Identification of Roughness in Compound Channels

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EXTENDED ABSTRACT

An accurate estimation of roughness coefficients is of vital importance in any open channel flow study. In flood routing in natural rivers, most channels have compound sections and the roughness values in main channel and flood plains are usually different. In order to have more accurate results, the roughness of main channel and flood plains should be considered separately. It is possible to identify the values of roughness using optimisation methods. However, studies on the inverse problem of estimating roughness values in compound channels are still limited.

In this study, some sets of synthetic data to work on the problem of identification of roughness values in a compound channel have been used. The writers adopt the implicit finite difference four-point box scheme to solve the Saint-Venant equations. The compound channel is treated as a divided channel section in which for any depth the conveyance of the compound section is then the sum of the main channel and floodplain conveyances. The algebraic equation system is linearised and solved by using the double sweep algorithm. The objective function of least square errors between observed and simulated discharge/stage is chosen for this inverse problem and solved by the Powell algorithm. The roughness values in compound channels are formulated as two quadratic polynomial functions of stage, one for main channel (n_c) and the other for flood plains (n_f). The performance of the model is evaluated for both cases when observed data are with and without noise. The computed results indicate that when observed data contain no noise the model can obtain very good results, especially for the main channel roughness. However, when the data contain noise although the computed main channel roughnesses are still very good there are some more biases of the computed floodplain roughness from the true value (see Figure 1).

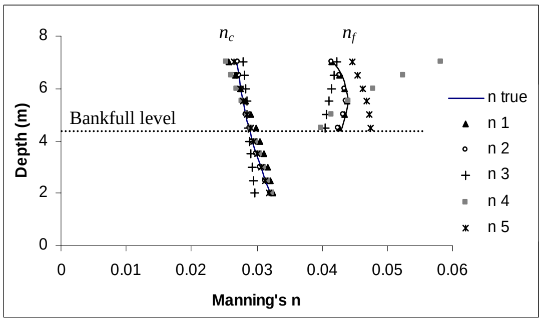


Figure 1. The true roughness and computed roughness versus depth when data contained noise (5 noisy data samples)

In the case when the cross-sections along the reach do not change much, an alternative approach using conveyance K is then tested. For this case the conveyance is presented as a cubic function of depth. Figure 2 shows the true conveyance which is calculated from the true roughness functions and the computed conveyance functions for both cases when observed data are with and without noise. From the figure it can be seen that there is a good agreement between the computed conveyance and the true one even when the data contain noise. The results indicate that using the conveyance function can give appropriate results and can reduce the number of variables. Solution results for illustrative problems indicate the potential applicability of the model for natural rivers.

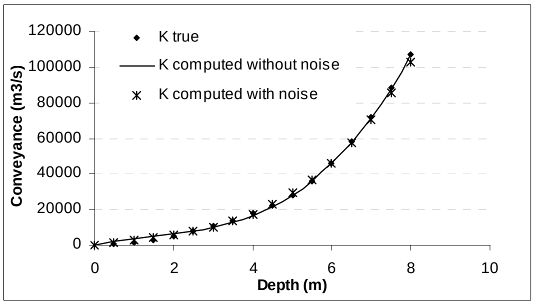


Figure 2. The true conveyance and computed conveyances versus depth when data with and without noise

1. INTRODUCTION

Flood routing in open channels is of vital importance to river engineers and managers. The basic equations can be derived from the principles of conservation of mass and momentum. The resulting equations are hyperbolic, non-linear differential equations known as the Saint-Venant equations. The channel roughness coefficients (Manning's *n*) as embedded in the momentum equation cannot be measured directly and therefore need to be estimated. As an empirical parameter, the estimation of this coefficient for a natural channel is not a trivial task as it depends on several factors, including surface roughness characteristics, vegetation, channel irregularity, bedform and flow conditions etc. (Chow 1959; Rouse 1965; Coon 1998) and the exact values are often uncertain.

