A method for defining conditional probability tables with link strength parameters for a Bayesian network

[1]Kokkonen, T., [1]H. Koivusalo, [1]H. Laine, [2]A. Jolma and [1]O. Varis

[1]Laboratory of Water Resources, Helsinki University of Technology, [2]Laboratory of Cartography and Geoinformatics, Helsinki University of Technology, E-Mail: Harri.Koivusalo@tkk.fi

Keywords: *Bayesian network; conditional probability table; expert elicitation; learning*

EXTENDED ABSTRACT

In a Bayesian network the state distribution of a node is conditionally independent of the set of all its nondescendents given the set of all its parents. The conditional probability table (CPT), which relates states of the parent nodes to those of a child node, includes entries for all possible combinations of the child and parent node states. Already with moderate dimensions the size of the CPT becomes so large that eliciting the required conditional probabilities from a panel of experts becomes impractical. Motivation of this paper is to make expert elicitation more feasible for populating CPTs in discrete state Bayesian networks. To achieve this, a method is presented that specifies how the CPT can be described with the aid of link strength parameters that are assigned to each link from a parent node to a child node, and that attain values from -1 to 1. In addition to aiding parameterisation of the CPT using expert knowledge, the link strength approach also provides a means to derive complete CPTs from training data when not all state combinations of parent nodes are present in the training data set.

The method presented in this paper relies on the concept of the link strength introduced in Varis and Kuikka (1994) and the generalised Noisy-Or model of Srinivas (1993). A link strength with a value of one characterises a perfect one-to-one relationship between a parent node and the child node, and a link strength with a value of zero indicates that the two nodes are completely independent. A negative link strength indicates that there is a negative relationship between the two nodes. The generalised Noisy-Or model provides a methodology for constructing the CPT for the child node given its parent nodes using a set of parameters whose number is equal to the sum of parent node states. This paper combines the link strength concept and the generalised Noisy-Or model to outline a method where the number of parameters required for defining the CPT is reduced to the number of parent nodes.

The link strength approach suggested in Varis and Kuikka (1994) has been modified to better suit for variables that are defined in the ratio scale statespace. In environmental studies concentrations and loads are typical examples of such variables. Then it is reasonable to assume that those states that are close to the most likely state have a higher probability of occurrence when compared to states that are far from it.

To ensure that the relationship between link strength parameters and the resulting CPT is intuitively reasonable, the following properties are targeted. First, inclusion of a parent node with a link strength value of zero should give an identical CPT to the case where that node is absent. Second, equal link strength values should result in an equally strong effect on the child node. Third, the effect of a negative link strength should have the same magnitude as the effect of an equally strong positive link, but be in the opposite direction. And finally, when all link strengths are zero, the CPT should be non-informative, i.e. all probabilities are equal to the inverse of the number of states in the child node. Simple examples are presented to demonstrate how link strength values are converted into CPTs, and the resulting CPTs are discussed in the light of the properties listed above.

The main contribution of this paper is to suggest a new method for describing the CPT with link strength parameters, whose number is equal to the number of parent nodes. This method is envisaged to aid in 1) defining CPTs based on the information acquired from an expert panel, and 2) deriving complete CPTs from training data when not all state combinations of parent nodes are present in the training data set.

1. INTRODUCTION

Bayesian networks provide a useful method for simplifying the assessment of the joint distribution of a set of random variables by exploiting conditional independencies among these variables. Also, the graphical nature of Bayesian networks is convenient as it facilitates the users to get a visual overview of the problem at hand. Bayesian inference has been extensively applied in many fields including medicine (e.g. Chevrolat *et al*. 1998), computer science (e.g. Heckerman *et al*. 1994), and environmental sciences (e.g. Ames 2002, Borsuk *et al*. 2004, Varis and Kuikka 1994).

