Stochastic Simulation of Settlement Prediction of Shallow Foundations Based on a Deterministic Artificial Neural Network Model

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EXTENDED ABSTRACT

The problem of estimating the settlement of shallow foundations on granular soils is complex and not yet entirely understood. In the past, many empirical and theoretical methods have been developed for predicting the settlement of shallow foundations on granular soils; however, these methods are far from accurate and consistent. In recent times, artificial neural networks (ANNs) have been used for settlement prediction of shallow foundations on granular soils and have shown to outperform the most commonly used traditional methods. However, despite the relative advantage of the ANN based approach, it is like most traditional methods in the sense that it is based on a deterministic approach that does not take into account the considerable level of uncertainty that may affect the magnitude of the predicted settlement. Thus, it provides single values of settlement with no indication of the level of risk associated with these values. In this paper, an alternative stochastic approach that considers the uncertainty associated with the predicted settlement from a deterministic ANN model is provided. The proposed stochastic approach is based on combining Monte Carlo simulation with the deterministic ANN model from which a set of stochastic design charts for settlement prediction of shallow foundations on granular soils is developed. The charts will enable the designer to make informed decisions regarding the level of risk associated with predicted settlements and consequently provide a more realistic indication of what the actual settlement might be.

1 INTRODUCTION

The settlement prediction of shallow foundations is often affected by a considerable level of uncertainty that may produce an unreliable estimation of the magnitude of settlement, while reliable settlement prediction is essential for design purposes. Uncertainty affecting settlement prediction is generally caused by one or two of the following sources: (a) input variable uncertainty, which is caused by the errors in measurements associated with the variables used for settlement prediction; and (b) prediction method uncertainty, which is caused by the inherent error associated with the modelling technique used to characterise settlement prediction (Cherubini and Greco 1991; Krizek *et al.* 1977). Most deterministic modelling methods for settlement prediction of shallow foundations on granular soils disregard the above uncertainties in their analysis and simulation. One way to include such uncertainties is to use stochastic simulation. Recently, artificial neural networks (ANNs) have been used successfully for settlement prediction of shallow foundations on granular soils and have been found to outperform the most commonly used traditional methods (Shahin *et al.* 2002b). However, ANNs, like most traditional methods of settlement prediction, are based on deterministic approaches that ignore the uncertainty that may affect the magnitude of the predicted settlement.

This paper is concerned with the application of stochastic analysis that incorporates prediction method uncertainty to deterministic ANN models for settlement prediction of shallow foundations on granular soils. Application of stochastic analysis that incorporates input variable uncertainty to ANN models for settlement prediction of shallow foundations on granular soils has been carried out by Shahin *et al.* (Shahin *et al.* 2005). Uncertainty associated with the prediction method is difficult to measure physically (Juang *et al.* 1991); however, if sufficient measured and predicted settlement data are available and assuming that the measured settlements are error free, then prediction method uncertainty can be quantified and used for the stochastic analysis of settlement prediction. This can be carried out by calculating the settlement ratio, *k*, which is defined as the ratio of the predicted settlement to the actual measured settlement (Cherubini and Greco 1991; Sivakugan and Johnson 2002). By utilising the above definition of *k*, and if a set of predicted and measured settlement is available, the settlement ratios can be calculated and used to obtain the probability density function (PDF) of *k*, which is a key issue for the stochastic analysis of settlement prediction, as will be described in Section 3. If the settlement prediction data used to estimate the PDF of *k* contain outliers (i.e. some cases within the available data have values of predicted settlements far from those of measured ones), the distribution of *k*, and consequently the final results of the stochastic predicted settlements, will be affected. As a result, excluding outliers from the settlement prediction data used to estimate the PDF of *k* will improve the stochastic analysis. In a previous work (Shahin *et al.* 2005) carried out by the authors of the current paper for the stochastic analysis of settlement prediction of shallow foundations, outliers of the data used to estimate the PDF of *k* were not excluded. The present paper is an extension to the work carried out previously by the authors where the stochastic analysis of settlement prediction of shallow foundations is improved by statistically analysing and excluding any possible outliers from the data used to estimate the PDF of *k*. Based on improved stochastic analysis, a set of stochastic design charts for settlement prediction of shallow foundations on granular soils is also developed and provided for routine use in practice.

