Comparing rainfall interpolation techniques for small subtropical urban catchments

[1]Y. Knight. [1]B. Yu, [1]G. Jenkins and [2]K. Morris

[1]School of Environmental Engineering, Griffith University, Nathan, Qld, 4111 Australia
[2]Brisbane City Council, Qld, 4000 Australia, E-Mail: yvonne.knight@bigpond.com

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EXTENDED ABSTRACT

Rainfall estimation is an integral component of hydrologic modelling for reliable flood forecasting and for assessing the impact of precipitation and runoff on water quality and ecosystem health in urbanised areas.

Spatial variability of rainfall adds to the complexity of estimating rainfall at the catchment scale and is a key factor that must be incorporated into estimations of rain fields. Rainfall interpolation from point measurements is one approach that hydrologists use to account for spatial variation in rainfall.

This study focuses on evaluating the suitability of three interpolation methods in terms of their accuracy, for small urban catchments. The Bridgewater Creek catchment in south Brisbane is the region of study. Thirteen storm events measured over 24-hours are interpolated using an inverse-distance weighted average method, thin plate smoothing spline, and an ordinary kriging technique. The delete-one validation method is used for comparing the 3 interpolation techniques.

The interpolation methods were applied using 3 different software packages. The inverse-distance weighted method (IDW) was implemented using Microsoft Excel. The spline interpolations were carried out with the ANUSPLIN software while the kriging method was completed using Geostatistical Analyst in ArcGIS. Since ANUSPLIN and Geostatistical Analyst both require a minimum number of data points for computation, it was necessary to consider a reduced set of storms for interpolation by the thin plate spline and kriging methods.

The thin plate spline and ordinary kriging interpolation techniques were found to have comparable estimation accuracy when individual storm events were considered. When compared to gauge-based rainfall measurements, the estimation error for these methods was approximately 23 mm, which corresponds to 30% of the observed mean rainfall. The IDW method was observed to display more frequent occurrence of extreme estimation error up to 98% for individual storms.

However, an analysis of an aggregated set of storm events for which all 3 methods could be applied showed that the IDW had the lowest estimation error and highest model efficiency of the three interpolation techniques for the storms selected.

1. INTRODUCTION

'The entire circulation of water in a catchment basin is governed by the spatial and temporal distribution of rainfall' (Bacchi and Kottegoda 1995). Rainfall data are required to determine the rate of accumulation of surface and groundwater, the infiltration dynamics of the soil surface of a catchment and the rate of evapotranspiration.

Hydrological modelling utilises rainfall data to predict flood events and determine ecosystem health of an area. Urban water quality models also rely on well-estimated rainfall data to accurately simulate pollutant release and subsequent transport mechanisms in catchments (Chaubey *et al* 1999). Rainfall is a key variable for all these types of modelling and thus it is important to have accurate estimations of rainfall. Without accurate rainfall estimation, hydrologic models will produce inaccurate results (Obled and Wendling 1991; Chaubey *et al* 1999; Berne *et al* 2004; Donohue *et al* 2005; Merritt *et al* 2005; Moore *et al* 2005; Pardo-Iguzquiza and Dowd 2005).

'Estimating the spatial distribution of rainfall from point estimates depends on the existing spatial relationships of the measured point values' (Bacchi and Kottegoda 1995). These are often statistical relationships, as physical knowledge is hard to obtain. For example, it is often found that two stations within a few kilometres of each other experience similar storm events. This suggests that areas in close proximity of each other have similar rainfall characteristics, which can be supported experimentally (Bacchi and Kottegoda 1995). Conversely, it has been noted that in some instances, a rain event may effect one gauge station and entirely by-pass a station within a few kilometres.

Spatial variability of rainfall adds to the complexity of estimating rainfall and is a key factor that must be incorporated into estimations. There are a number of approaches that hydrologists use to account for spatial variation in rainfall estimates.

Meteorological radars are one method used to measure spatially distributed rainfall. Radars give a large-scale vision of precipitation fields compared to scattered point estimates from rainfall gauges (Bacchi and Kottegoda 1995). However, radar technologies are not available in all catchment areas due to the sophisticated and costly equipment that is involved. In addition, resolution of radar data is often too coarse for small urban catchments.

A second and more viable approach for hydrologists and meteorologists to determine mean areal rainfall is to interpolate data from irregularly spaced rain gauges within a catchment (Dirks *et al* 1998). There are many interpolation methods that can be used, each of which varies in their degree of complexity and predictive accuracy.

Furthermore, the accuracy of rainfall computed from spatially averaged rainfall using point sources compared to rainfall fields using radars, varies depending on the scale of the basin in question and the rainfall variability (Arnaud *et al* 2002).