In unsteady open channel flow modelling, direct or explicit parameter determination using empirical methods such as Chow (1959) and Urquhart (1975) is not adequate. Therefore, the values of roughness parameters are often estimated through a trial-and-error procedure based on visual comparison of simulated and observed values. This approach suffers from subjectivity, and is tedious and time-consuming. To overcome this problem automatic optimisation methods may be applied to identify the roughness values by minimizing a chosen objective function. Becker and Yeh (1972, 1973) used the influence coefficient approach by minimising the sum of squares of differences between observed data and numerically simulated values to estimate the parameters. Wiggert et al. (1976) employed a conjugate gradient method and formulated the objective function by using the sum of the absolute difference between observed and simulated stages and discharges at intermediate sections. Fread and Smith (1978) used a modified Newton-Raphson search technique for estimating the roughness parameter as a function of stage and discharge. They minimized the sum of the absolute value of the difference between observed and computed stages and discharges. Their method required breaking down the river into a number of single channel reaches before calibrating each reach separately. Wormleaton and Karmegam (1984) formulated the objective function in terms of relative errors using both depth and discharge values and identified the parameters with the influence coefficient algorithm and also a nonlinear least-square technique. Khatibi et al. (1997) used a nonlinear least square technique with three types of objective function by a modified Gauss-Newton method. They investigated the statistical behaviour of the errors induced in the identified parameter in response to Gaussian noise as normally contained in the observed data. Atanov et al. (1999) introduced a variational approach of Lagrangian multipliers using a least square errors criterion to estimate roughness coefficients. However, the algorithm can be applied only to simple prismatic channels. The Sequential Quadratic Programming Algorithm was used by Ramesh et al. (2000) to minimize the objective function based on the least square error criterion. Recently, a Limited-memory quasi-Newton method was used by Ding et al. (2004) to identify Manning's *n* in shallow water flows and applied to the East Fork River.

At present, studies on the roughness identification problem are still sparse and the above studies have just considered roughness parameters in the in-bank channel. However, in flood routing in natural rivers, most channels have compound sections and the roughness values in main channel and flood plains are usually different. As indicated by Wormleaton and Karmegam (1984) this problem needs to extend to over-bank flow where flood plain roughness will obviously have to be considered. Some problems identified roughness in compound channels where the roughness in main channel and flood plains are constants, as done by Nguyen and Fenton (2004), however in some natural channels the roughness varies considerably with stage or discharge, hence the constant values of roughness may not be adequate to represent the roughness in these channels.

Therefore, in this study, the roughness identification problem has been extended for compound channels where the roughness values are formulated as two separate polynomial functions of stage, one for main channel and the other for flood plains. Moreover, in cases when the cross-sections along the reach do not change much, the more practical concept of conveyance K is tested.

2. METHODOLOGY

2.1. Governing Equations

The unsteady one-dimensional open-channel equations can be derived from the principles of conservation of mass and momentum resulting in equations known as the Saint-Venant equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad (1)$$

$$\frac{\partial Q}{\partial t}+2\beta\frac{Q}{A}\frac{\partial Q}{\partial x}+\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{\partial Z}{\partial x}$$
$$-\beta\frac{Q^2B}{A^2}S_0+gAS_f-u_q q=0 \tag{2}$$

where A is the wetted cross-sectional area; Q is the discharge; Z is the water stage or surface water elevation; q is the lateral inflow per unit length of channel; B is the channel width at the surface water; β is the momentum correction factor; g is the acceleration of gravity; S_0 is the channel bed slope; S_f is the friction slope; u_q is the x direction velocity component of the lateral inflow; x and t are space and time variables respectively.

The friction slope S_f is given by Manning's equation. For compound channels, the critical assumption is that friction slope is constant in main channel and floodplains. The conveyance is computed using divided section method in which for any depth the conveyance of the compound section is the sum of the main channel and floodplain conveyances. Then:

$$S_f=\frac{Q_c|Q_c|}{K_c^2}=\frac{Q_f|Q_f|}{K_f^2}=\frac{Q|Q|}{(\sum K_i)^2} \tag{3}$$

where Q_c, Q_f and Q are the discharge of main channel and flood plains and the total discharge of the section, K_c and K_f are the conveyances of main channel and floodplains and which are determined as follows:

$$K_i=\frac{A_i R_i^{2/3}}{n_i} \tag{4}$$

where K_i, A_i, R_i and n_i are conveyance, area, hydraulic radius and Manning's roughness coefficient of sub-cross-section i respectively.