2 DETERMINISTIC ARTIFICIAL NEURAL NETWORK MODEL

The present study uses an artificial neural network (ANN) model to obtain deterministic settlement predictions of shallow foundations on granular soils. The ANN model was developed by Shahin *et al.* (2002b) and uses feedforward multi-layer perceptrons (MLPs) that are trained with the back-propagation algorithm (Rumelhart *et al.* 1986). The software package *Neuframe* Version 4.0 (Neusciences 2000) was used for this purpose. Details of the ANN model development are beyond the scope of this paper and are given by Shahin *et al.* (2002b). The model has five inputs representing the footing width, *B*, net applied footing load, *q*, average blow count obtained using a standard penetration test (SPT) over the depth of influence of the foundation, *N*, (this is used as a measure of soil compressibility), footing geometry, *L/B*, and footing embedment ratio, D_f/B. The single model output is foundation settlement, S_m. The database used for model development comprises a total of 189 individual cases, which is almost the largest data set used to develop such models. The data were obtained from the literature and span a wide range of the input and output data cases, as summarised in Table 1. The available data were divided randomly into three sets: training, testing and validation, in such a way that they are statistically consistent and thus represent the same statistical population (Masters 1993). The training set was used to adjust the model free parameters (i.e. connection weights), the testing set was used to decide when to stop training to avoid overfitting, and the validation set was used to test the predictive ability of the model in real-world situations. In total, 80% of the data were used for training and 20% were used for validation. The training data were further divided into 70% for the training set and 30% for the testing set. The optimum model geometry was determined using a trial-and-error approach in which ANN models were trained with one hidden layer and 1, 2, 3, 5, 7, 9 and 11 hidden layer nodes, respectively. The optimal network parameters were obtained by training the ANN model with different combinations of learning rates and momentum terms. A model with 2 hidden layer nodes, a learning rate of 0.2, and a momentum term of 0.8 was found to perform best. Details of the above parameters are given by Shahin *et al*. (2002b). The performance of the ANN model is summarised in Table 2. It can be seen that the model performs well, as it has high correlation coefficients, *r*, and low root mean squared errors (RMSE) and mean absolute errors (MAE) between the measured and predicted settlements for all three data sets (i.e. training, testing and validation). A comparison carried out by Shahin *et al.* (2002b) on the validation set, and utilising the ANN model and three of the most commonly used traditional methods indicated that the ANN method provides more accurate predic-

Table 1. Data ranges used for the ANN model

Model variables	Min.	Max.
Footing width, *B* (m)	0.8	60.0
Net applied load, *q* (kPa)	18.3	697.0
SPT blow count, *N*	4.0	60.0
Footing geometry, *L/B*	1.0	10.5
Embedment ratio, D_f/B	0.0	3.4
Settlement, S_m (mm)	0.6	121.0

tions than the traditional methods. In order to facilitate the ANN technique for deterministic settlement prediction of shallow foundations on granular soils, the information obtained from the ANN model was translated into a relatively simple hand-calculation formula (Shahin *et al.* 2002a) and also into a set of design charts (Shahin 2003) suitable for practical use.

Table 2. Performance of the ANN model

Data set	*r*	RMSE (mm)	MAE (mm)
Training	0.930	10.01	6.87
Testing	0.929	10.12	6.43
Validation	0.905	11.04	8.78

3 STOCHASTIC SETTLEMENT PREDICTION

In order to determine the impact of the prediction method uncertainty on predicted settlements obtained from the ANN technique, Monte Carlo simulation is applied to the deterministic ANN model described previously. Monte Carlo simulation attempts to generate a random set of values from known or assumed probability distributions of some variables involved in a certain problem. Full details of the Monte Carlo technique are given by many authors (e.g. Hammersley and Handscomb 1964; Rubinstein 1981). As mentioned previously, prediction method uncertainty for settlement of shallow foundations can be examined by calculating the settlement ratio, *k*, for a set of predicted versus measured settlements and obtaining the probability density function (PDF) of *k*, which is used for the stochastic analysis, as will be described below. The distribution of *k* is obtained using the 189 case records used by Shahin et al. (2002b) for the development of the deterministic ANN model. The values of *k* are found to lie in the range 0.25 to 10.4. The mean value of *k* is 1.4 and the standard deviation is 1.3. As mentioned previously, possible outliers in the data used for the estimation of the PDF of *k* will result in false evaluation of its distribution, which in turn will affect the final results of the stochastic predicted settlements. Consequently, The box plot method (Kotzais *et al.* 1990), as proposed by Cherubini (2000), is used to eliminate any possible outliers from the data used for the estimation of the PDF of *k*. As part of the method, the central tendency of *k* is indicated by the median, whereas its spread is indicated by the lower (Q_L) and upper (Q_U) quartiles. Points whose values are either less than (Q_L–1.5*IQD*) or greater than (Q_U+1.5*IQD*), where *IQD* is the interquartile distance and is equal to (Q_U–Q_L), are considered to be outliers. Such a plot is shown in Figure 1 for the available data. It can be seen that some points are greater than Q_U+1.5*IQD*. Consequently, these data points may be considered to be outliers and are omitted from the data used to estimate the PDF of *k*. The number of outliers are found to be 20 out of 189 data records, resulting in 169 data records that are used to obtain the PDF of *k*. The PC-based software *@Risk* (Palisade 2000) is used to determine the PDF that provides the best fit to the remaining 169 data points. For a given set of data values, @Risk can identify the probability distribution that best fits these values from 38 candidate distributions and provides statistical properties that describe the distribution. The theoretical distribution that is found to best match the actual distribution of *k* is the Weibull distribution (Johnson and Leone 1964), as shown in Figure 2. The statistical properties of the Weibull distribution obtained are given in Table 3.

