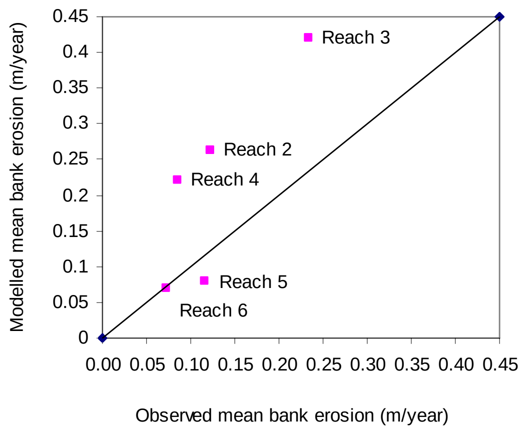

Figure 2. 1:1 plot of observed versus modelled mean bank erosion rate by the river reaches.

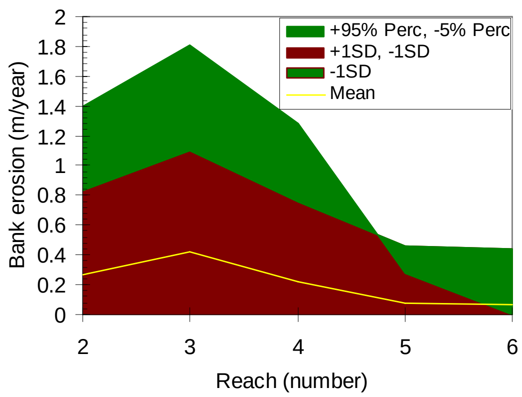

Figure 3. Summary plot showing the bands of the bank erosion at the reaches (BE_s at ± 1 standard deviation, 95th and 5th (always zero) percentile values).

8. SENSITIVITY OF THE MODEL

In order to see what controls the model behaviour, the sensitivity of the model is tested. Preliminary sensitivity analysis had suggested that predicted bank erosion rate is most sensitive to bank angle (Jha et al 2004). However, more detailed sensitivity analyses suggest that other variables are also important in explaining the variability of the model predictions.

In the Monte Carlo framework, the sensitivity of the model to particular variables can be assessed by evaluating the correlation between the various inputs and the model output. Spearman rank correlation coefficients are used. The results are displayed using a correlation Tornado plot (figure 4), which plots the correlation coefficients for the various inputs in order of magnitude (i.e. from largest to smallest sensitivity). This analysis was conducted for each reach and it shows that in reaches two, three and four, the bank angle (θ), soil erodibility coefficient (M), and duration of bankfull flow (t) are the most sensitive variables. In the reach five, the critical shear stress (τ_c) replaces *M*, among the most sensitive variables, while only t and θ are important in Reach 6.

To further explore the sensitivity, an analytical method called scenario analysis is performed. Scenario analysis examines what input variable values are associated with particular ranges (subsets) of output values (eg. outputs in the fourth quartile). Unlike the general sensitivity analysis above and the advanced sensitivity analysis below, this identifies the key variable groups and their values associated with particular outcomes (e.g. high simulated erosion). It is based on comparing the median of an input variable for the subset of realizations of interest with the median of the same input variable for all realisations. For each input, when the absolute difference between the overall median and the subset median is greater than half the standard deviation of that input variable (over all realizations) then the input is considered to be sensitive.

Three scenarios are considered – the lower quartile, upper quartile and the upper decile of simulated bank erosion rates. Table 1 shows the most sensitive variables by reach for the three scenarios considered. θ, M, τ_c and t remain the most sensitive variables.

The sensitivity results from the above two analyses are limited in that these methods have considered only those inputs which vary from realisation to realisation. Some variables, namely the riparian tree cover density (*Rd*), upstream catchment area (*CA*) (a predictor of stream flow), stream bed slope (*S*) and all other GIS layers are held constant during the model simulations. In order to understand the sensitivity of the model to all input variables, another approach called Advanced Sensitivity Analysis, is carried out that has incorporated these fixed variables in the analysis.

In the advanced sensitivity analysis, the base values of all the input variables of the model are changed by various amounts (-10%, -5%, 0%, +5%, and +10% of the base value) and the sensitivity of the model is calculated using the change in the mean BE_s in the five reaches. For this analysis 15,000 realisations were used. The results of the Advance Sensitivity Analysis indicate that the S is also one of the most important variables in reaches 2, 3 and 4, but not in reaches 5 and 6 which have very low slopes (0.0257% and 0.0001% respectively).

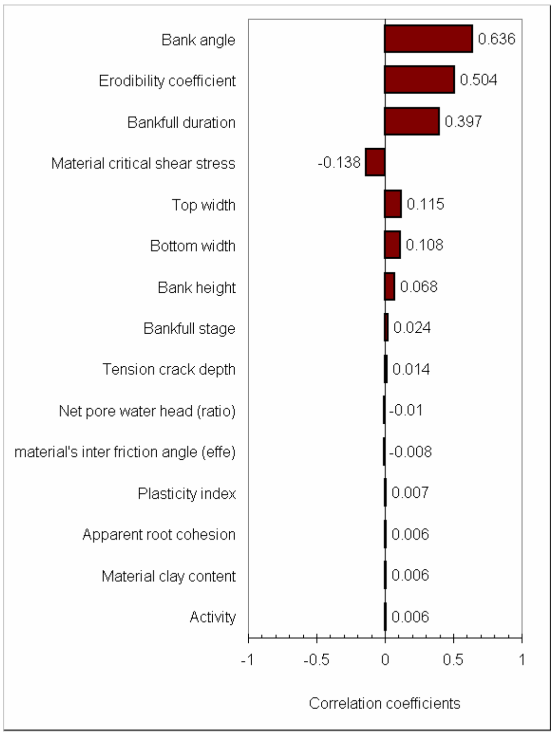

Figure 4. Correlation Tornado plot between the inputs and the bank erosion rate for the reach two. Longer bars at the top show the most sensitive variables.