In this study, the Saint-Venant equations are solved by the implicit finite difference Preissmann box scheme. The algebraic equation system is linearised and solved by using the double sweep algorithm (Liggett and Cunge 1975, Cunge et al. 1980).

2.2. Roughness Identification Procedure

The capability for the identification of the roughness coefficient of the model river is based on minimising a chosen objective function. The procedure starts with initial estimated parameters and performs a completed simulation run. The objective functions are evaluated by comparing the observed data against the simulated ones by the model. If the value of the function is above the prescribed tolerance value, the process is continued iteratively through computing a correction to the parameters by using an optimisation. In this study Powell's optimisation algorithm (Press *et al*. 1992, p. 409) is applied. The advantage of using this algorithm is that it does not need to calculate the derivatives of the objective function. Also, the upper and lower constraints are introduced to restrict the coefficients to physically realistic values.

The selection of objective functions is one of the factors affecting the quality of identification problem. Nguyen and Fenton (2004) investigated the effect of three main types of objective function and showed that least square objective function had the best performance. Khabiti *et al*. (1997) indicated that the selection of objective function was found to be prone to undue biases affecting the identified parameters, which could be avoided through a careful consideration of the problem. They considered the sum of square of errors using absolute errors and relative errors with respect to observed values and relative errors with respect to simulated values. They concluded that the formulation of the objective function using relative errors seems to induce an undue bias that increase with increasing noise level. Therefore, in this study the objective function sum of square of absolute errors between observed and simulated stages/discharges is considered as follows:

$$\min\sum_{j=1}^{N}\sum_{i=1}^{M}(Y_{Oi,j}-Y_{Si,j})^2 \tag{5}$$

where the subscripts i, j correspond respectively to the values at different times and locations, M is number of observation times, N is number of observation stations, Y_O is observed discharge or stages, Y_S is simulated discharge or stage.

In this study, the roughness coefficients of the main channel and flood plains are formulated as second order polynomial functions of water stage as follows. For main channel roughness:

$$n_c=a_0+a_1(Z-Z_0)+a_2(Z-Z_0)^2 \tag{6}$$

For main floodplains roughness:

$$n_f=b_0+b_1(Z-Z_f)+b_2(Z-Z_f)^2 \tag{7}$$

where: n_c and n_f are roughness coefficients of main channel and floodplains respectively, Z is the water stage, Z_0 is the minimum water level at a certain cross section at which the cross section characteristics are initially tabulated in the input data, Z_f is elevation of floodplains, and a_0, a_1, a_2, b_0, b_1 and b_2 are coefficients of the roughness functions that need to be identified.

Moreover, an alternative approach for the case where the cross-sections along the reach are not changed much, the more practical concept of conveyance K is used. Several types of conveyance functions were tested and a cubic function of depth was found to be an appropriate function to present for K of a compound channel:

$$K = K_1(Z - Z_b) + K_2(Z - Z_b)^2 + K_3(Z - Z_b)^3 \quad (8)$$

where Z_b is the bed elevation, and K_1, K_2 and K_3 are coefficients of conveyance function that need to be identified.

3. SYNTHETIC DATA

The model for identification of roughness coefficients n was tested using synthetic data in a compound channel where the roughness functions are defined. Because in the field the true value of roughness is not known, the advantage of using synthetic data is that it is possible to make comparisons between the estimated n with the true n. This can provide the abilities to evaluate the performance of the model.

The observed gauge station was located at intermediate section of the channel. The observation data for these cases were simulated by solving governing Equations (1) and (2) with the true values n. Identical initial and boundary conditions were applied while obtaining the simulated observation data and while solving the optimisation models.

A model channel has a length of 40 km with slope of 0.0004. The geometry of the compound cross section is illustrated in Figure 3. The side slopes of the cross section for both main channel and flood plain are 1.5. The downstream boundary condition is the stage hydrograph at 40 km point of a 80 km long channel. The downstream boundary condition for the 80 km channel, which has the same channel properties and upstream boundary condition as the 40 km channel, is a steady uniform rating function. The upstream boundary condition is the synthetic hydrograph generated by:

$$Q(t) = Q_b + (Q_p - Q_b)\left[\frac{t}{t_p}\exp\left(1 - \frac{t}{t_p}\right)\right]^{\beta} \quad (9)$$

where Q_b is initial discharge, Q_p is peak discharge and t_p is time to peak, β is a constant. In this case $Q_b = 200$ m^3/s, $Q_p = 1500$ m^3/s, $t_p = 4$ hours and $\beta = 5$.