In a Bayesian network, the Markov condition requires that the state distribution of a node is conditionally independent of the set of all its nondescendents given the set of all its parents. The conditional probability table (CPT), which relates states of the parent nodes to those of a child node, includes entries for all possible combinations of the child and parent node states. Now, if a child node has p parent nodes, and both the child node and its parents have s states, the total number of entries in the CPT becomes s^{p+1}. Already with moderate dimensions (e.g. p = 3 and s = 5), the size of the CPT is so large (625 entries) that eliciting the required conditional probabilities from a panel of experts can become difficult and time-consuming. This paper presents a method for specifying how the CPT can be described with the aid of link strength parameters that are assigned to each link from a parent node to a child node, and that attain values from -1 to 1. The method applies to Bayesian networks where nodes have discrete states.

In addition to parameterising the CPT using expert knowledge, the link strength approach provides a means to populate also those rows in the CPT that remain empty after initial application of training data. This is necessary as the training data may not include all combinations of parent node states. Assume, for example, that a lake water quality variable is explained by the amount of runoff and the percentage of peat land in the catchment draining to the lake. Now, if there are data from a set of lakes, it can happen that in the most peat dominated catchment there has not been a dry summer during the measurement period, and hence the row in the CPT corresponding to a high peat percentage and a low runoff remains empty even after using training data.

2. METHODOLOGICAL BACKGROUND

2.1. Noisy-Or model

Pearl (1988) presented a method which considers the case where any one of a set of conditions is likely to cause the same effect in the child node, and the likelihood of the effect does not diminish when several of these conditions are present simultaneously. This method is called the Noisy-Or model (Figure 1), and it presumes each node to be Boolean having only two states: true or false.

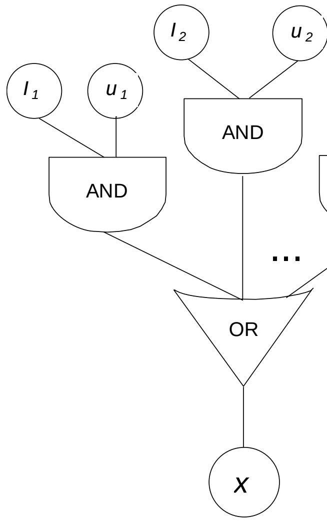

Figure 1. Schematic of the Noisy-Or model. Notation: I_i is the i^{th} inhibitor mechanism, u_i is the state of the i^{th} parent node, and x is the state of the child node.

In the Noisy-Or model it is sufficient to determine the conditional probabilities between the child node and each of its parent nodes (causes) separately. These probabilities are typically much easier to identify than the conditional probabilities between a child node and the set of all its parents. Moreover, given one parent at a time the number of conditional probabilities increases linearly with an increasing number of parent nodes (Neapolitan 2004), as opposed to an exponential increase in the case when the dependency between the child node and all its parents is defined concurrently.

2.2. Generalised Noisy-Or model

Srinivas (1993) generalised the Noisy-Or model to the case where both the parent and child nodes can have any number of discrete states. Figure 2 shows the schematic of the generalised Noisy-Or model. The state of each parent node u_i is passed through a line failure device N_i that yields as an output u_i', which can (in case of failure) be different to u_i.

The symbol i is the index of a parent node. The function F maps the output from all failure devices, $\mathbf{u}' = [u_1', u_2', \ldots, u_n']$, to the state of the child node x where n is the number of parent nodes. The aim is to create the CPT $P(x|\mathbf{u})$ relating the states of the parent nodes $\mathbf{u}$ and the states of the child node x with aid of the failure devices $\{N_i\}$ and the mapping function F.

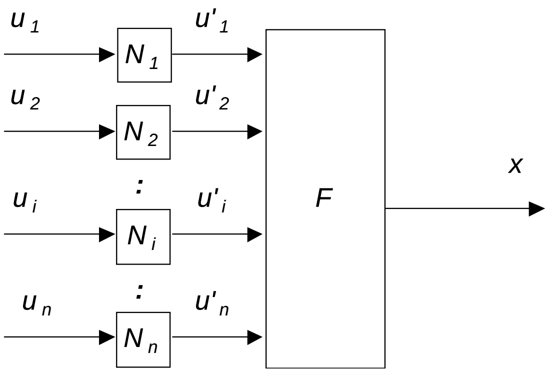

Figure 2. Schematic of the generalized Noisy-Or model.