The focus of this study is to evaluate interpolation methods for determining rainfall in a small subtropical urban catchment with a relatively dense rain gauge network. Brisbane is serviced by a dense network of gauges similar to that in many Australian capital cities. Hence, it is appropriate that interpolation techniques rather than estimation using meteorological weather radars be used to estimate rainfall for an area surrounding the Bridgewater Creek catchment, Brisbane. A network of 11 rainfall gauges in a region bounded by the Gateway Arterial road, South-East freeway and the Brisbane River was used for interpolation. The specific locations are as illustrated in Figure 1. The code names used by Brisbane City Council are indicated in Figure 1 for each site.

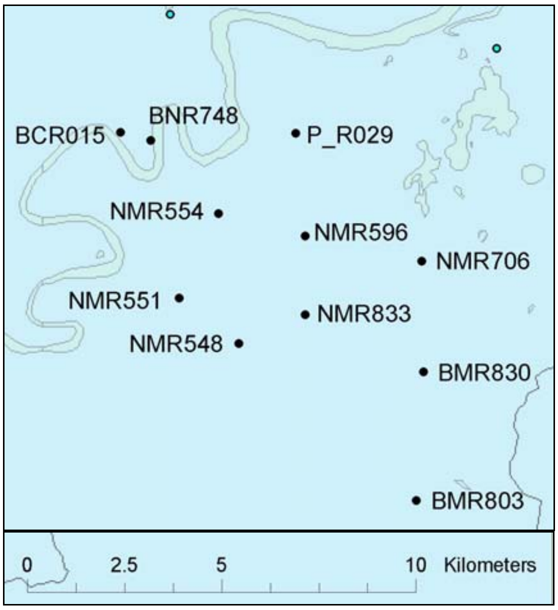

Figure 1: Map showing locations of rain gauges in Southeast Brisbane used for interpolation.

The objective of the study was to identify the most accurate method for interpolation for the small urban catchment in Brisbane. Three interpolation methods will be investigated using the 'delete-one' validation technique. These include an inverse-distance weighting method, thin plate smoothing splines and an ordinary kriging technique.

2. METHODOLOGY

2.1. Data and Storm Selection

Rainfall data were collated for 11 gauges in the Brisbane area. The rain data obtained were recorded with tipping-bucket rain gauges having a 1-mm resolution. Processed rainfall data were supplied by the City Design department of Brisbane City Council for this study.

Thirteen storm events since 1995 were selected for interpolation. The storms selected had recorded the 13 highest maximum 1-hour rainfall at the Norman Creek (East Brisbane) rain gauge (NMR554). NMR554 is located in the Bridgewater creek catchment. The response time of small catchments such as Bridgewater creek is assumed to be approximately 1 hour due to their small area (Morris 2005). Storm events prior to 1995 were not considered, as the rain gauge network in Brisbane before 1995 was not extensive.

The 'cleaned' raw rainfall data consisted of rainfall measurements for every 1 minute interval over 24-hours (midnight to midnight). These were summed for each storm to give the total rainfall depths recorded at each gauge location in a 24-hour period surrounding the storm event. The 24-hour rainfall varied from 9mm to 222 mm among the 11 x 13 site-events.

2.2. Delete-one Cross Validation

The delete-one method is a cross-validation process that eliminates bias in interpolation methods which generally exists when all data points (*i=1,2… n)* are used to predict a value at a point *i* (Wang and Zidek 2004). The delete-one method involves removing one observation point (*j*) at a time from the whole data set to estimate a value of rainfall at *j* from the remaining (*n-1)* data points (Tomczak 1998).

2.3. Qualitative Measures of Estimation Accuracy

Four statistics are used to characterize the performance of interpolation methods in this study. They are: RMSE, RMSE as a percentage of the mean observed rainfall, bias and model efficiency. These statistics are described in equations (1.1), (1.2), (1.3) and (1.4).

Model Efficiency (E):

$$E = 1 - \frac{\sum_{i=1}^{M}(O_i - E_i)^2}{\sum_{i=1}^{M}(O_i - \bar{O})^2} \qquad (1.1)$$

Where:
M = no. storm events
O_i = Observed rain at i
E_i = Estimated rain at i
$\bar{O}$ = Mean observed rainfall for (i=1,2…M)

Root mean square error (*RMSE*) in millimetres:

$$RMSE = \sqrt{\frac{1}{M}\sum(O_i - E_i)^2} \qquad (1.2)$$

Root mean square error as percentage of the mean:

$$\frac{RMSE}{\bar{O}} \times 100\% \qquad (1.3)$$

Bias:

$$Bias = \frac{\bar{E}}{\bar{O}} \qquad (1.4)$$

Standard error of interpolated surfaces are often expressed as a percentage of the mean, due to the nature of the distribution of precipitation (Hutchinson 2004). The RMSE of estimated rainfall is based on the size of the storm and hence it is useful to quote the RMSE as a percentage of the average rainfall.