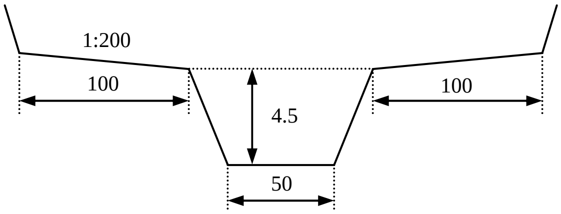


Figure 3. The cross section of the compound channel

The true roughness function of main channel (n_c) and flood plains (n_f) were presented as function of stage as follows:

For main channel roughness:

$$n_f = 0.032 + 0.0015(Z - Z_f) - 0.0001(Z - Z_f)^2 \quad (10)$$

For main floodplains roughness:

$$n_f = 0.043 + 0.002(Z - Z_f) - 0.001(Z - Z_f)^2 \quad (11)$$

In practice, flow measurement data usually contain observation errors/noise. Also, in mathematical modelling the other error sources are “model errors” and “numerical errors”. “Model errors” are associated with imperfections of the governing equation and some restricted assumptions to simplify the physical processes. “Numerical errors” include rounded errors, truncation errors related to the finite different methods. These random errors are generally thought to be normally distributed (Khabiti et al. (1997)). In this study, the noise was introduced into the simulated observed discharges or depths (noise free) Y_o as follows:

$$Y_o^n = Y_o + \varepsilon Y_o \quad (12)$$

where Y_o^n is the simulated observed data with noise level σ, $\varepsilon = N(\mu, \sigma)$ is a random error term sampled from a normal distribution of zero mean and standard deviation of σ.

4. RESULTS AND DISCUSSION

The identification procedure starts with initial assumed coefficients in Equations (6) and (7). The results indicate that when there is no noise contained in the observed data, the model can identify properly the roughness functions. Although the computed coefficients are different from the coefficients of the true roughness functions (Equations (10) and (11)) the computed roughness curves are very close to the true ones especially for the main channel roughness, as illustrated in Figure 4. There is some deviation of the flood plain values, and this may be attributed to the compensation of conveyance between main channel and floodplains because when the water level is greater than bank-full level at a cross section there are two important roughness values that can offset each other.

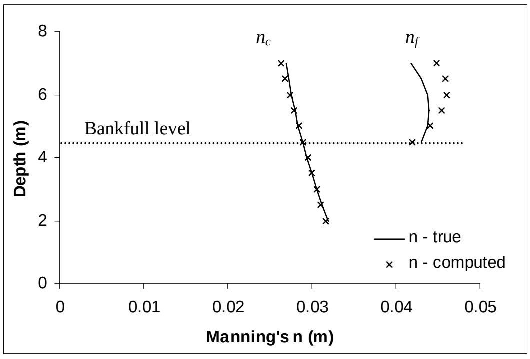


Figure 4. The true roughness and computed roughness versus depth when data contain no noise

To consider the case when data contain noise, 5 samples noise of $\sigma = 0.05$ were generated from different seeding random numbers. Figure 5 shows the true roughness and computed roughness n_1, n_2, n_3, n_4 and n_5 with five noisy observed data sets respectively. Although the main channel roughness can be identified rather accurately (the maximum deviation from the true one is less than 8%) the floodplain roughness values are biased from the true ones. The computed results also indicate that there is some compensation of floodplain roughness because of some small deviation of main channel roughness.