Each line failure device N_i is parameterised with line failure probabilities $\mathbf{p}_i^{fail}$ for all states of the parent node. The reader is referred to Srinivas (1993) for a detailed description on how the conditional probabilities $P(\mathbf{u}'|\mathbf{u})$ are determined by the line failure probabilities. Table 1a shows an example of $P(\mathbf{u}'|\mathbf{u})$ where a parent node has three states, and each of these states has a zero line failure probability. In another example in Table 1b all three states have an equal line failure probability of 1/3. Clearly, Table 1a depicts a situation where the state u_i determines with certainty the state u'_i, whereas in the case presented in Table 1b knowledge about the state u_i does not convey any information about the state u'_i.

In case of the original Boolean Noisy-Or model, the F function yields state *true* for the child node if any of the parent nodes is in state *true*. Srinivas (1993) suggested the following form of the F function for the generalised case where nodes may take any of a number of discrete states:

$$F(\mathbf{u}') = x\left(ceiling\left[(m_x - 1)\left(\frac{1}{n}\sum_{i=1}^{n}\frac{I(u'_i)}{(m_i - 1)}\right)\right]\right) \qquad (1)$$

where $F(\cdot)$ gives the state of the child node x, $I(u_i')$ ($\in \{1,2,\ldots m_i\}$) is the index of the state u_i', m_x is the number of states of the child node, m_i is the number of states in the i^{th} parent node, and $ceiling[\cdot]$ is a roundup function to the closest integer. The notation $x(j)$ denotes the j^{th} state of the child node x. The above F function is in essence a weighted average over the state indices of $\mathbf{u}'$, and it degenerates to the Boolean Or function when the inputs and outputs have only two states.

Table 1. Conditional probabilities $P(\mathbf{u}'|\mathbf{u})$ for the cases (a) where all line failure probabilities $\mathbf{p}_i^{fail}$ are zero and (b) where all three states have an equal line failure probability ($\mathbf{p}_i^{fail}$ **=** **[**1/3, 1/3, 1/3]). Analogically to $x(j)$, $u_i(j)$ and $u_i'(j)$ denote the j^{th} state of u_i and u_i', respectively.

a)

	$u_i(1)$	$u_i(2)$	$u_i(3)$
$u_i'(1)$	1	0	0
$u_i'(2)$	0	1	0
$u_i'(3)$	0	0	1

b)

	$u_i(1)$	$u_i(2)$	$u_i(3)$
$u_i'(1)$	0.33	0.33	0.33
$u_i'(2)$	0.33	0.33	0.33
$u_i'(3)$	0.33	0.33	0.33

Srinivas (1993) derived the following equation for the conditional probabilities between the states of the child node and the states of the parent nodes

$$\begin{aligned} P(x\,|\,\mathbf{u}) &= \sum_{\mathbf{u}':x=F(\mathbf{u}')} P(\mathbf{u}'|\,\mathbf{u}) \\ &= \sum_{\mathbf{u}':x=F(\mathbf{u}')} \prod_{\mathbf{u}'} P(u'_i\,|\,u_i) \end{aligned} \qquad (2)$$

2.3. Concept of link strength in fully connected belief networks

To parameterise the conditional probabilities $P(\mathbf{u}'|\mathbf{u})$ in the generalised Noisy-Or model, line failure probabilities need to be determined for each state in each parent node. The link strength approach of Varis and Kuikka (1994) provides a way to describing a CPT relating two nodes with aid of a single link strength parameter, and this method can be used to define $P(\mathbf{u}'|\mathbf{u})$ with just one parameter for each parent node.