Exact interpolation methods are indicated by RMSE and RMSE/$\bar{O}$ values of zero. Accordingly, the most accurate interpolation methods are indicated by RMSE and RMSE/$\bar{O}$ values closest to zero. It should be noted that for interpolation of precipitation data, estimation error is unlikely to be zero. Hutchinson (2004) states that standard errors of fitted surfaces should be approximately 10% for monthly mean precipitation data when adequate gauge networks are available.

Model efficiency (E) equals 1 when observations and estimations are in prefect agreement. Model efficiency can be less than zero when the model estimations are worse than using the average of observed rainfall as an estimator. For high model efficiency, the mean squared error (MSE) of the rainfall estimations will need to be small relative to the standard deviation of the observed rainfall.

Interpolation results that indicate no bias have a bias statistic value of 1. That is, the average rainfall estimate is equal to the average observed rainfall. Bias values greater than one indicate that the estimated rainfall is generally overestimated, while values less than one suggest that the interpolation method resulted in underestimation.

2.4. Inverse Distance Weighted Average

The inverse distance weighted average method (IDW) assumes that rainfall at a gauged point compared to an ungauged point is inversely proportional to the distance between the two points (Chow *et al* 1988). An exponent of one was used for the inverse distance relationship.

The IDW method was executed in MS Excel spreadsheets for each of the 13 storm events. The interpolation method was carried out manually.

2.5. Thin Plate Smoothing Splines

After evaluation of a number of options, the program chosen for fitting thin plate smoothing splines was ANUSPLIN 4.3 developed by M.F. Hutchinson (Hutchinson 2004). ANUSPLIN 4.3 is a set of FORTRAN programs developed at Australian National University that calculates and optimises thin plate smoothing splines to data sets of unlimited size and distribution (Price *et al* 2000). ANUSPLIN is widely used and has been applied in a number regions including Australia, New Zealand, Europe, South America, Africa and China (Hutchinson 1991). ANUSPLIN has been used to interpolate daily rainfall and other weather variables to create and update climate databases at a 0.05^{o} resolution for Australia (Jeffrey *et al* 2001). Other available programs include ArcGIS, which was used for the kriging method, but it was considered desirable to use ANUSPLIN as an alternative methodology, with runtime advantages since it runs under DOS rather than Windows.

SPLINA is one of the FORTRAN programs and is suitable for fitting thin plate smoothing spline functions to data sets with up to 2000 points. LAPPNT calculates the values and error estimates of the fitted spline at specified points.

Hutchinson (2004) describes the method as a 'generalised multivariate linear regression in which the parametric model is replaced by a suitably smooth non-parametric function'. ANUSPLIN is based on smoothing splines as described by Wahba (1990), Hutchinson (1984) and Wahba and Wendelberger (1980).

2.6. Ordinary Kriging

ArcGIS Geostatistical Analyst was chosen as the tool to implement the kriging interpolation method. Geostatistical Analyst is an extension of ArcMap used to generate surfaces from point data. The software is a powerful, user-friendly package and is flexible for implementation.

Surface fitting using Geostatistical Analyst involves exploratory spatial data analysis, calculation and modelling surface properties of nearby locations, and surface estimation and assessment of results (Johnston *et al* 2003). The ordinary kriging interpolation method does not rely on data being normally distributed.

2.7. Reduced data set

Both the thin plate spline and kriging methods resulted in predicting rainfall for a reduced set of storm events due to the computational requirements of the ANUSPLIN and ArcGIS programs. Thus, an additional approach for the comparison of the three interpolation methods was introduced to analyse model performance. A set of 4 storm events, which could be investigated using all of the interpolation methods, was considered. The 4 storm events were plotted together to give a representation of the overall model performance. It is anticipated that the findings from combining the points of all storm events to estimate model efficiency may vary from the results obtained by examining the model efficiency associated with individual storm events.

3. RESULTS AND DISCUSSION

Table 1 shows the average error statistics for each of the three interpolation methods based on the all storm data available. The numbers of individual storm events on which these statistics are based are indicated for each interpolation method.

Table 1: Error statistics for 3 methods using all data available.

Method	*RMSE (mm)*	*RMSE /O (%)*	*Bias*	*E*	No. Storms
IDW	21.5	42.4	0.941	-0.51	13
Spline	22.5	31.4	1.004	-0.26	8
Kriging	25.5	29.3	1.017	0.18	4

Table 2 shows the quantitative measures of errors for the three interpolation methods based on a consistent set of 4 storms. The errors are aggregated for the four storm events.