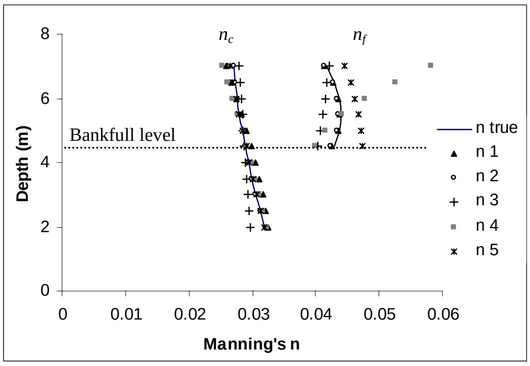


Figure 5. The true roughness and computed roughness versus depth when data contained noise (5 noisy data samples)

An alternative approach using conveyance K was then tested, where instead of identifying the coefficients of roughness functions, the coefficients of the conveyance function (Equation (8)) were identified. The advantage of using conveyance function is that it can reduce the number of unknown coefficients and therefore it can reduce computation time.

Figure 6 shows the true conveyance which is calculated from the true roughness functions (Equations (10) and (11)) of the given channel above and the computed conveyance functions for both cases when observed data are with and without noise. From the figure it can be seen that there is a good agreement between the computed conveyance and the true one even when data contain noise.

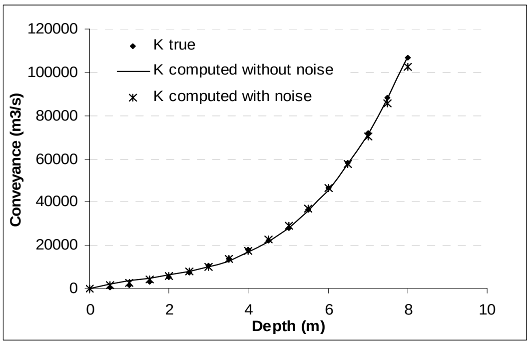
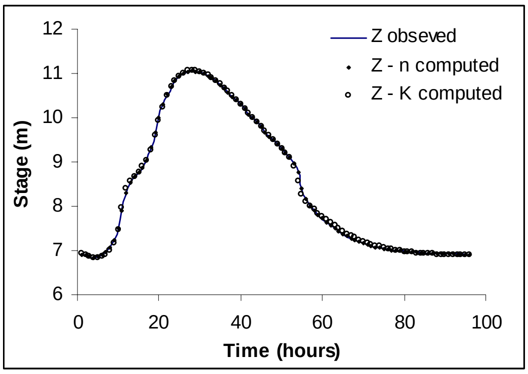


Figure 6. The true conveyance and computed conveyances versus depth when data with and without noise

Figure 7 shows the observed hydrographs (using true roughness functions), the computed hydrographs using computed roughness functions and computed conveyance function at 10 km from the downstream end of the reach. From the figure it can be seen that the hydrographs using the computed roughness functions are closer to the observed hydrographs than the other. Although the simulated hydrograph obtained by using the conveyance function approach is not as accurate as using roughness functions for practical purposes it is still acceptable.



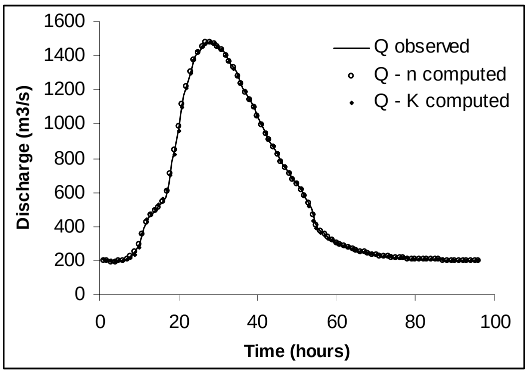


Figure 7. Observed hydrographs computed from the true roughness function and simulated hydrographs computed from the computed roughness functions and computed conveyance function

5. CONCLUSIONS

In this study, the inverse problem of estimating roughness values has been studied for compound channels by using synthetic data. The true values of roughness in the channel were presented as two quadratic polynomial functions, one for the main channel and the other for the flood plain. The performance of the model was evaluated for both cases when observed data were with and without noise. The computed results indicate that the model can identify properly the value of roughness in the main channel even when observed data contain noise. However, there are some biases of the computed floodplain roughness from the true one. An alternative approach using conveyance *K* was tested. The results indicate that for prismatic channels using the conveyance function can give appropriate results and can reduce the number of variables. Solution results for illustrative problems indicate the potential applicability of the model for the natural rivers.

6. ACKNOWLEDGMENTS

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