When the link strength η increases from zero to one, the diagonal of the CPT increases from $1/m$ (in the case of m states) to 1, and the remaining probability mass is distributed evenly over the off-diagonal elements of the CPT:

$$P[u_i'(r)\,|\,u_i(c)] = \frac{1}{m} + \eta\left(1 - \frac{1}{m}\right) \quad , r = c \qquad (3a)$$

$$P[u_i'(r)\,|\,u_i(c)] = \frac{1}{m-1}\left[1 - \frac{1}{m} - \eta\left(1 - \frac{1}{m}\right)\right] \quad , r \neq c \qquad (3b)$$

The notations $u_i(j)$ and $u_i'(j)$ denote the j^{th} state of the u_i and u_i', respectively. A negative link strength indicates that there is a negative relationship between the two nodes. A link strength with a value of one characterises a perfect one-to-one relationship (Table 1a), and a link strength with a value of zero indicates that the two nodes are completely independent (Table 1b). These extreme cases are identical to the conditional probabilities $P(\mathbf{u'}|\mathbf{u})$ shown in Table 1. Table 2 illustrates the CPT when the link strength is 0.5 or -0.5.

Table 2. The CPT in the fully connected networks of Varis and Kuikka (1994) for the cases where the link strength parameter is (a) 0.5 or (b) –0.5.

a)

	$u_i(1)$	$u_i(2)$	$u_i(3)$
$u_i'(1)$	0.67	0.17	0.17
$u_i'(2)$	0.17	0.67	0.17
$u_i'(3)$	0.17	0.17	0.67

b)

	$u_i(1)$	$u_i(2)$	$u_i(3)$
$u_i'(1)$	0.17	0.17	0.67
$u_i'(2)$	0.17	0.67	0.17
$u_i'(3)$	0.67	0.17	0.17

3. METHOD

3.1. Conditional probability $P(\mathbf{u'}|\mathbf{u})$

While this study adopts the concept of link strength in determining the conditional probabilities $P(\mathbf{u'}|\mathbf{u})$, the method for constructing these probabilities used here has modified that of Varis and Kuikka (1994).

The modification was introduced as it was seen desirable to assign a higher probability to those states that are close to the most likely state when compared to states that are far from it. This is a reasonable assumption when the studied variables are defined in the ratio scale statespace. In environmental studies concentrations and loads are typical examples of such variables. Instead of distributing the probability that the parent nodes after line failure are in different states equally across the off-diagonal states, regardless of distance between states, the inverse of the squared distance between state indices is proposed as a weighting criterion.

The conditional probabilities $P(\mathbf{u'}|\mathbf{u})$ were constructed according to the following equation

$$P[u_i'(r) \mid u_i(c)] = \frac{1}{m_i} + |\eta_i| K \quad , r = c \tag{4a}$$

$$P[u_i'(r) \mid u_i'(c)] = \left(|\eta_i| \frac{\frac{1}{(c-r)^2}}{\sum_{j=1..m_i | j \neq r} \frac{1}{(j-r)^2}} + \frac{1-|\eta_i|}{m_i - 1} \right) \left(1 - \frac{1}{m_i} - |\eta_i| K \right) \quad , r \neq c \tag{4b}$$

where r is the index of the state u_i', c is the index of the state u_i, η_i is the link strength of the i^{th} node, and $K = 1 - 1/m_i$, and $|\cdot|$ is the absolute value function. In Equation (4b), the second term of the product is probability of parent states being affected by line failure, and the first term of the product is a weighting coefficient. The value of the weight depends on the inverse of the squared distance between parent node state before and after line failure, $1/(c-r)^2$, and on the value of the link strength. The distance dependency was introduced in order to increase the probability of the state u_i' when its index is close to that of state u_i.

3.2. Desired properties of the conditional probabilities $P(x|\mathbf{u})$

To guarantee that the relationship between link strength parameters and the resulting conditional probabilities $P(x|\mathbf{u})$ is intuitively reasonable, the following properties need to be satisfied:

Property 1. Inclusion of a parent node with a link strength value of zero should give an identical result to the case where that node is absent. For example, when one link has the value of one, and all others are zero, then the index of the state x should be with certainty equal to the index of state of the variable with link strength one.

Property 2. Equal link strength values should result in an equally strong effect on $P(x|\mathbf{u})$.

Property 3. The effect of a negative link strength should have the same magnitude as the effect of an equally strong positive link, but be in the opposite direction.