Table 2: Error statistics for 3 methods using consistent data set for 4 storm events for comparison

Method	*RMSE (mm)*	*RMSE /O (%)*	*Bias*	*E*
IDW	25.0	30.7	0.996	0.40
Spline	28.8	35.4	1.001	0.20
Kriging	25.5	29.3	1.017	0.18

Figures 2, 3 and 4 are comparisons of estimated and observed rainfall for 4 storms events and all gauge estimations. The figures represent the IDW, thin plate spline and kriging methods respectively.

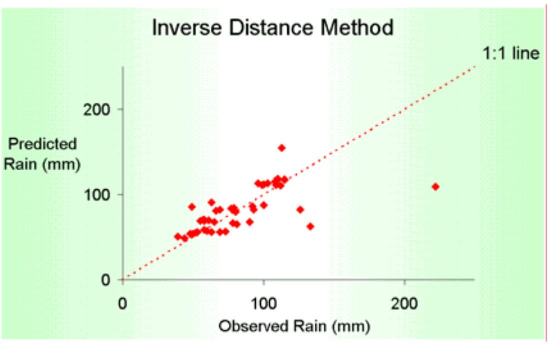

Figure 2: Comparison of estimated and observed rainfall for 4 storms events and all gauge estimations using the IDW method.

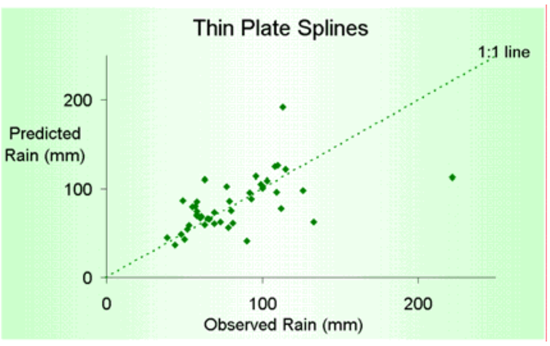

Figure 3: Comparison of estimated and observed rainfall for 4 storms events and all gauge estimations using the thin plate spline method.

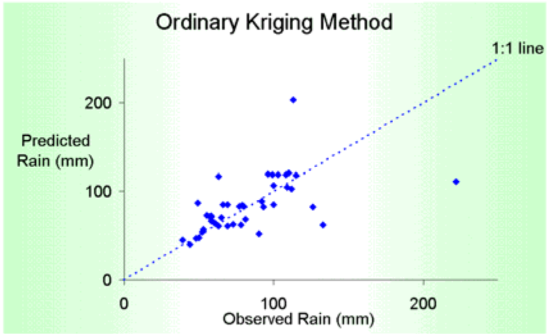

Figure 4: Comparison of estimated and observed rainfall for 4 storms events and all gauge estimations using the ordinary kriging method.

Estimation errors of individual storm events were considerably different from errors calculated using a set of 4 aggregated storms as indicated in tables 1 and 2.

Consideration of individual storm events shows that the spline and kriging methods provide more accurate estimations than the IDW method. More frequent occurrence of extreme error was observed with the IDW method. The spline and kriging methods produced errors of approximately 30% of the mean rainfall. Estimated rainfall using these 2 methods was found to be relatively unbiased.

For the aggregated storm set of 4 events, the IDW and kriging methods were found to perform comparably to each other. Contrastingly, the spline method had the highest percentage RMSE. The estimation error for the IDW and kriging method was approximately 30% of the mean. The IDW method was found to have the greatest model efficiency of all three methods.

Accordingly, it appears that when larger sample size of measurements is selected, the IDW method improves substantially in comparison to the spline and kriging methods.

4. CONCLUSIONS

Three interpolation methods have been investigated in this study in the context of small urban catchments. Their performance in terms of their accuracy in rainfall estimation has been analysed in detail.

The inverse-distance weighted average interpolation technique was found to involve the most frequent occurrence of extreme error for rainfall estimation for individual storm events. The average RMSE for the method was calculated to be 22mm or 42% of the observed rainfall mean. For individual storm events, the spline and kriging methods have similar predictive capacity producing errors of approximately 30% of the mean.

Analysis of an increased sample size of estimated and observed rainfall by aggregating data from a set of 4 storms, showed that the reduction of random error associated with IDW estimations served to improve model efficiency. RMSE values of 26mm (30% of the mean of the observed events) were recorded for interpolation using ordinary kriging. Model efficiency was greatest for the IDW method. The thin plate spline method using ANUSPLIN software was found to produce errors of 29mm, i.e. 35% of the mean the observed events.

Relatively high estimation error for all three interpolation methods is attributed to the time scale used in this study, namely 24-hour event rainfall. Daily rainfall is noted to be more variable than monthly and annual measurements, thus making it harder to estimate.

In the context of Brisbane's small urban catchments, the IDW method was found to be the most accurate and reliable interpolation method to be used. The IDW method was considered superior to the kriging method for this application due to the small set of gauges analysed in the study.

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