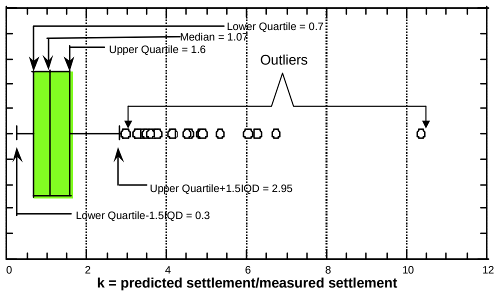

Figure 1. Box plot for 189 data record of *k*

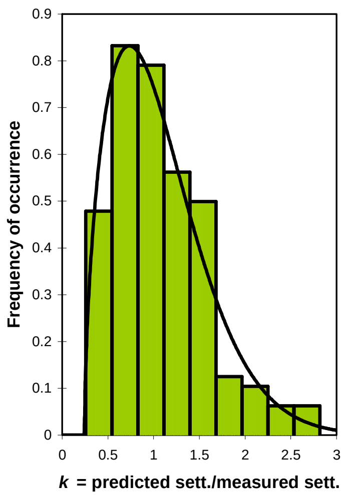

Figure 2. Weibull distribution of *k*

Table 3. Weibull distribution parameters of *k*

Statistical parameter	Value
Minimum	0.25
Maximum	2.80
Mean	1.06
Standard deviation	0.53
Shape parameter (α)	1.59
Scale parameter (β)	0.91

It can be seen from Table 3 that removing the outliers from the analysis of k resulted in a reduction in the mean and standard deviation of k to 1.06 and 0.53, respectively. A Monte Carlo simulation can then be conducted to estimate the uncertainty associated with the predicted settlements. The detailed procedure is as follows:

1. The PDF of k is estimated using a set of predicted and measured settlements, as described above;
2. For an individual case of settlement prediction, the deterministic settlement is calculated using the ANN model developed by Shahin *et al.* (2002b), hand-calculation formula (Shahin *et al.* 2002a) or deterministic design charts (Shahin 2003);
3. A random value of k is generated from the PDF of k obtained in Step 1;
4. From the definition of k, the deterministic predicted settlement in Step 2 is divided by the generated random value of k from Step 3 and the corresponding actual settlement is calculated;
5. Steps 3 and 4 are repeated for many iterations (Monte Carlo simulation); and
6. The settlements obtained as part of the Monte Carlo simulation are used to estimate the cumulative distribution function (CDF) or to plot the cumulative probability distribution from which the probability of non-exceedance ($P_{N/E}$), or level of risk, associated with a certain settlement prediction, can be estimated.

4 NUMERICAL EXAMPLE

The following case study is examined. A rectangular footing, the dimensions of which are 2.5 × 4.0 m, is founded at a depth of 1.5 m below the ground surface. The soil beneath the footing is sand that extends to a depth in excess of two times its width. The net applied footing load is 350 kPa and the average SPT blow count is 16.