Property 4. When all link strengths are zero, the CPT should be non-informative, i.e. all probabilities are equal to the inverse of the number of states in x.

3.3. Mapping of child node states

For the sake of simplicity, it is now assumed that all parent nodes and the child node have the same number of states, i.e. $m_i = m_x = m$ for all i. In order to satisfy Properties 1 to 3, Equation 1 was modified and the F function was written in the following form

$$F(\mathbf{u}') = x\left(\frac{1}{\sum_i |\eta_i|}\sum_i \left[|\eta_i| H(u'_i)\right]\right) \tag{5}$$

where

$$H(u'_i) = I(u'_i) \qquad , \eta_1 \geq 0$$

$$H(u'_i) = m_i + 1 - I(u'_i) \quad , \eta_1 < 0$$

Use of the absolute values of link strengths to weight the indices of the states $\mathbf{u}_i$' in Equation 5 ensures that a parent node having a link strength of zero is equal to not having that variable in the network at all (Property 1).

As the ceiling function in Equation (1) always rounds the index of the child node state up to the next integer, the resulting index of the child node state becomes biased upwards. This leads to a violation of Property 2, and therefore the ceiling function was removed. Now, if the index of the child node state, $I(x)$, attains a non-integer value, both states around that value are assigned with weights w that are linearly related to the difference between the indices of these states and $I(x)$. For example, if $I(x) = 2.8$ then states $I(x) = 2$ and $I(x) = 3$ are assigned with weights 0.2 and 0.8, respectively.

Property 3 is satisfied by introducing the function H that in case of a negative link strength mirrors the index of the state u_i' over the median index.

3.4. Conditional probabilities $P(x|\mathbf{u})$

Computation of the conditional probabilities $P(x|\mathbf{u})$ according to Equations (2), (4) and (5) leads to a result that is in violation of Property 4. In cases where all link strengths are weak, and close to each other, the distribution of the index of state x is not uniform over the dimension of x as required. This is due to the fact that the weighted average calculated in the F function is not uniformly distributed, although the indices of the parent node states are (nearly) uniformly distributed.

Therefore, in order to satisfy Property 4, the values of the conditional probabilities $P(x|\mathbf{u})$ were adjusted with a correction in such a manner that $P(x|\mathbf{u})$ becomes non-informative when all link strengths approach zero. For two parents and three states the correction c_2 is derived from the following equation.

$$c_2 = \frac{|\eta_{\min}|}{|\eta_{\max}|} c_1 + \left(1 - \frac{|\eta_{\min}|}{|\eta_{\max}|}\right) w \tag{6a}$$

$$c_1 = |\eta_{\max}| w + \left(1 - |\eta_{\max}|\right) b_i w \tag{6b}$$

where w is the weight resulting from a non integer state index (see Section 3.3.), $|\eta_{\max}|$ and $|\eta_{\min}|$ are the higher and the lower absolute values of the two link strengths, respectively, and b_i is a coefficient for an i^{th} state ($b_1 = 1.5$, $b_2 = 0.6$, $b_3 = 1.5$). The values of b_i are based on the distribution of the arithmetic average of uniformly distributed variables. The rationale behind Equation (6) is that correction is only necessary when both link strengths are weak and close to each other. Equation (6b) ensures that when any of the link strengths is strong the correction is low. And Equation (6a) specifies that the closer the two strengths are to each other, the more correction is applied. The correction attains its maximum when all link strengths approach zero, and no correction is applied when any of the (absolute values of) link strengths approaches one.

4. RESULTS

Conditional probabilities $P(x|\mathbf{u})$ with varying link strength values are demonstrated in the following simple examples, where the Bayesian network has a child node with two parents, and all nodes have three states. Table 3 lists the combinations of link strengths that will be discussed.

Table 3. Link strength combinations (η_1, η_2) discussed in this section.