In the above analyses, riparian trees have been assumed to affect bank stability and failure but not the fluvial erosion processes. In those scenarios tree density did not have an important effect of bank erosion rates. Some literature (eg Millar and Quick 1998) suggests that τ_c can increase by a factor of almost three under well vegetated bank conditions. To investigate this effect some additional model runs incorporating a dependence between *Rd* and τ_c were undertaken. While the sensitivity to riparian vegetation increased, the model still showed fairly low sensitivity to *Rd*.

The final step in exploring the models sensitivity involved checking the dependence of the mean BE_s on the subjective (assumed) distributions. The change in the mean BE_s resulting from changes in the shape of the subjective distributions of the insensitive variables was checked [e.g. uniform and triangular (subjective) distributions for tension crack depth (*K*) and riparian tree weight stress (*wt*); the shape of clay content of bank material (*Cl*) distribution ~ BetaGeneral (3,3, 5%, 20, 95%, 40)] by re-running the detailed Sensitivity Analysis for reach two as detailed above. This analysis showed that the mean BE_s only changes slightly (less than 1%) and that the results are robust to the selection of the shape of these distributions.

Table 1: Scenario analysis of bank erosion simulations based on conditional median.

Reach	Scenario	Bank angle	Erodibility coefficient	Bankfull duration	Critical shear stress
Two	≤ 25 %	-0.55	-0.68	-0.59	--
	≥ 75 %	0.92	0.76	0.55	--
	≥ 90 %	1.67	0.83	0.62	--
Three	≤ 25 %	--	-0.75	-0.67	--
	≥ 75 %	0.91	0.657	--	--
	≥ 90 %	1.54	0.65	--	--
Four	≤ 25 %	-0.57	-0.64	-0.53	--
	≥ 75 %	0.94	0.73	0.58	--
	≥ 90 %	1.79	0.88	0.68	--
Five	≤ 25 %	-0.53	--	-0.64	0.61
	≥ 75 %	0.99	--	--	-0.77
	≥ 90 %	2.25	0.64	--	-0.71
Six	≤ 25 %	--	--	-1.05	--
	≥ 75 %	0.89	--	1.05	--
	≥ 90 %	2.31	--	1.08	--

These sensitivity explorations (in five reaches of the Goulburn River) confirm that only a limited number of the model's variables are important in determining BE_s. These variables include θ, M, S, τ_c and t (not necessarily in order). The variability of these inputs needs to be well represented for the model to produce realistic simulations of bank erosion and its variability.

Additional sensitivity analyses of the model in other streams and creeks within the Goulburn River basin has also found that θ, M, τ_c, S and t are the most important variables in the model (when the bank soil texture is loam, as in the above cases). The sensitivity scenario, however, is not consistent for other bank materials.

There are significant differences in the mean BE_s (Figures 2 & 3) between the five reaches with reach three having the highest BE_s and reach six the minimum. Of the five most sensitive variables (θ, M, S, τ_c and t), only S varies between the reaches. The statistical distributions of the others are the same for each reach. Reach three has the steepest estimated bed slope (0.025%) and it exceeds the estimated slopes of reaches 2, 4, 5, and

6 by factors of 1.7, 2.0, 5.5, and 1430 respectively. Some other variables are also different but they do not have much effect on the mean BE_s.

9. SUMMARY AND CONCLUSION

A Monte Carlo approach is used to both identify and estimate uncertainty in a cross-section-scale stream bank erosion model and to scale the results up to the reach-scale. The erosion model is based on an understanding of cross-section scale fluvial erosion and mass failure processes and a stochastic upscaling. The model input variables are divided into relatively well known and poorly known variables. Reach slope, catchment area and riparian tree density are considered to be relatively well known, while the rest of the input variables (cross-section geometry, geotechnical properties of the bank materials, riparian tree density, and bankfull flow duration) are considered poorly known. Poorly known input variables are treated stochastically using either empirically-based stochastic prediction models, fitted parametric distributions or subjectively chosen statistical distributions. A Monte Carlo framework is used to combine the inputs and the erosion model in the software @RISK 4.5. The final output includes both an estimate of the reach-scale long-term average bank erosion rate and an estimate of the combination of actual variability and input data related uncertainty of cross-section scale bank erosion rates within the reach.

The model is tested against an independent dataset of long term average bank erosion rates (m/year) along 40 km (five reaches) of the Goulburn River downstream of Eildon Dam, Victoria, Australia. Given the absence of any model calibration, the results are very promising with erosion rates being estimated within a factor of 2. However, further testing against data sets for other rivers is required before the typical accuracy of the estimated erosion rates.

A variety of sensitivity analyses have been carried out to find what the most important controls on the model behaviour are. The results of the analyses suggest that five variables namely θ, M, S, τ_c and t (not necessarily in order) are the most important variables (for medium textured bank materials). Good knowledge of these is indispensable for reliable prediction of bank erosion. It should be noted that these observations are drawn from limited exploration. More detailed exploration of other bank materials and environmental settings is underway that will allow conclusions to be drawn about the key variables that control bank erosion in systems dominated by fluvial erosion and mass failure.

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