Solution: The steps described previously for the inclusion of prediction method uncertainty are used as follows. The PDF of k for the ANN method is obtained (Step 1) and was found to fit a Weibull distribution, as described previously. The deterministic single solution of settlement prediction is obtained from the ANN model given by Shahin *et al.* (2002b) and is found to be 13.3 mm (Step 2). From the statistical properties of the Weibull distribution obtained in Step 1, random values of k are generated (Step 3). The numerical example is re-calculated by dividing the settlement predicted in Step 2 by the generated value of k obtained from Step 3 and a corresponding actual settlement is calculated (Step 4). Steps 3 and 4 are repeated many times (Monte Carlo simulation) until a convergence criterion is achieved (Step 5). In order to determine whether convergence has been achieved, the statistics describing the distribution of the predicted settlements are calculated at fixed numbers of simulations and compared with the same statistics at previous simulations. Convergence is deemed to have occurred if the change in the statistics describing the distribution of predicted settlement is 1% or less. It was found that 1,400 simulations are sufficient to achieve convergence. The predicted settlements obtained for the 1,400 simulations are used to plot the cumulative probability distribution curve from which different probabilities of non-exceedance are obtained (Step 6). The results are shown in Figure 3, which also includes the deterministic single settlement value of 13.3 mm, and summarised in Table 4. It can be seen from Figure 3 that there is a probability of approximately 62% that the settlement will not exceed the deterministic single estimate of 13.3 mm, which means that there is 38% probability that the settlement could be higher than the deterministic estimate of 13.3 mm. This result indicates that prediction method uncertainty can affect settlement and thus, should not be neglected in the analysis and simulation of settlement prediction. In addition, there are probabilities of 75%, 80%, 85%, 90% and 95% (i.e. probability levels that may be needed for design purposes) that the settlement will not exceed 15.4, 16.1, 17.0, 18.0 and 19.3 mm, respectively (Table 4).

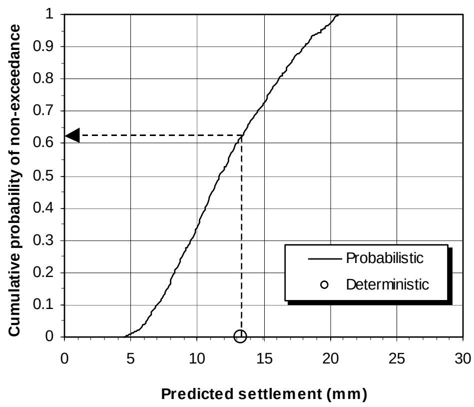

Figure 3. Cumulative probability distribution incorporating model uncertainty for the numerical example

Table 4. Predicted settlement accounting for model uncertainty for the numerical example