	Parent 1 (η_1)	Parent 2 (η_2)
Case 1	0	0
Case 2	1	0
Case 3	1	1
Case 4	0.5	0.5
Case 5	0.5	-0.5

In Case 1 neither one of the two parent nodes have control over the child node, and all CPT entries attain a value of 1/3. In Case 2 the state of the first parent determines the state of the child node with certainty regardless of the state of the second parent. This leads to a CPT where the information shown in Table 1a is repeated three times. This is in line with Property 1 discussed earlier. Table 4 presents the CPT for Case 3 where both parent nodes have link strengths of one. Two characteristics are worth noting here. Firstly, in the third row the first and the second parent would set the child node in state 1 and state 3, respectively. As both parents have equal link strengths of one, the child node attains with certainty the average state of these two (i.e. state 2). Secondly, in the second row the probability is split between two states of the child node. Here the first and the second parent are in states 1 and 2, respectively, and both have equal link strength values. In order to satisfy Property 2, both parents should have an equal control over the child node state, which requires the probability mass to be split between the child node states of 1 and 2.

Table 4. CPT for the case where the child node has two parent nodes, all nodes have three states, and both link strength parameters have the value of one.

		$x(1)$	$x(2)$	$x(3)$
$u_1(1)$	$u_2(1)$	1	0	0
$u_1(1)$	$u_2(2)$	0.5	0.5	0
$u_1(1)$	$u_2(3)$	0	1	0
$u_1(2)$	$u_2(1)$	0.5	0.5	0
$u_1(2)$	$u_2(2)$	0	1	0
$u_1(2)$	$u_2(3)$	0	0.5	0.5
$u_1(3)$	$u_2(1)$	0	1	0
$u_1(3)$	$u_2(2)$	0	0.5	0.5
$u_1(3)$	$u_2(3)$	0	0	1

Table 5 shows the CPT for Case 4 where the two parent nodes have link strength values of 0.5. The form of the CPT is similar to Case 3 (Table 4), but the spread of the probability mass is greater in each row. Note that in row 2, according to Property 2, the probabilities of child node states 1 and 2 should be equal, but there is a slight difference between them (0.44 vs. 0.46). This difference is partly contributed to the non-uniform distribution of the probability mass to the off diagonal of $P(u_i'|u_i)$, and partly to an inaccuracy in the correction algorithm outlined in Section 3.4. In Case 5 the negative link strength for the second parent merely causes a rearrangement of rows as compared to Case 4. Rows 1 and 3 (as well as 4 and 6, and 7 and 9) change places. This is in accordance with Property 3.

Table 5. CPT for the case where the child node has two parent nodes, all nodes have three states, and both link strength parameters have the value of 0.5.

		$x(1)$	$x(2)$	$x(3)$
$u_1(1)$	$u_2(1)$	0.68	0.28	0.04
$u_1(1)$	$u_2(2)$	0.44	0.46	0.10
$u_1(1)$	$u_2(3)$	0.21	0.57	0.21
$u_1(2)$	$u_2(1)$	0.44	0.46	0.10
$u_1(2)$	$u_2(2)$	0.19	0.62	0.19
$u_1(2)$	$u_2(3)$	0.10	0.46	0.44
$u_1(3)$	$u_2(1)$	0.21	0.57	0.21
$u_1(3)$	$u_2(2)$	0.10	0.46	0.44
$u_1(3)$	$u_2(3)$	0.04	0.28	0.68

5. DISCUSSION

The method presented above provides a means to describe the CPT that relates states of the parent nodes to those of a child node with link strength parameters. It was demonstrated that the proposed method produces CPTs that are in line with the desired properties outlined in Section 3.2.

The mapping function relating parent node states to the states of the child node is based on the average of state indices of parent nodes. Therefore, the CPT fails to be non-informative when all link strengths approach zero unless a correction is applied. This is a result of the central limit theorem stating that the sum of any distributions ultimately approaches the normal distribution, and hence the average of uniformly distributed states of **u’** also approaches the normal distribution. The current correction scheme gives satisfactory results for the cases of two and three parent nodes, but it is not applicable to networks having a greater number of parent nodes for any one child node.

The motivation behind the presented approach to describing conditional probability tables with link strength parameters is two-fold. Firstly, it facilitates determining conditional probabilities based on the information acquired from a panel of experts, which otherwise easily becomes impractical if the number of parent nodes or the number of node states is large. Various studies have shown that the link strength approach allows a transparent, pragmatic and communicative means for elicitating expert judgment values for Bayesian network models. Varis and Kuikka (1994) demonstrated how link strengths can be elicited from a panel of experts in assessing the impact of climate change for southern part of Finland. Pellikka *et al*. (2005) analyzed a palette of options of wildlife management in Finland on the basis of an expert panel. Varis and Keskinen (2005) performed a policy analysis of the Tonle Sap Lake, Cambodia using the same approach. Secondly, when data are available to populate a subset of the CPT, the presented method provides a tool for deriving complete CPTs from training data. Koivusalo *et al*. (2005) report a study where conditional frequencies obtained from environmental simulation models are used to define CPTs of a Bayesian network using the method presented in this paper. In the above study, strength values were optimized against the conditional frequencies derived from the training data.

6. CONCLUSIONS

A new method for discrete state Bayesian networks was presented for describing the CPT that relates parent node states to child node states with aid of link strength parameters. The number of link strength parameters is equal to the number of parent nodes. This method is envisaged to aid in 1) defining CPTs based on the information acquired from an expert panel, and 2) deriving complete CPTs from training data when not all state combinations of parent nodes are present in the training data set.

7. ACKNOWLEDGMENTS

This study was mainly funded by the European Commission's Environment and Sustainable Development Programme under contract EVK1-CT-2002-00121 (CLIME), and by the Academy of Finland research position of the first author. Additional funding was received from the Maj and Tor Nessling Foundation. Support provided by Prof. Pertti Vakkilainen is greatly acknowledged. Constructive criticism and suggestions given by the anonymous reviewers are much appreciated.

8. REFERENCES

Ames, D.P. (2002), Bayesian decision networks for watershed management, PhD thesis, Utah State University, USA, 250 p.

Borsuk, M.E., C.A. Stow and K.H. Reckhow (2004), A Bayesian network of eutrophication models for synthesis, prediction, and uncertainty analysis. *Ecological Modelling* 173: 219-239.

Chevrolat, J.-P., J.-L. Golmard, S. Ammar, R. Jouvent and J.-F. Boisvieux (1998), Modelling behavioral syndromes using Bayesian networks. *Artificial Intelligence in Medicine* 14: 259–277.

Heckerman, D., J. Breese and K. Rommelse, (1994), Troubleshooting under uncertainty, Technical Report MSR-TR-94-07, Microsoft Research, Redmond, Washington, USA, 1994.

Koivusalo, H., T. Kokkonen, H. Laine, A. Jolma and O. Varis (2005), Exploiting simulation model results in parameterising a Bayesian network – A case study of dissolved organic carbon in catchment runoff. *International Congress on Modelling and Simulation*, Melbourne, December 2005.

Neapolitan, R.E. (2004), Learning Bayesian Networks, Prentice Hall, Upper Saddle River, USA.

Pellikka, J., S. Kuikka, H. Lindén and O. Varis (2005), The role of game management on wildlife populations – uncertainty analysis of expert knowledge, *European Journal of Wildlife Research* 51: 48-59.

Pearl, J. (1988), Probabilistic Reasoning in Intelligent Systems. Palo Alto, CA: Morgan Kaufmann, 1988.

Srinivas, S. (1993), A generalization of the Noisy-OR model, in Heckerman, D., Mamdani, A. (Eds.): *Uncertainty in Artificial Intelligence*; *Proceedings of the Ninth Conference*, Morgan Kaufmann, San Mateo, California, USA.

Varis, O. and M. Keskinen. (2005), Policy Analysis for the Tonle Sap Lake, Cambodia – A Bayesian network model approach. To appear: *International Journal of Water Resources Development*.

Varis, O. and S. Kuikka (1994), BeNe-EIA: A Bayesian approach to expert judgment elicitation with case studies on climate change impacts on surface waters. *Climatic Change* 37: 539-563.