$P_{(N/E)}$ (%)	Settlement (mm)
75	15.4
80	16.1
85	17.0
90	18.0
95	19.3

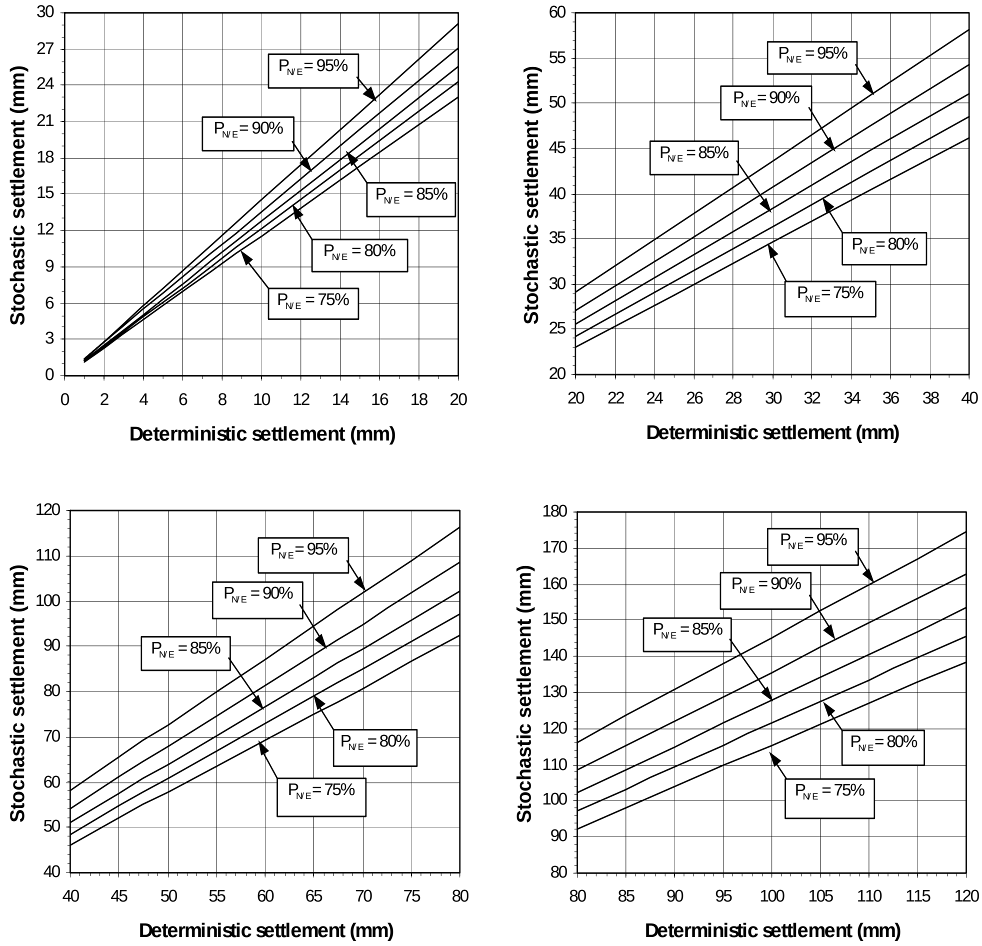

Figure 4. Stochastic ANN-based design charts for settlement prediction (**$P_{N/E}$** = Probability of non-exceedance)

5 STOCHASTIC SETTLEMENT PREDCITION DESIGN CHARTS

The stochastic simulation proposed in this study that incorporates the ANN prediction method uncertainty is used to develop a generic set of stochastic design charts based on the ANN model. The charts are expected to be a useful tool for practitioners, from which the level of risk associated with predicted settlement can be readily obtained. Since the ANN model predicts the most accurate settlement estimates to date (Shahin *et al.* 2002b), the subsequent stochastic design charts are considered to be the most reliable of those currently available. The procedure that is used to develop the charts is as follows:

1. A random synthetic value of predicted settlement, which accounts for an individual case of settlement prediction, is generated between the ranges given in Table 1;
2. The approach, outlined previously, that incorporates prediction method uncertainty for obtaining a stochastic settlement prediction is applied to the settlement predicted in the previous step and the corresponding CDF is obtained;
3. From the above CDF, the 75%, 80%, 85%, 90% and 95% probabilities of non-exceedance are determined;
4. Another random synthetic value of predicted settlement is generated by increasing the value generated in Step 1 by 5% of the total range between the minimum and maximum values given in Table 1;
5. Steps 2 to 4 are repeated until the maximum synthetic value of predicted settlement is reached; and
6. For each probability level of non-exceedance, the synthetic deterministic settlements are plotted against stochastic settlements and a set of design charts is generated, as shown in Figure 4.

For any individual case of settlement prediction within the ranges of the data shown in Table 1, the deterministic single settlement prediction can be obtained from the ANN model given by Shahin *et al.* (2002b), hand-calculation formula (Shahin *et al.* 2002b) or the deterministic design charts (Shahin 2003), and the corresponding stochastic settlement can be readily obtained from Figure 4, accounting for a certain desired probability of non-exceedance. For example, if the deterministic ANN model predicts a settlement of 22 mm and reliability levels (i.e. probabilities of non-exceedance) of 90% and 95% are required, the corresponding design stochastic settlements (Figure 4) are 32 and 34 mm, respectively. It should be noted that the applicability of these charts is constrained by the range of the 169 data records used to characterise the uncertainty associated with the settlement ratio, *k* (Table 1), as described previously. However, the range of applicability of the approach can be extended in future, by re-training the ANN model and re-generating the design charts should additional data records become available. It should also be noted that, as mentioned earlier, the stochastic solution that incorporates prediction method uncertainty relies on the estimation of the PDF of *k* and consequently, as many case records of settlement prediction as possible are needed in order to obtain a reliable estimate of the PDF of *k*. For further verification of the stochastic design charts, values obtained using the charts need to be compared with corresponding actual measured settlements of some additional case records, once available.

6 CONCLUSIONS

The results of the numerical example that incorporates prediction method uncertainty in the analysis of settlement of shallow foundations on granular soils indicated that there was a probability of approximately 38% that the settlement could be higher than the deterministic single estimation. This result indicated that prediction method uncertainty affects settlement and thus should not be neglected in the analysis and simulation of settlement prediction. It was shown in this work that the developed stochastic charts can be used to predict settlements for a certain desired reliability level given the deterministic settlement predicted from the ANN model developed by Shahin *et al.* (2002b), which is believed to be a useful tool in the design of shallow foundations on granular soils.

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